DESIGN OF MACHINE ELEMENTS

VOLUME I

Second Edition

T. KRISHNA RAO

Design of Machine Elements

(VOLUME - I)

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I.K. International Publishing House Pvt. Ltd.
NEW DELHI • BANGALORE

Published by

I.K. International Publishing House Pvt. Ltd.
S-25, Green Park Extension
Uphaar Cinema Market
New Delhi–110 016 (India)
E-mail:info@ikinternational.com
Website: www.ikbooks.com

ISBN 978-93-81141-37-3

© 2011 I.K. International Publishing House Pvt. Ltd.

Reprint 2015

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Published by Krishan Makhijani for I.K. International Publishing House Pvt. Ltd., S-25, Green Park Extension, Uphaar Cinema Market, New Delhi–110 016 and Printed by Rekha Printers Pvt. Ltd., Okhla Industrial Area, Phase II, New Delhi–110 020.

Dedicated to
My beloved parents
Sri T. Sreeramamurthy
and
Smt. T. Atchuthamma

Preface

Design engineers always endeavour to design machine members for safety, durability economically. To accomplish this activity, the designer has to identify the types of loads, materials properties, safety requirements and failure modes for a particular situation. These days a large number of textbooks are available on this subject. The student has to refer more than one book to meet his requirements. Design of Machine Elements – (Vol - I) covers all the features like coverage of topics, variety of solved problems, simplicity in presentation and understanding, variety of exercise problems, etc.

This book is aimed mainly for mechanical engineering students of BE/B. Tech and AMIE and various other competitive examinations. Diploma students and practising designers will also find the book useful. The following are some of the salient features of this book.

- Concise and compact, covering all major important topics.
- Large number of solved problems in each topic.
- Hints and notes wherever required for easy understanding.
- Good number of exercise problems.
- Inclusion of relevant standards tables and graphs.
- Simple language and explanations to facilitate easily understandable even by average students.
- Step-by-step procedures for each type of problem and step-by-step approach in solving problems.

Some mistakes might have crept in this text either in concept, calculation or printing. The author shall be highly grateful to the readers, teachers, students and other users of this book for their valuable comments and pointing out mistakes to improve the quality of this book in future editions.

T. Krishna Rao

Acknowledgements

I express my sincere thanks to the management, Gokula Education Foundation, Shri M.R. Seetha Ram, Director, Dr. K. Rajanikanth, Principal and Dr. R. Chandrasekhar, HoD, (Mechanical), M.S. Ramaiah Institute of Technology, Bangalore for their support and encouragement.

I also express my thanks to Dr. K. Rajagopal, Vice Chancellor, J.N.T. University, Hyderabad and Dr. N.V.R. Naidu, Professor and Head, Dept. of IEM, MSRIT, Bangalore for their valuable guidance and suggestions during the preparation of this book.

I am indebted to all the friends, colleagues and my family members who have encouraged me to author this book. My thanks are specially due to my wife Lakshmi and daughter Meghana for their help and support.

I thank I.K. International Publishing House and respected authors (V.K. Jadon & Suresh Verma) for according me necessary permission to use some material from their *Machine Design Data Book*.

I am thankful to Miss Sumalatha for helping in proof-reading, editing and correcting the book at various stages.

T. Krishna Rao

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CHAPTER

1 Introduction

1.1 MACHINE DESIGN

Machine design is the art of developing new ideas for the construction of machines and exposing those ideas in the form of plans and drawings. For a machine to be better designed, the components must be strong enough to perform the functions and must be economical to manufacture.

1.2 SHIGLEY'S MODEL FOR DESIGN

The Shigley's design model is presented below in Fig. 1.1 that comprises six phases.

Fig. 1.1

Recognition of need is the beginning of any design process. The designer may recognize a need on his own or it may be communicated to him by his employer or customer.

Definition of the problem must include all the specifications for the thing that is to be designed. The specifications are the input and output quantities, the characteristics and dimensions, limitations etc.

The synthesis of a scheme connecting possible system elements is sometimes called conceptualisation. During this, the specifications are formulated.

The design is an iterative process in which we synthesize several components of a system, analyse and optimise them. Mathematical models may be used to simulate the real physical system.

Evaluation is a significant phase of the total design process. This involves the testing of prototype in the laboratory to see whether it meets all needs and satisfies all the specifications or not.

Presentation is selling the product to users. After thorough testing, the flaws in design may be rectified and the modifications are implemented. Then the products are manufactured and introduced in the market.

1.3 MATERIALS AND PROPERTIES

Engineering materials include ferrous and non-ferrous materials. The commonly used ferrous materials are: cast iron, carbon steals, alloys steels and non-ferrous materials are copper, aluminium, lead, robber, nylon, FRP etc.

The important mechanical properties of engineering materials are strength, hardness, toughness, ductility, resilience, wear resistance. Impact strength, Poisson ratio etc. These properties are listed in various hand books, ASTM, ISO, BIS and other standards.

1.4 DESIGN CONSIDERATIONS

The important design considerations are:

1. Strength	2. Stiffness	3. Wear	4. Safety
5. Reliability	6. Cost	7. Weight	8. Ease of manufacture
9. Size and Shape	10. Thermal properties	11. Hardness	12. Utility.

1.5 CODES AND STANDARDS

A standard is a set of specifications for parts, materials or processes intended to achieve uniformity, efficiency and specified quality. One of the important purposes of a standard is to place a limit on the number of items in a particular specification and to reduce inventory.

A code is a set of specifications for the analysis, design, manufacture and construction of something. The purpose of a code is to achieve a specific degree of safety, efficiency and performance or quality.

There are several organisations who have established specifications for standards and safety or design codes. Some of them are:

ISO : International Standards Organisation
BIS : Bureau of Indian Standards (ISI)
SAE : Society of Automotive Engineers
ASME : American Society of Mechanical Engineers
ASM : American Society for Metals
ASTM : American Society for Testing Materials
JIS : Japan International Standards
DIN : German Institute for Standards (Deutsche Industrie Norma).

1.6 ENGINEERING STRESS AND STRAIN

When any external force acts on any body, resisting force is developed internally and the body is said to be in a state of stress.

Stress is defined as resisting force acting on unit area of cross section (c/s).

$$\text{Stress} = \frac{\text{Resisting force}}{\text{Area of cross section}}, \qquad \sigma = \frac{F}{A} \qquad \ldots\left(\frac{\text{EQ 2.1}}{\text{P 2.2}}\right)$$

Strain is defined as the deformation per unit length.

$$\text{Strain} = \frac{\text{Change in length}}{\text{Original length}}$$

$$\epsilon = \frac{\delta l}{l} \qquad \ldots\left(\frac{\text{E 2.3}}{\text{P 2.2}}\right)$$

1.7 SIMPLE STRESS

1.7.1 Axial Stress

Axial stress is of two types:

(i) **Tensile stress:** When a body is acted upon by a tensile force along the axis, then the stress produced is known as tensile stress.

$$\text{Tensile stress } \sigma = \frac{\text{Tensile force}}{\text{Area of c/s}} = \frac{F}{A} \qquad \ldots(2.1)$$

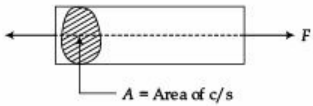


Fig. 1.2

(ii) **Compressive stress:** When a body is acted upon by a compressive force along the axis, then the stress produced is known as compressive stress.

Fig. 1.3

$$\text{Compressive stress, } \sigma = \frac{\text{Compressive force}}{\text{Area of c/s}} = \frac{F}{A}$$

Since these two are acting along axis, they are called axial stresses.

1.7.2 Shear Stress

Shear stress is the stress induced in a body when subjected to two equal and opposite forces acting tangentially. There are two cases of shear, namely, single shear and double shear.

Figure 1.4 shows a riveted joint connecting two plates. The plates carry a tensile load *F*.

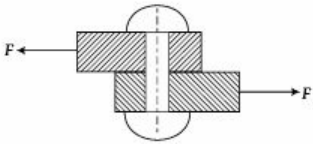


Fig. 1.4

This equal and opposite force '*F*' acting on the rivet at the line of separation causes rivet to shear. This is known as single shear.

$$\text{Shear stress, } \tau = \frac{\text{Force}}{\text{Area of c/s}} = \frac{F}{A} \qquad \ldots\left(\frac{25}{P\,2.3}\right)$$

Figure 1.5 shows another riveted joint with two cover plates and the plates are subjected to a tensile force *F*.

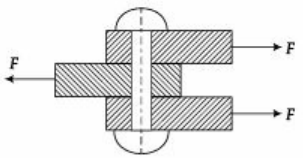


Fig. 1.5

This force causes the rivet to shear at two places or in other words two areas are resisting this force. Hence this is known as double shear.

Shear stress for double shear, $\tau = \dfrac{F}{2A}$.

1.7.3 Bending Stress

The simple way to understand the bending stress is either a cantilever subjected to a transverse force as shown in Fig. 1.6 (a) or a simply a supported beam subjected to a transverse force as shown in Fig 1.6(b).

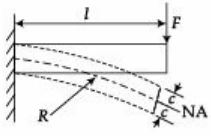


Fig. 1.6(a)

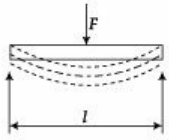


Fig. 1.6(b)

The bending equation is

$$\frac{M_b}{I} = \frac{\sigma}{C} = \frac{E}{R} \qquad \ldots\left(\frac{2.90}{\text{P } 2.15}\right)$$

where M_b = bending moment = Force $\times \perp lr$ distance
I = moment of inertia
C = distance from neutral axis (NA) to the extreme fibre
E = modulus of elasticity
R = radius of curvature

$\therefore$ Bending stress $\sigma = \dfrac{M_b . C}{I} = \dfrac{M_b}{(I/C)} = \dfrac{M_b}{Z}$ $\qquad \ldots\left(\dfrac{2.91}{\text{P } 2.15}\right)$

$$Z = \frac{I}{C} = \text{Section modulus}$$

1.7.4 Torsional Shear Stress

Figure 1.7 shows a round bar subjected to a twisting moment (torque) M_t.

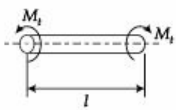


Fig. 1.7

The equation of torsion is

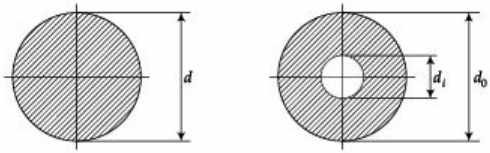
$$\frac{M_t}{J} = \frac{\tau}{r} = \frac{G\theta}{l} \quad \ldots\left(\frac{2.86}{P\,2.14}\right)$$

where M_t = twisting moment or torque

J = polar moment of inertia

$= \frac{\pi}{32} d^4$ for solid circular shaft

$J = \frac{\pi}{32}(d_o^4 - d_i^4)$. Hollow circular shaft



(a) Solid shaft (b) Hollow shaft

Fig. 1.8 Types of shafts.

$r = \frac{d}{2}$ for solid shaft

$r = \frac{d_0}{2}$ for hollow shaft

G = rigidity modulus

θ = angle of twist in radians

l = length

$\therefore$ Shear Stress, $\tau = \frac{M_t \cdot r}{J}$

Power: Power, $P = \frac{2\pi n M_t}{60000}$ kW

where n = speed in rpm

M_t = torque in N-m

$\therefore$ $M_t = \frac{60000P}{2\pi n} = \frac{9550P}{n}$ N-m $\quad \ldots\left(\frac{2.87\,b}{P\,2.15}\right)$

$= \frac{9550 \times 10^3 P}{n}$ N-mm

$$M_t = 9.55 \times 10^6 \frac{P}{n} \text{ N-mm}$$

P = Power in kW

n = Speed in rpm.

1.8 STRESS-STRAIN DIAGRAM

Figure 1.9 shows the stress-strain diagram for mild steel specimen subjected to a tensile load tested on UTM. Stress and strain are calculated at various loads and a graph is plotted. The various salient points are explained below.

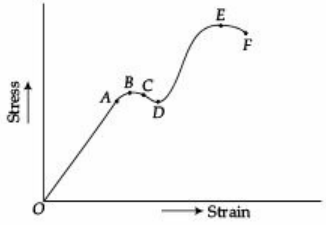


Fig. 1.9 Stress-strain diagram.

Point A: Proportionality limits. Upto this point the stress-strain curve is a straight line and the stress is proportional to strain. The Hooke's law is obeyed upto this point A.

Point B: Elastic limit. When load is applied, a member deforms. If the member regains its original shape after removal of the load, then it is said to be with in elastic limit. This is very close to proportionality limit A.

Points C and D: Upper and Lower yield points. Beyond elastic limit, there is increase in strain without appreciable increase in load. The point C is known as upper yield point and D is known as lower yield point. The increase of strain without increase of load is known as yielding. The yield stress is important for materials and for designers.

Point E: Ultimate stress. If the load is further increased, then the curve follows a parabolic shape and reaches maximum stress at E. This maximum stress is known as ultimate stress.

Point F: Breaking stress. Finally the member breaks at F and the corresponding stress is known as breaking stress.

1.9 FACTOR OF SAFETY

Factor of safety is the ratio of strength of a material to the design stress. The strength could be yield strength or ultimate strength.

$$\text{Factor of safety, FOS} = \frac{\text{Yield stress}}{\text{Design stress}} \text{ or } \frac{\text{Ultimate stress}}{\text{Design stress}}$$

The usual values of FOS are 1.5 to 3 based on yield stress and 3 to 6 based on ultimate stress. The various factors that affect the selection of factor of safety are:

1. Uncertainty of loading,
2. Uncertainty of material properties,
3. Human life at risk,
4. Severity of damage due to the failure,
5. Uncertainty of working conditions etc.

The design stress is also known as safe stress, working stress, maximum stress, permissible stress, limiting stress or allowable stress.

$$\text{Design stress} = \frac{\text{Yield stress}}{\text{FOS}} \text{ or } \frac{\text{Ultimate stress}}{\text{FOS}}$$

$$\sigma = \frac{\sigma_y}{\text{FOS}} \text{ or } \frac{\sigma_u}{\text{FOS}}$$

1.10 SYMBOLS

F = force, N
A = area of cross section, mm^2
d = diameter, mm
d_o = outside dia, mm
d_i = inside dia, mm
b = width mm
h = depth or height mm
l = length, mm
P = power, kW
n = speed in rpm
M_b = bending moment, N-mm
M_t = torque or twisting moment, N-mm
E = modulus of elasticity, MPa
G = modulus of rigidity, GPa

$(1 \text{ MPa} = 10^6 \text{ Pa} = \text{N/mm}^2)$
$(1 \text{ GPa} = 10^9 \text{ Pa} = 10^3 \text{ MPa})$

σ_y = yield stress, MPa or N/mm^2
σ_u = ultimate stress, MPa
τ_y = yield shear stress, MPa

τ_u = ultimate shear stress MPa

$\sigma_{-1} = \sigma_e$ = Endurance stress, MPa

σ = allowable or design stress, MPa = $\frac{\sigma_y}{FOS}$ or $\frac{\sigma_u}{FOS}$

τ = allowable shear stress = $\frac{1}{2}\sigma$ (in general)

FOS = Factor of safety

WKT : We know that

PROBLEMS

Problem 1: A round steel rod is subjected to a tensile load of 90 kN. Taking the yield stress for the steel as 328.6 MPa and factor of safety as 1.8, determine suitable diameter for the rod.

Given data: Load, F = 90 kN = 90000 N

Yield stress, σ_y = 328.6 MPa

FOS = 1.8

$\therefore$ allowable stress, $\sigma = \frac{\sigma_y}{FOS} = \frac{328.6}{1.8} = 182.56$ MPa

We know that for axial load, $\sigma = \frac{F}{A}$ ...(2.1)

$$182.56 = \frac{90000}{A}$$

$\therefore$ $$A = \frac{90000}{182.56} = 493.0 \text{ mm}^2$$

For circular rod $$A = \frac{\pi d^2}{4} = 493$$

$\therefore$ diameter, d = 25.054 mm $\simeq$ 25 mm.

Problem 2: Figure 1.10 shows a machine member of 60 mm diameter, 300 mm long and supported at one end, subjected to a tensile load of 30 kN. Find the tensile and shear stresses induced in the member.

Given data: Load, F = 30 kN = 30000 N

diameter d = 60 mm

$\therefore$ $$A = \frac{\pi d^2}{4} = \frac{\pi(60)^2}{4} = 2827.433 \text{ mm}^2$$

WKT Tensile stress, $\sigma = \frac{F}{A} = \frac{30000}{2827.433} = 10.61$ MPa

Max. shear stress, $\tau = \frac{1}{2}\sigma = 5.305$ MPa

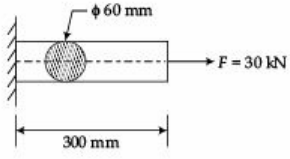


Fig. 1.10

Problem 3: Determine the max. stress induced in a link loaded as shown in the Fig. 1.11.

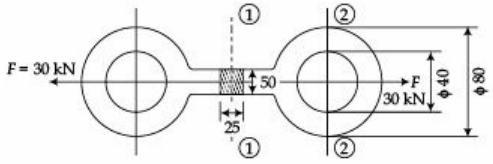


Fig. 1.11

Given data: Load $F = 30$ kN $= 30000$ N

Section 1-1 is a rectangular c/s, $A_1 = b.h$

Area of c/s, $A_1 = 25 \times 50 = 1250$ mm^2

$\therefore$ Tensile stresss, $\sigma_1 = \dfrac{F}{A_1} = \dfrac{30000}{1250} = 24$ MPa

Section 2-2 is a hollow as shown in Fig. 1.11a.

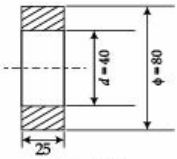


Fig. 1.11a

Area of c/s, $A_2 = 25\ (80 - 40) = 1000$ mm^2

$\therefore$ Tensile stress, $\sigma_2 = \dfrac{F}{A_2} = \dfrac{30000}{1000} = 30$ MPa

$\therefore$ Max. tensile stress = 30 MPa at **Section 2-2**

Problem 4: Figure 1.12 shows a machine link made of steel ($\sigma_y = 234$ MPa) subjected to a tensile load of 45 kN. Find the required thickness of the link taking factor of safety as 3.

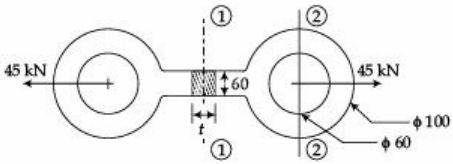


Fig. 1.12

Given data: Load, $F = 45$ kN $= 45000$ N

Yield stress $= \sigma_y = 234$ MPa

FOS $= 3$

$\therefore$ Allowable tensile stress, $\sigma = \dfrac{\sigma_y}{\text{FOS}} = \dfrac{234}{3} = 78$ MPa

Consider section 1-1. Area of c/s $A_1 = 60t$

WKT for tensile load stress, $\sigma = \frac{F}{A}$ ...(2.1)

$$78 = \frac{45000}{60t} \Rightarrow \text{thickness } t = 9.615 \simeq 10 \text{ mm.}$$

Consider section 2-2, Area of c/s, $A_2 = (100 - 60)t = 40t$

$\therefore$ $\sigma = \frac{F}{A}$

$$78 = \frac{45000}{40t} = \text{thickness } t = 14.43 \simeq 15 \text{ mm}$$

$\therefore$ Recommended thickness, $t = 15$ mm (bigger of these 2 values).

Problem 5: A circular rod made of steel ($\sigma_y = 420$ MPa) is subjected to a compressive load of 120 kN. Find the suitable diameter of the rod taking factor of safety as 2. Neglect column effect.

Given data: Yield stress, $\sigma_y = 420$ MPa

Load, $F = 120$ kN $= 120 \times 10^3$ N

FOS = 3

$\therefore$ Allowable stress, $\sigma = \frac{\sigma_y}{FOS} = \frac{420}{3} = 140$ MPa

WKT Compressive stress, $\sigma = \frac{F}{A}$

$$140 = \frac{120 \times 10^3}{\left(\frac{\pi}{4}d^2\right)} \Rightarrow \text{diameter } d = 33 \text{ mm}$$

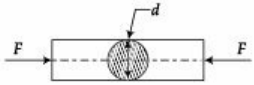


Fig. 1.13

Problem 6: A 120 mm wide rectangular plate carries a compressive load of 75 kN. Find the required thickness of the plate if the allowable stress is 90 MPa.

Given data: Load, $F = 75$ kN $= 75 \times 10^3$ N

Width, $b = 120$ mm

Allowable stress, $\sigma = 90$ MPa

Area of c/s, $A = bt = 120t$

WKT Compressive stress, $\sigma = \frac{F}{A}$

$$90 = \frac{75 \times 10^3}{120t} = \text{thickness, } t = 6.94 \simeq 7 \text{ mm}$$

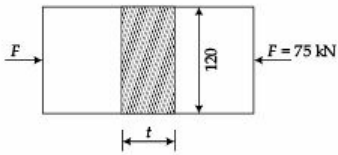


Fig. 1.14

Problem 7: A tube of 100 mm outside diameter is subjected to a compressive load of 90 kN. If the allowable stress for the tube material is 60 MPa, find the suitable inside dia of the tube.

Given data:

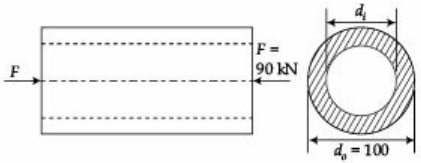


Fig. 1.15

Load, $F = 90 \text{ kN} = 90 \times 10^3 \text{ N}$

Outside dia of tube, $d_o = 100$ mm

Allowable stress, $\sigma = 60$ MPa

Let inside dia be $= d_i$

$$\text{Area of c/s, } A = \frac{\pi}{4}(d_o^2 - d_i^2)$$

$$\text{WKT for compressive load, } \sigma = \frac{F}{A}$$

$$60 = \frac{90 \times 10^3}{(\pi/4)\,(100^2 - di^2)} \Rightarrow d_i = 89.94 \simeq 89 \text{ mm}$$

(**Note:** Inside dia is to be rounded off to lower value).

Problem 8: Four steel ropes of each 50 mm diameter are used in a 120 kN passenger lift. The area of cross section is $0.38\,d^2$, where d is the diameter of the rope. Check the design and give your comment, if the allowable stress in the rope is 45 N/mm^2.

Given data:

Dia of rope, $d = 50$ mm

No. of ropes = 4

Total load $= 120 \text{ kN} = 120 \times 10^3 \text{ N}$

Area of c/s of each rope $F = 0.38\,d^2$

Allowable stress, $\sigma_{all} = 45$ MPa

$$\text{Load on each rope } F = \frac{\text{Total load}}{\text{No. of ropes}} = \frac{120 \times 10^3}{4} = 30 \times 10^3 \text{ N}$$

$$\text{Area of c/s, } A = 0.38\,d^2 = 0.38 \times 50^3$$

$$= 950 \text{ mm}^2$$

Ropes are subjected to tensile stress.

$\therefore$ Stress induced in each rope, $\sigma = \dfrac{F}{A}$

$$\sigma = \frac{30000}{950} = 31.58 \text{ MPa} < 45 \text{ MPa}$$

$\because$ Induced stress 31.58 MPa is less than allowable stress 45 MPa. Hence the design is safe.

Problem 9: A machine weighing 100 kN is placed on the 4 levelling screws. The allowable stress for the screw material may be taken as 120 N/mm^2. Find the suitable core diameter of the screw required.

Given data: Total load, $W = 100 \text{ kN} = 100 \times 10^3$ N

No. of screws = 4

$\therefore$ Load on each screw $= \dfrac{W}{4} = \dfrac{100 \times 10^3}{4}$

$\therefore$ $F = 25000$ N

Allowable stress in screw, $\sigma = 120$ MPa

The bolts are subjected to compressive stress, since the machine weight is applied on them.

WKT, $\sigma = \dfrac{F}{A}$

$\therefore$ $A = \dfrac{F}{\sigma} = \dfrac{25000}{120}$ mm^2

But core area of bolt $= \dfrac{\pi}{4} dc^2$

where dc = core or root diameter of bolt

$$\frac{\pi}{4} dc^2 = \frac{25000}{120}$$

$\therefore$ $dc = 16.29$ mm.

Problem 10: Figure 1.16 shows a riveted joint subjected to a tensile load of 60 kN. Find suitable diameter of rivet if it is made of steel having tensile yield stress of 328.6 MPa.

Given data: Local, $F = 60 \text{ kN} = 60 \times 10^3$ N

Yield stress in tensile, $\sigma_y = 328.6$ N/mm^2

Assume factor of safety, FOS = 2.5

$$\sigma = \frac{\sigma_y}{FOS} = \frac{3286}{2.5} = 131.44 \text{ MPa}$$

$\therefore$ Assume allowable shear stress, $\tau = 0.5\sigma$

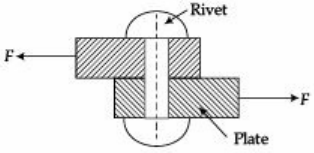


Fig. 1.16

Tensile load on the plate is shear load for bolts.

The rivet is subjected to single shear.

$\therefore$ WKT, $\tau = \dfrac{F}{A} \Rightarrow 65.72 = \dfrac{60 \times 10^3}{\frac{\pi}{4} d^2}$

$\therefore$ Dia of rivet, $d = 34.09 \simeq 35$ mm.

Problem 11: Figure 1.17 shows a bolted joint subjected to a tensile load of 120 kN. Find the root dia of bolt taking allowable shear stress as 60 MPa.

Given data: Total load = 120 kN = 120×10^3 N

No of blots = 3 (shown in Fig.)

$\therefore$ Load on each bolt = $\dfrac{120 \times 10^3}{3}$

$F = 40 \times 10^3$ N

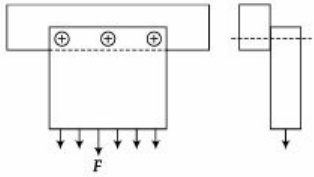


Fig. 1.17

Allowable shear stress, $\tau = 60$ MPa

Tensile load on the plate is shear load for bolts.

The bolts are under single shear.

Shear stress, $\tau = \dfrac{F}{A}$ for single shear

$$60 = \frac{40 \times 10^3}{\frac{\pi}{4} dc^2} \qquad \left(A \text{ of bolt} = \frac{\pi}{4} dc^2\right)$$

$\therefore$ Core dia, $dc = 29.13$ mm.

Problem 12: Figure 1.18 shows a knuckle joint carrying a tensile load of 90 kN. Find suitable diameter of rods and knuckle pin taking allowable shear and tensile stresses as 72 MPa and 120 MPa respectively.

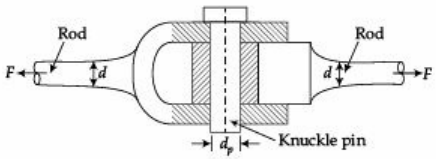


Fig. 1.18

Given data: Load, $F = 90$ kN = 90×10^3 N

Allowable shear stress, $\tau = 72$ MPa

Allowable tensile stress, $\sigma = 120$ MPa

The rods are subjected to tensile stress and the knuckle pin to double shear.

For the rods in tension, tensile stress, $\sigma = \dfrac{F}{A_{rods}}$

$\therefore$ $$A_{rods} = \frac{F}{\sigma}$$

$$\frac{\pi}{4}d^2 = \frac{90\times10^3}{120} \Rightarrow \text{dia of rods } d = 30.9 \simeq 31 \text{ mm}$$

For the knuckle pin in double shear,

$$\text{Shear stress, } \tau = \frac{F}{2A_{pin}}$$

$\therefore$ $$A_{pin} = \frac{F}{2\tau}$$

$$\frac{\pi}{4}d_p^2 = \frac{90\times10^3}{2\times72}$$

$$\text{Dia of pin, } d_p = 28.21 \simeq 29 \text{ mm.}$$

Problem 13: A centrifuge has a small bucket weighing 0.2 kg with contents. It is supported on a manganese bronze pin at the end of a horizontal arm. The pin is 18 mm in diameter and is in double shear under the action of the centrifugal force. The arm rotates at 10,000 rpm and the centre of gravity (c.g.) of the bucket is at 270 mm from the axis of rotation. Determine the shear stress induced in the pin. If the yield shear stress for the pin material is 300 MPa, find the factor of safety adopted.

Given data: Dia of pin, $d = 18$ mm. And in double shear due to centrifugal force.

Speed of arm, $n = 10000$ rpm.

Radius of rotation = distance to cg $r = 270$ mm $= 0.27$ m

Yield shear stress, $\tau_y = 300$ MPa

Mass of bucket, $m = 0.2$ kg

Centrifugal force, $F = m.a$

$$F = m.r\,w^2$$

where m is mass in kg, r = radius in m.

w = angular velocity $= \dfrac{2\pi n}{60}$

$$F = 0.2 \times 0.27 \times \left(\frac{2\pi\times10000}{60}\right)^2$$

$$F = 59217.63 \text{ N}$$

∴ Shear stress in pin due to double shear,

$$\tau = \frac{F}{2A} = \frac{59217.63}{2\frac{\pi}{4}\times 18^2} = 116.35 \text{ N/mm}^2$$

∴ $$\text{FOS adopted} = \frac{\tau_y}{2A} = \frac{300}{116.35} = 2.58.$$

Problem 14: A beam of uniform rectangular cross section is fixed at one end and carries a transverse load of 1.8 kN at a distance of 0.9 m from the fixed end. The material used is C30 steel (σ_y = 294.2 MPa) and factor of safety is 2.5. Find the width and depth of c/s if the depth is twice the width.

Given data:

Load, F = 1.8 kN = 1800 N

Length, l = 0.9 m = 900 mm

Yield stress, σ_y = 294.2 MPa

FOS = 2.5

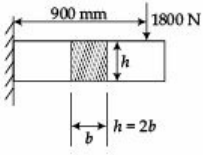


Fig. 1.19

∴ $$\text{Allowable stress, } \sigma = \frac{\sigma_y}{\text{FOS}} = \frac{294.2}{2.5} = 117.68 \text{ MPa}$$

depth of c/s, $h = 2b$

The beam is subjected to bending moment.

WKT equation of bending is $\frac{M_b}{I} = \frac{\sigma}{C}$ ...(2.90)

where M_b = bending moment = $F \cdot l$

$= 1800 \times 900 = 1.62 \times 10^6$ N-mm

I = moment of inertia = $\frac{bh^3}{12} = \frac{b(2b)^3}{12} = \frac{8b^4}{12} = 0.667\, b^4$

C = distance from NA to outer fibre = $\frac{h}{2} = \frac{2b}{2} = b$

σ = allowable stress = 117.68 MPa

Substituting, $$\frac{1.62\times 10^6}{0.667b^4} = \frac{117.68}{b} \Rightarrow b = 27.43 \simeq 28 \text{ mm}$$

∴ Width, b = 28 mm.

Thickness, $h = 2b$ = 56 mm.

Problem 15: A circular rod made of C40 steel is simply supported between bearings 1.8 m apart. It carries a central load of 6 kN. Find the suitable diameter of the rod. Taking factor of

safety = 2.5 based on yield stress of 328.6 MPa. The deflection is not to exceed 3 mm at the middle.

Given data: Load, $F = 6$ kN $= 6 \times 10^3$ N

$l = 1.8$ m $= 1800$ mm

Yield stress, $\sigma_y = 328.6$ MPa

FOS = 2.5

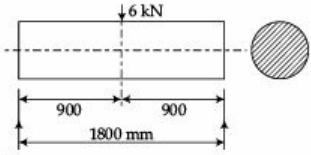


Fig. 1.20

$\therefore$ Allowable stress, $\sigma = \dfrac{\sigma_y}{\text{FOS}} = \dfrac{328.6}{2.5} = 131.44$ MPa

Deflection, $y = 3$ mm

The rod is subjected to bending moment

$$\text{WKT, } \frac{M_b}{I} = \frac{\sigma}{C} \qquad \text{...(2.90)}$$

where bending moment, $M_b = \dfrac{Fl}{4}$ ...(Fig. 4, T 2–8 / Pg. 2–34)

for simply supported beam with central load

$$M_b = \frac{6000 \times 1800}{4} = 2.7 \times 10^6 \text{ N-mm}$$

I = moment of Inertia $= \dfrac{\pi d^4}{64}$ (for circular c/s)

C = distance from NA to outer fibre $= \dfrac{d}{2}$

σ = allowable stress = 131.44 MPa

Substituting, $\dfrac{2.7 \times 10^6}{\pi d^4/64} = \dfrac{131.44}{(d/2)}$

$\therefore$ dia, $d = 59.367 \simeq 60$ mm

To limit the deflection to 3 mm, consider deflection for simply supported beam with central load $y = \dfrac{Fl^3}{48EI}$ (Fig. 4, T 2-8, Pg. 2-34)

E = Modulus elasticity may be taken as = 206.8 GPa = 206.8×10^3 MPa for steels

Substituting $3 = \dfrac{6000 \times 1800^3}{48 \times 206.8 \times 10^3 \times (\pi d^4/64)}$

$\therefore$ Dia, $d = 69.95$ mm $\simeq 70$ mm

Recommend the bigger of these two, diameter = $d = 70$ mm.

Problem 16: A 1 mm thick steel blade is bent into a circular arc of 500 mm radius. Determine the bending stress induced and the bending moment required to bend the blade of width 15 mm. Take E = 210 GPa.

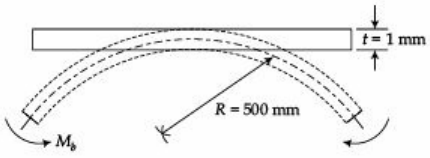


Fig. 1.21

Given data: Thickness, t = 1 mm

Width, b = 15 mm

Radius, R = 500 mm

E = 210 GPa = 210×10^3 MPa

For the blade under bending,

$$\text{WKT, } \frac{M_b}{I} = \frac{\sigma}{C} = \frac{E}{R} \qquad \text{...(2-90)}$$

where M_b = bending moment

I = moment of Inertia $\frac{bt^3}{12}$

$$= \frac{15 \times 1^3}{12} = 1.25 \text{ mm}^4$$

σ = bending stress

C = distance from NA to outer fibre

$$= \frac{t}{2} = \frac{1}{2} = 0.5 \text{ mm}$$

Consider $\frac{\sigma}{C} = \frac{E}{R}$

$\therefore$ Bending stress, $\sigma = \frac{E}{R} \cdot C$

$$\sigma = \frac{210 \times 10^3}{500} \times 0.5 = 210 \text{ MPa}$$

Consider $\frac{M_b}{I} = \frac{\sigma}{C}$

$\therefore$ Bending moment, $M_b = \frac{\sigma}{C} \cdot I = \frac{210}{0.5} \times 1.25$

M_b = 525 N-mm.

Problem 17: Figure 1.22 shows a machine member in the form of cantilever, of circular c/s 50 mm diameter, 250 mm long subjected to a transverse load of 3 kN at its free end. Find the max. bending end shear stresses induced in the member.

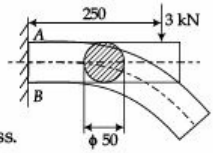


Fig. 1.22

Given data: Load, $F = 3$ kN $= 3000$ N

Length, $l = 250$ mm

Dia, $d = 50$ mm

Point A is subjected to tensile stress and point B to compressive stress.

WKT for bending, $\frac{M_b}{I} = \frac{\sigma}{C}$

where M_b = bending moment $= Fl = 3000 \times 250$

$= 750 \times 10^3$ N-mm

I = moment of inertia $= \frac{\pi d^4}{64}$

$I = \frac{\pi \times 50^4}{64} = 306.796 \times 10^3$ mm^4

$C = \frac{d}{2} = \frac{50}{2} = 25$ mm

$\therefore$ Bending stress, $\sigma = \frac{M_b \cdot C}{I} = \frac{750 \times 10^3 \times 25}{306.796 \times 10^3}$

$\sigma = 61.115$ MPa

This is tensile at A and compressive at B

$\therefore$ Maximum shear stress, $\tau_{max} = 0.5\ \sigma_{max}$

$\tau_{max} = 0.5 \times 61.115 = 30.558$ MPa.

Problem 18: Determine the dia of solid steel shaft required to transmit 30 kW of power at 900 rpm. The shear stress should be limited to 60 MPa and angle of twist is not to exceed 0.75°/m length. Rigidity modulus may be assumed to be 81 GPa.

Given data: Power, $P = 30$ kW

Speed, $n = 900$ rpm

Allowable shear stress, $\tau = 60$ MPa

Angle of twist, $\theta = 0.75° = 0.75 \times \frac{\pi}{180}$ radians

Length of shaft, $l = 1$ m $= 1000$ mm

(θ is given per 1 m length, hence, $l = 1$ m)

Rigidity modulus, $G = 81$ GPa $= 81 \times 10^3$ MPa

$$\text{Torque, } M_t = \frac{9.55 \times 10^6 P}{n} = \frac{9.55 \times 10^6 \times 30}{900}$$

$$M_t = 318.33 \times 10^3 \text{ N-mm}$$

For solid shaft polar moment of Inertia

$$J = \frac{\pi d^4}{32} \text{ and } r = \frac{d}{2}$$

(i) Consider $\frac{M_t}{J} = \frac{\tau}{r}$

$$\frac{318.33 \times 10^3}{(\pi d^4/32)} = \frac{60}{(d/2)} = d = 30 \text{ mm}$$

(ii) Consider $\frac{M_t}{J} = \frac{G\theta}{l}$

$$\frac{318.33 \times 10^3}{(\pi d^4/32)} = \frac{81000 \times (0.75 \times \pi/180)}{1000}$$

$$d = 41.818 \text{ mm}$$

Minimum dia required to satisfy both conditions is bigger of the above two values

$\therefore$ $d = 41.818 \simeq 42$ mm.

Problem 19: A steel shaft 60 mm diameter rotates at 1200 rpm. It is made of steel having allowable shear stress as 60 MPa. Find the safe power that can be transmitted by the shaft. Also find the angle of twist in a length of 1 m taking rigidity modulus as 80 GPa.

Given data:

Dia, $d = 60$ mm

Speed, $n = 1200$ rpm

Allowable shear stress, $\tau = 60$ MPa

Length, $l = 1$ m $= 1000$ mm.

Rigidity modulus, $G = 80$ GPa $= 80 \times 10^3$ MPa.

To find power, let us first find torque capacity of the shaft.

(i) WKT $\frac{M_t}{J} = \frac{\tau}{r}$

$\therefore$ $\text{Torque, } M_t = \tau \frac{J}{r}$

where $J = \frac{\pi d^4}{32}$ and $r = \frac{d}{2}$ for solid shaft

$$\therefore \quad M_t = \frac{60\left(\frac{\pi \times 60^4}{32}\right)}{(60/2)} = 2.545 \times 10^6 \text{ N-mm}$$

$$= 2.545 \times 10^3 \text{ N-m}$$

$$\text{WKT power, } P = \frac{2\pi n M_t}{60000} \text{ kW with } M_t \text{ in N-m}$$

$$P = \frac{2 \times \pi \times 1200 \times 2.545 \times 10^3}{60000}$$

$$P = 319.81 \text{ kW.}$$

(ii) Angle of twist

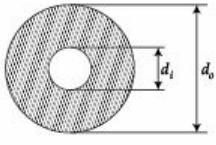
$$\text{consider } \frac{M_t}{J} = \frac{G\theta}{l}$$

$$\frac{2.545 \times 10^6}{\left(\frac{\pi \times 60^4}{32}\right)} = \frac{80 \times 10^3 \times \theta}{1000} \Rightarrow \theta = 0.025 \text{ radians}$$

$$\therefore \quad \theta = 0.025 \times \frac{180}{\pi} = 1.43 \text{ deg.}$$

Problem 20: A hollow machine shaft with diameters ratio of 0.6 is required to transmit 42 kW at 1500 rpm. The allowable shear stress is 60 MPa. Angle of twist is not to exceed 1°/m length. Find suitable diameters of the shaft.

Given data: Hollow shaft



$$\text{Diameters ratio } \frac{d_i}{d_o} = 0.6 \Rightarrow d_i = 0.6\, d_o$$

$$\left\{\begin{matrix}\text{If the ratio is} < 1, \text{ it is } d_i/d_o \\ \text{If the ratio is} > 1, \text{ it is } d_o/d_i\end{matrix}\right\}$$

Power, $P = 42$ kW, speed $n = 1500$ rpm

Allowable shear stress, $\tau = 60$ MPa

$$\text{Angle of twist, } \theta = 1° = 1 \times \frac{\pi}{180} \text{ radians}$$

length, $l = 1$ m $= 1000$ mm.

Assume rigidity modulus, $G = 80$ GPa $= 80 \times 10^3$ MPa

$$\text{For torsion, WKT } \frac{M_t}{J} = \frac{\tau}{r} = \frac{G\theta}{l} \qquad \ldots(2.86)$$

where $$\text{Torque, } M_t = \frac{9.55 \times 10^6 P}{n} = \frac{9.55 \times 10^6 \times 42}{1500}$$

$$M_t = 267.4 \times 10^3 \text{ N-mm}$$

for $$\text{Hollow shaft, } J = \frac{\pi}{32}(d_o^4 - d_i^4)$$

$$J = \frac{\pi}{32}[d_o^4 - (0.6\, d_o^4)] = \frac{\pi d_o^4}{32}(1 - 0.6^4)$$

$$= \frac{\pi}{32} d_o^4 (0.8704)$$

$$r = \frac{d_o}{2}$$

(i) Consider $$\frac{M_t}{J} = \frac{\tau}{r} \Rightarrow \frac{267.4 \times 10^3}{\frac{\pi}{32} d_o^4 (0.8704)} = \frac{60}{d_o/2}$$

$\therefore$ $$d_o = 29.65 \text{ mm}$$

(ii) Again consider $$\frac{M_t}{J} = \frac{G\theta}{l}$$

Assume rigidity modulus $G = 80$ GPa $= 80 \times 10^3$ MPa (considering as steel)

$$\frac{267.4 \times 10^3}{\frac{\pi}{32} d_o^4 (0.8704)} = \frac{80 \times 10^3 \times \left(\frac{1 \times \pi}{180}\right)}{1000}$$

$\therefore$ $$d_o = 38.69 \text{ mm}$$

Minimum dia required, $d_o = 38.69$ mm

(Bigger of the above two)

Say, $d_o \simeq 39$ mm

$\therefore$ $$d_i = 0.6\, d_o = 0.6 \times 39 = 23.4 \simeq 23 \text{ mm}$$

(Inner dia is to be rounded off to lower value)

Problem 21: A hollow shaft 60 mm outside diameter and 30 mm inside diameter is 600 mm long and rotates at 1800 rpm. It is made of steel having allowable shear stress of 90 MPa and $G = 80$ GPa. Find the power capacity of this shaft if the angle of twist is not to exceed 0.7°.

Given data:

Outside dia, $d_o = 60$ mm

Inside dia, $d_i = 30$ mm

Length, $l = 600$ mm

Speed, $n = 1800$ rpm

Allowable shear stress, $\tau = 90$ MPa, angle of twist $\theta = 0.7° = 0.7 \times \frac{0.7 \times \pi}{180}$ radians

Rigidity modulus, $G = 80$ GPa $= 80 \times 10^3$ MPa

$$\text{WKT } \frac{M_t}{J} = \frac{\tau}{r} = \frac{G\theta}{l} \qquad \text{...(2.86)}$$

First consider $\frac{M_t}{J} = \frac{\tau}{r}$ find M_t and power.

Again consider $\frac{M_t}{J} = \frac{G\theta}{l}$ find M_t and power.

Recommend the smaller one as maximum capacity.

(i) $$\text{Consider, } \frac{M_t}{J} = \frac{\tau}{r} \Rightarrow M_t = \tau \cdot \frac{J}{r}$$

$$\text{For hollow shaft, } J = \frac{\pi}{32}(d_o^4 - d_i^4)$$

$$J = \frac{\pi}{32}(60^4 - 30^4) = 1.1928 \times 10^6 \text{ mm}^4$$

$$r = \frac{d_o}{2} = \frac{60}{2} = 30$$

$$\therefore \quad M_t = \frac{90 \times 1.1928 \times 10^6}{30} = 3.578 \times 10^6 \text{ N-mm.}$$

$$\therefore \quad M_t = 3.578 \times 10^3 \text{ N-m}$$

$$\therefore \quad \text{Power, } P = \frac{2\pi n M_t}{60000} \text{ kW } (M_t \text{ in N-m})$$

$$= \frac{2\pi \times 1800 \times 3.578 \times 10^3}{60000} = 674.53 \text{ kW}$$

(ii) $$\text{Consider } \frac{M_t}{J} = \frac{G\theta}{l}$$

$$\therefore \quad M_t = G\theta \cdot \frac{J}{l} = \frac{80000 \times \left(0.7 \times \frac{\pi}{180}\right) \times 1.1928 \times 10^6}{600}$$

$$= 1.943 \times 10^6$$

$$\text{N-mm} = 1.943 \times 10^3 \text{ Nm}$$

$$\therefore \quad \text{Power, } P = \frac{2 \times \pi \times 1800 \times 1.943 \times 10^3}{60000} = 366.25 \text{ kW}$$

Recommended capacity, $P = 366.25$ kW.

1.11 PRINCIPAL STRESSES OR COMPOUND STRESSES

1.11.1 Unidirectional Stress

Figure 1.23 shows a member subjected to unidirectional stress = σ_x acting on face AB. (tensile or compressive).

The normal stress on a plane AC inclined at θ.

$$\sigma_\theta = \sigma_x \cos^2 \theta \quad \text{and} \quad \left(\frac{2-13}{P\,2.3}\right)$$

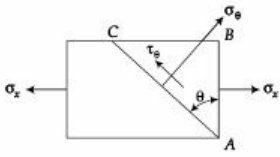


Fig. 1.23

shear stress acting along plane AC inclined at θ.

$$\tau_\theta = \frac{\sigma_x}{2} \sin 2\theta \qquad \left(\frac{2-14}{P\,2.4}\right)$$

$$\text{Max shear stress, } \tau_{max} = \frac{\sigma_x}{2} \text{ at } \theta = 45°. \qquad ...(2.17)$$

1.11.2 Biaxial Stresses

Figure 1.24 shows an element subjected to biaxial stresses σ_x along x direction (tensile or compressive) and σ_y along y direction $\perp lr$ to x. (tensile or compressive)

Normal stress on the plane AC inclined at θ.

$$\sigma_\theta = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right) \cos 2\theta \qquad ...\left(\frac{2-28}{P\,2.5}\right)$$

Shear stress on the plane AC inclined at θ.

$$\tau_\theta = \left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta. \qquad ...\left(\frac{2-29}{P\,2.5}\right)$$

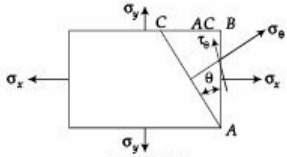


Fig. 1.24

Maximum principal stress, $\sigma_1 = \sigma_x$

Minimum principal stress, $\sigma_2 = \sigma_y$

$$\text{Maximum shear stress} = \frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma_x - \sigma_y}{2}. \qquad ...\left(\frac{2-31}{P\,2.5}\right)$$

1.11.3 Pure Shear

Figure 1.25 shows an element subjected to pure shear stress τ_{xy}.

Normal stress on the plane AC inclined at θ.

$$\sigma_\theta = \tau_{xy} \sin 2\theta \qquad ...\left(\frac{2-22}{P\,2.5}\right)$$

Shear stress on the plane AC inclined at θ.

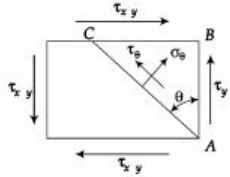


Fig. 1.25

$$\tau_\theta = -\tau_{xy} \cos 2\theta \quad \ldots\left(\frac{2-23}{P\,2.5}\right)$$

Maximum principal stress, $\sigma_1 = \tau_{xy}$

Minimum principal stress, $\sigma_2 = -\tau_{xy}$ ...(2-24)

Maximum shear stress, $\tau_{max} = \dfrac{\sigma_1 - \sigma_2}{2} = \tau_{xy}$. ...(2-26)

1.11.4 Biaxial Stresses Combined with Shear Stress

Figure 1.26 shows an element acted upon by biaxial stresses combined with shear stress.

σ_x = stress along x direction, tensile or compressive

σ_y = stress along y direction, tensile or compressive

τ_{xy} = shear stress on xy plane.

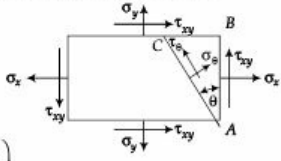


Fig. 1.26

Normal stress along a plane AC inclined at θ.

$$\sigma_\theta = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right) \cos 2\theta + \tau_{xy} \sin 2\theta \quad \ldots\left(\frac{2-32}{P\,2.6}\right)$$

Shear stress along a plane AC inclined at θ.

$$\tau_\theta = \left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta - \tau_{xy} \cos 2\theta \quad \ldots\left(\frac{2-33}{P\,2.6}\right)$$

Maximum principal stress

$$\sigma_1 = \frac{1}{2}\left[(\sigma_x + \sigma_y) + \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}\right] \quad \ldots(2\text{-}34)$$

Minimum principal stress

$$\sigma_2 = \frac{1}{2}\left[(\sigma_x + \sigma_y) - \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}\right] \quad \ldots(2\text{-}35)$$

Maximum shear stress

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \quad \ldots(2.37)$$

The angles of principal planes

$$\theta_{1,2} = \frac{1}{2} \tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) \quad \ldots(2\text{-}36)$$

θ_1 and θ_2 are at 90° apart.

The angle of max shear planes

$$\theta_\tau = \frac{1}{2} \tan^{-1}\left[\frac{\sigma_x - \sigma_y}{-2\tau_{xy}}\right] \quad \ldots(2.38)$$

1.11.5 Unidirectional Stress Combined with Shear Stress

Figure 1.27 shows an element acted upon by an unidirectional stress σ_x (tensile or compressive) along with a shear stress τ_{xy}.

There is no stress acting along y direction $\perp lr$ to x.

Hence, $\sigma_y = 0$.

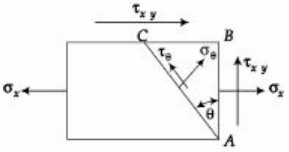


Fig. 1.27

$\therefore$ Maximum principal stress.

$$\sigma_1 = \frac{1}{2}\left[\sigma_x + \sqrt{\sigma_x^2 + 4\tau_{xy}^2}\right]$$

Maximum shear stress

$$\tau_{max} = \frac{1}{2}\sqrt{\sigma_x^2 + 4\tau_{xy}^2} \qquad \text{...(P 2.40)}$$

(For details and derivations, the readers are advised to refer to textbooks on the strength of materials).

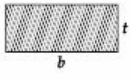
Note: Tensile stresses are considered +ve and compressive stresses are regarded as –ve values in all the above equations.

Problems on principal stresses

Problem 22: An element is subjected to an unidirectional force of 9 kN. Determine the cross sectional dimensions of rectangular c/s taking width as twice the thickness. Also find the normal and shear stresses along a plane inclined at 30°. The allowable stress for the material may be taken as 120 MPa.

Given data: Force, $F = 9$ kN $= 9000$ N

Rectangular c/s



Width = 2 · thickness = $b = 2t$

Allowable stress, $\sigma = 120$ MPa.

Inclination of plane, $\theta = 30°$

We know that, $\sigma = \frac{F}{A}$

$$120 = \frac{9000}{b \cdot t} = \frac{9000}{2t \cdot t}$$

$\therefore$ $t = 6.12$ mm $\simeq 7$ mm

$\therefore$ Width, $b = 2t = 14$ mm

Actual stress induced: $\sigma_x = \frac{F}{A} = \frac{9000}{7 \times 14}$

$$\sigma_x = 91.84 \text{ MPa}$$

Normal stress along a plane inclined at θ

$$\sigma_\theta = \sigma_x \cos^2 \theta = 91.84 \cos^2 30 = 68.88 \text{ MPa}$$

Shear stress along a plane inclined at θ

$$\tau_\theta = \frac{\sigma_x}{2} \sin 2\theta = \frac{91.84}{2} \sin (2 \times 30)$$

$$\tau_\theta = 39.77 \text{ MPa}.$$

Problem 23: A rectangular bar of cross section 220 mm × 50 mm is subjected to a tensile load F as shown in Fig. 1.28. The allowable normal and shear stresses on the oblique plane BC are 9 MPa and 4.5 MPa respectively. Determine the safe value of load F.

Given data: Width = 220 mm thickness = 50 mm

$\therefore$ Area of cross section, $A = 220 \times 50 = 11000 \text{ mm}^2$

Normal stress on plane BC, $\sigma_\theta = 9$ MPa

Shear stress on plane BC, $\tau_\theta = 4.5$ MPa

θ is the angle wrt the plane of F i.e., vertical

$$\theta = 90 - 60 = 30°$$

WKT, $\sigma_\theta = \sigma_x \cos^2 \theta$

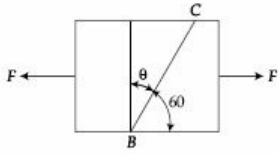


Fig. 1.28

$\therefore$ $$\sigma_x = \left(\frac{\sigma_\theta}{\cos^2 \theta} \right) = \frac{9}{\cos^2 30} = 12 \text{ MPa}$$

and $$\tau_\theta = \frac{\sigma_x}{2} \cdot \sin 2\theta$$

$$\sigma_x = \frac{2 \cdot \tau_\theta}{\sin 2\theta} = \frac{2 \times 4.5}{\sin 2(30)} = 10.39 \text{ MPa}$$

Taking the smaller value, $\sigma_x = 10.39$ MPa

$\therefore$ Load, $F = \sigma_x \cdot A$

$$= 10.39 \times 11000$$

$$F = 114.32 \times 10^3 \text{ N}$$

Problem 24: A machine member is subjected to the following stresses.

Determine $\sigma_x = 90$ MPa, $\sigma_y = 60$ MPa.

(i) Normal and shear stresses along a plane inclined at 30°,

(ii) Principal stresses and

(iii) Maximum shear stress.

Given data: $\sigma_x = 90$ MPa

$\sigma_y = 60$ MPa

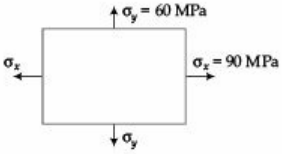


Fig. 1.29

(i) Normal stress on a plane inclined at θ

$$\sigma_\theta = \left(\frac{\sigma_x - \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta$$

$$= \left(\frac{90+60}{2}\right) + \left(\frac{90-60}{2}\right)\cos(2\times 30)$$

$$= 82.5 \text{ MPa}$$

Shear stress on a plane inclined at θ

$$\tau_\theta = \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta$$

$$= \left(\frac{90-60}{2}\right)\sin 2\times 30$$

$$= 12.99 \text{ MPa.}$$

(ii) Maximum principal stress

$$\sigma_1 = \text{Max of } \sigma_x \text{ or } \sigma_y = 90 \text{ MPa}$$

Minimum principal stress

$$\sigma_2 = \text{Min of } \sigma_x \text{ or } \sigma_y = 60 \text{ MPa}$$

(iii) Maximum shear stress

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{90-60}{2} = 15 \text{ MPa.}$$

Problem 25: A certain element is subjected to pure shear stress of 60 MPa. Find the normal and shear stresses acting on plane inclined at 25°. Also find the principal stresses and maximum shear stress.

Given data: $\tau_{xy} = 60$ MPa

$\theta = 25°$

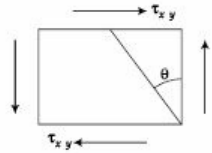


Fig. 1.30

(i) Normal stress on inclined plane, $\sigma_\theta = \tau_{xy} \sin 2\theta$

$$= 60 \sin(2\times 50) = 45.96 \text{ MPa}$$

Shear stress on inclined plane, $\tau_\theta = \tau_{xy} \cos 2\theta$

$$= 60 \cos(2\times 25) = 38.57 \text{ MPa}$$

(ii) Max. principal stress, $\sigma_1 = \tau_{xy} = 60$ MPa

Min. principal stress, $\sigma_2 = -\tau_{xy} = -60$ MPa

Max. shear stress, $\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \tau_{xy} = 60$ MPa.

Problem 26: An element is acted upon by the following stresses.

$\sigma_x = 120$ MPa, $\sigma_y = 90$ MPa, $\tau_{xy} = 30$ MPa

(i) Compute the stresses on a plane inclined at 20°.

(ii) Find principal stresses and their location.

(iii) Find maximum shear stress and its location.

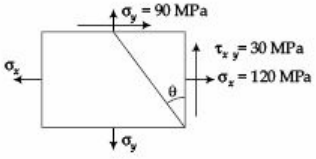


Fig. 1.31

Given data:

$\sigma_x = 120$ MPa

$\sigma_y = 90$ MPa

$\tau_{xy} = 30$ MPa

$\theta = 20°$

(i) (a) Normal stress on the plane inclined at θ

$$\sigma_\theta = \left(\frac{\sigma_x - \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy} \sin 2\theta$$

$$= \left(\frac{120 - 90}{2}\right) + \left(\frac{120 - 90}{2}\right)\cos (2 \times 20) + 30 \sin (2 \times 20)$$

$$\sigma_\theta = 135.77 \text{ MPa}$$

(b) Shear stress on the plane inclined at θ

$$\tau_\theta = \left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta - \tau_{xy} \cos 2\theta$$

$$= \left(\frac{120 - 90}{2}\right) \sin (2 \times 20) - 30 \cos (2 \times 20) = -13.34 \text{ MPa}$$

(ii) (a) Max. principal stress, $\sigma_1 = \frac{1}{2}\left[(\sigma_x + \sigma_y) + \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}\right]$

$$\sigma_1 = \frac{1}{2}\left[(120 + 90) + \sqrt{(120 - 90)^2 + 4(30)^2}\right]$$

$$= 138.54 \text{ MPa}$$

(b) Min. principal stress, $\sigma_2 = \frac{1}{2}\left[(\sigma_x - \sigma_y) - \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}\right]$

$$\sigma_2 = \frac{1}{2}\left[(120 + 90) - \sqrt{(120 - 90)^2 + 4(30)^2}\right]$$

$$= 71.47 \text{ MPa}$$

(c) Angle or planes of principal stresses or principal planes,

$$\theta_{1,2} = \frac{1}{2}\tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{2\times 30}{120-90}\right) = 31.72°$$

$\therefore$ Plane of max. principal stress,

$$\theta_1 = 31.72°$$

Plane of min. principal stress,

$$\theta_2 = 90 + \theta_1 = 121.72°$$

(iii) (a) Max. shear stress, $\tau_{max} = \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$

$$\left(\text{or } \tau_{max} = \frac{\sigma_1 - \sigma_2}{2}\right)$$

$$\tau_{max} = \frac{1}{2}\sqrt{(120-90)^2 + 4(30)^2} = 33.53 \text{ MPa}$$

(b) Angle of max. shear stress,

$$\theta_\tau = \frac{1}{2}\tan^{-1}\left(\frac{\sigma_x - \sigma_y}{-2\tau_{xy}}\right)$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{120-90}{-2\times 30}\right)$$

$$\theta_\tau = -13.28°.$$

Problem 27: A machine member is subjected to the following stresses.

$\sigma_x = 90$ MPa and $\tau_{xy} = 25$ MPa. Determine:

(i) Normal and shear stresses along a plane inclined at 30°,

(ii) Principal stresses and max. shear stress.

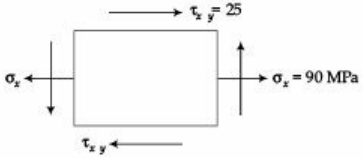


Fig. 1.32

Given data: $\sigma_x = 90$ MPa, $\tau_{xy} = 25$ MPa, $\sigma_y = 0$ (not given)

(i) (a) Normal stress on plane inclined at θ

$$\sigma_\theta = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{90}{2} + \frac{90}{2}\cos(2 \times 30) + 25\sin(2 \times 30)$$

$$= 89.15 \text{ MPa}$$

(b) Shear stress on plane inclined at θ

$$\tau_\theta = \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta - \tau_{xy}\cos 2\theta$$

$$= \frac{90}{2}\sin(2 \times 30) - 25\cos(2 \times 30)$$

$$= 26.47 \text{ MPa}$$

(ii) (a) Maximum principal stress,

$$\sigma_1 = \frac{1}{2}\left[\sigma_x + \sqrt{\sigma_x^2 + 4\tau_{xy}^2}\right]$$

$$= \frac{1}{2}\left[90 + \sqrt{90^2 + 4(25)^2}\right] = 96.48 \text{ MPa}$$

(b) Minimum principal stress,

$$\sigma_2 = \frac{1}{2}\left[\sigma_x - \sqrt{\sigma_x^2 + 4\tau_{xy}^2}\right]$$

$$= \frac{1}{2}\left[90 - \sqrt{90^2 + 4(25)^2}\right] = -6.48 \text{ MPa}$$

(c) Maximum shear stress,

$$\tau_{max} = \frac{1}{2}\sqrt{\sigma_x^2 + 4\tau_x^2}$$

$$= \frac{1}{2}\sqrt{90^2 + 4(25)^2} = 51.48 \text{ MPa.}$$

Problem 28: A machine member of diameter 60 mm, in the form of a cantilever and 300 mm long, carries an axial load of 18 kN and a transverse load of 3 kN at its free end. Determine the maximum and minimum principal stresses and maximum shear stress induced in the member.

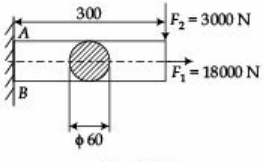


Fig. 1.33

Given data: Tensile force $F_1 = 18$ kN = 18000 N

Transverse force, $F_2 = 3$ kN $= 3000$ N

Consider two critical points A and B.

F_1 induces tensile stress in the c/s both at A and B.

$$\therefore \quad \text{Direct tensile stress, } \sigma_d = \frac{F_1}{A} = \frac{18000}{\frac{\pi}{4}d^2} = \frac{18000}{\left(\frac{\pi}{4}60^2\right)}$$

$$\sigma_d = 6.37 \text{ MPa (tensile both at } A \text{ \& } B).$$

The member is subjected to bending under the effect of transverse load F_2 (cantilever with end load). This induces tensile stress at A and compressive stress at B.

$$\text{Bending stress, } \sigma_B = \frac{M_b \cdot C}{I}$$

where

$$M_b = \text{bending moment} = F_2 \cdot l = 3000 \times 300$$

$$M_b = 900 \times 10^3 \text{ N-mm}$$

$$I = \text{moment of inertia} = \frac{\pi d^4}{64} = \frac{\pi \times 60^4}{64} = 636.17 \times 10^3 \text{ mm}^4$$

$$C = \frac{d}{2} = \frac{60}{2} = 30 \text{ mm}$$

$$\therefore \quad \sigma_B = \frac{900 \times 10^3 \times 30}{636.17 \times 10^3} = 42.44 \text{ MPa}$$

$$\therefore \quad \sigma_{B \text{ at } A} = 42.44 \text{ MPa (tensile)}$$

$$\sigma_{B \text{ at } B} = -42.44 \text{ MPa (compressive)}$$

For point *A*: Maximum principal stress,

$$\sigma_{max} = \text{combined stress at } A = \sigma_A + \sigma_B$$

$$= 6.37 + 42.44 = 48.81 \text{ MPa}$$

Minimum principal stress,

$$\sigma_{min} = 0$$

$\therefore$ Maximum shear stress,

$$\tau_{max} = \frac{\sigma_{max}}{2} = \frac{48.81}{2} = 24.405 \text{ MPa}$$

For point *B*: Maximum principal stress = 0

Minimum principal stress,

$$\sigma_{min} = \text{combined stress at } B = \sigma_A - \sigma_B$$

$$= 6.37 - 42.44 = -36.07 \text{ MPa}$$

$\therefore$ Maximum shear stress,

$$\tau_{max} = \frac{-36.07}{2} = -18.035 \text{ MPa.}$$

Problem 29: A machine member of dia 60 mm is subjected to a transverse load of 3 kN and a twisting moment of 1.2 kNm as shown in Fig. 1.34. Determine the principal stress and shear stress at the critical points.

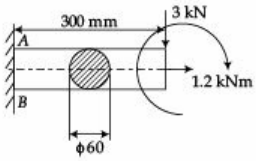


Fig. 1.34

Given data: $F = 3$ kN $= 3000$ N

Torque, $M_t = 1.2$ kNm $= 1.2 \times 10^6$ N-mm

Bending moment due to F

$$M_b = Fl = 3000 \times 300 = 900 \times 10^3 \text{ N-mm}$$

$$\text{Bending stress, } \sigma_x = \frac{M_b \cdot C}{I}$$

$$C = \frac{d}{2} = \frac{60}{2} = 30 \text{ mm}$$

$$I = \frac{\pi d^4}{64} = \frac{\pi \times 60^4}{64} = 636.17 \times 10^3 \text{ mm}^4$$

$$\therefore \quad \sigma_x = \frac{900 \times 10^3 \times 30}{636.17 \times 10^3} = 42.44 \text{ MPa}$$

(This is tensile at A and compressive at B)

$$\text{Shear stress, } \tau_{xy} = \frac{M_t r}{J}$$

$$r = \frac{d}{2} = \frac{60}{2} = 30 \text{ mm}$$

$$J = \frac{\pi d^4}{32} = \frac{\pi \times 60^4}{32} = 1.272 \times 10^6 \text{ mm}^4$$

$$\therefore \quad \tau_{xy} = \frac{1.2 \times 10^6 \times 30}{1.272 \times 10^6} = 28.29 \text{ MPa}$$

$$(\sigma_y = 0)$$

$\therefore$ Maximum principal stress at A

$$\sigma_1 = \frac{1}{2}\left[\sigma_x + \sqrt{\sigma_x^2 + 4\tau_{xy}^2}\right]$$

$$= \frac{1}{2}\left[42.44 + \sqrt{42.44^2 + 4(28.29)^2}\right]$$

$$= 56.58 \text{ MPa}$$

Maximum shear stress at A

$$\tau_{max} = \frac{1}{2}\sqrt{\sigma_x^2 + 4\tau_{xy}^2}$$

$$= \frac{1}{2}\sqrt{42.44^2 + 4(28.29)^2} = 35.36 \text{ MPa.}$$

Problem 30: Find the maximum normal and shear stresses at the critical section for a machine member diameter 60 mm subjected to a tensile load of 18 kN and a twisting moment of 1.2 kNm as shown in the figure 1.35.

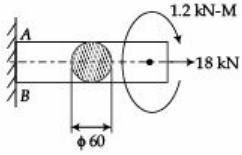


Fig. 1.35

Given data: Axial force, $F = 18$ kN $= 18000$ N

Torque, $M_t = 1.2$ kNm $= 1.2 \times 10^6$ N-mm

Dia, $d = 60$ mm

$$\text{Direct tensile stress, } \sigma_d = \sigma_x = \frac{F}{A} = \frac{18000}{\frac{\pi}{4}(60)^2} = 6.37 \text{ MPa}$$

This is tensile both at A and B.

Shear stress due to torque, $\tau = \tau_{xy} = \dfrac{M_t r}{J}$

$$\tau_{xy} = \frac{1.2 \times 10^6 \times \frac{60}{2}}{\frac{\pi}{32}(60)^4} = 28.29 \text{ MPa}$$

$\therefore$ Maximum principal stress at A,

$$\sigma_1 = \frac{1}{2}\left[\sigma_x + \sqrt{\sigma_x^2 + 4\tau_{xy}^2}\right]$$

$$= \frac{1}{2}\left[6.37 + \sqrt{6.37^2 + 4(28.29)^2}\right]$$

$$= 31.65 \text{ MPa}$$

Maximum shear stress,

$$\tau_{max} = \frac{1}{2}\sqrt{\sigma_x^2 + 4\tau_{xy}^2}$$

$$= \frac{1}{2}\sqrt{6.37^2 + 4(28.29)^2} = 28.46 \text{ MPa.}$$

Problem 31: A machine member 60 mm diameter is subjected an axial tensile load of 18 kN, a transverse load of 3 kN and a turning moment of 1.2 kNm as shown in Fig. 1.36. Determine the max. principal and shear stresses induced in the member.

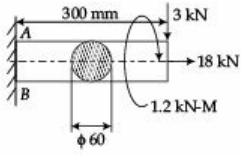


Fig. 1.36

Given data: Axial load, $F_1 = 18 \text{ kN} = 18000 \text{ N}$

Transverse load, $F_2 = 3 \text{ kN} = 3000 \text{ N}$

Torque, $M_t = 1.2 \text{ kNm} = 1.2 \times 10^6 \text{ N-mm}$.

Bending moment due to transverse load, $M_b = F_2 \cdot l$

$$M_b = 3000 \times 300 = 900 \times 10^3 \text{ N-mm}$$

Direct tensile stress, $\sigma_d = \dfrac{F_1}{A} = \dfrac{18000}{\left(\dfrac{\pi}{4}60^2\right)} = 6.37 \text{ MPa}$

This is tensile both A and B.

Bending stress due to transverse load, $\sigma_B = \dfrac{M_b \cdot C}{I}$

$$C = \frac{d}{2} = \frac{60}{2} = 30 \text{ mm}$$

$$I = \frac{\pi d^4}{64} = \frac{\pi \times 60^4}{64} = 636.17 \times 10^3 \text{ mm}^4$$

$\therefore$ $$\sigma_B = \frac{900 \times 10^3 \times 30}{636.17 \times 10^3} = 42.44 \text{ MPa}$$

This is tensile at A and compressive at B.

Total tensile stress at A.

$\therefore$ $$\sigma_A = \sigma_d + \sigma_B = 6.37 + 42.44 = \sigma_x$$

$$\sigma_x = 48.81 \text{ MPa}$$

Shear stress, $\tau = \tau_{xy} = \dfrac{M_t \cdot r}{J}$

$$r = \frac{d}{2} = \frac{60}{2} \text{ mm}, \qquad J = \frac{\pi d^4}{32} = \frac{\pi (60)^4}{32} \text{ mm}^4$$

$\therefore$ $$\tau_{xy} = \frac{1.2 \times 10^6 \times (60/2)}{\left(\dfrac{\pi (60)^4}{32}\right)} = 28.29 \text{ MPa}$$

∴ Maximum principal stress,

$$\sigma_1 = \frac{1}{2}\left[\sigma_x + \sqrt{\sigma_x^2 + 4\tau_{xy}^2}\right]$$

$$= \frac{1}{2}\left[48.81 + \sqrt{48.81^2 + 4(28.29)^2}\right]$$

$$\sigma_1 = 61.77 \text{ MPa}$$

Maximum shear stress, $\tau_{max} = \frac{1}{2}\sqrt{\sigma_x^2 + 4\tau_{xy}^2}$

$$= \frac{1}{2}\sqrt{48.81^2 + 4(28.29)^2} = 37.36 \text{ MPa.}$$

Problem 32: A machine member is subjected to a twisting moment of 1.5 kNm and a bending moment of 3 kNm. Find suitable dia of the shaft if the normal and shear stresses are 120 MPa and 75 MPa respectively.

Given data: Twisting moment, $M_t = 1.5$ kNm $= 1.5 \times 10^6$ N–mm

Bending moment, $M_b = 3$ kNm $= 3 \times 10^6$ N–mm

Allowable shear stress, $\tau_{max} = 75$ MPa

Allowable normal stress, $\sigma_{max} = 120$ MPa

Shear stress due torque, $\tau = \tau_{xy} = \frac{M_t r}{J} = \frac{1.5 \times 10^6 \times \frac{d}{2}}{\left(\frac{\pi d^4}{32}\right)}$

$$\tau_{xy} = \frac{7.639 \times 10^6}{d^3} \text{MPa}$$

Bending stress, $\sigma = \sigma_x = \frac{M_b \cdot C}{I}$

$$\sigma_x = \frac{3 \times 10^6 \times d/2}{(\pi d^4/64)} = \frac{30.56 \times 10^6}{d^3} \text{ MPa}$$

$$\sigma_y = 0 \text{ (Not given)}$$

∴ Max. normal stress, $\sigma_{max} = \frac{1}{2}\left[\sigma_x + \sqrt{\sigma_x^2 + 4\tau_{xy}^2}\right]$

$$120 = \frac{1}{2}\left[\frac{30.56 \times 10^6}{d^3} + \sqrt{\left(\frac{30.56 \times 10^6}{d^3}\right)^2 + 4\left(\frac{7.639 \times 10^6}{d^3}\right)^2}\right]$$

$$d = 64.61 \text{ mm}$$

$$\text{Max. shear stress, } \tau_{max} = \frac{1}{2}\sqrt{\sigma_x^2 + 4\tau_{xy}^2}$$

$$75 = \frac{1}{2}\sqrt{\left(\frac{30.56 \times 10^6}{d^3}\right)^2 + 4\left(\frac{7639 \times 10^6}{d^3}\right)^2}$$

$$d = 61.07 \text{ mm}$$

$\therefore$ Minimum dia required, $d = 64.61$ mm (bigger of the above two).

Problem 33: A round rod made of steel ($\sigma_y = 300$ MPa) is subjected to a tensile load of 30 kN and a bending moment of 1.8 kNm. Taking factor of safety as 2.5, determine the suitable diameter for the rod.

Given data: Axial load, $F = 30$ kN $= 30000$ N

Bending moment, $M_b = 1.8$ kNm $= 1.8 \times 10^6$ N mm

Yield stress, $\sigma_y = 300$ MPa. FOS $= 2.5$

$\therefore$ $$\text{Allowable tensile stress, } \sigma = \frac{\sigma_y}{\text{FOS}} = \frac{300}{2.5} = 120 \text{ MPa}$$

$$\text{Direct tensile stress, } \sigma_d = \frac{F}{A} = \frac{30000}{\left(\frac{\pi}{4}d^2\right)} = \frac{38.2 \times 10^3}{d^2}$$

$$\text{Bending stress, } \sigma_B = \frac{M_b C}{I} = \frac{1.8 \times 10^6 \times \frac{d}{2}}{\left(\frac{\pi d^4}{64}\right)} = \frac{18.335 \times 10^6}{d^3}$$

$\therefore$ $$\text{Combined stress, } \sigma = \sigma_d + \sigma_B = \frac{38.2 \times 10^3}{d^2} + \frac{18.335 \times 10^6}{d^3}$$

$$\text{Equating this to allowable stress, } \sigma = 120 = \frac{38.2 \times 10^3}{d^2} + \frac{18.335 \times 10^6}{d^3}$$

$$120d^3 - 38.2 \times 10^3 d - 18.335 \times 10^6 = 0$$

Solving, $d = 55.44$ mm $\simeq 56$ mm.

Problem 34: Determine normal and shear stresses induced at section *AA* when a load of 12 kN is applied at the centre of crank pin as shown in Fig. 1.37.

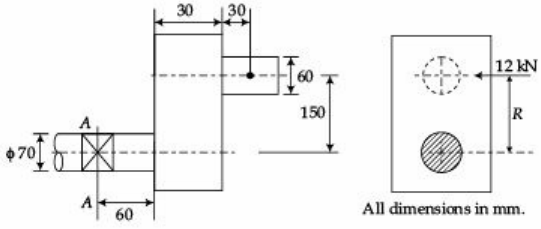


Fig. 1.37

Given data: Load, $F = 12 \text{ kN} = 12 \times 10^3 \text{ N}$

Dia of the shaft at critical section, $d = 70$ mm

The given load acting on crank pin has two effects at section *AA*.

(i) Bending stress due to cantilever subjected to bending moment = $F \cdot l$

where l = distance from section *AA* to load

$= 60 + 30 + 30 = 120$ mm

$F = 12000$ N

$\therefore \quad M_b = F \cdot l = 12000 \times 120 = 1.44 \times 10^6$ N-mm

$\therefore$ Bending stress, $\sigma_b = \dfrac{M_b \cdot C}{I} = \dfrac{1.44 \times 10^6 \times d/2}{\left(\dfrac{\pi d^4}{64}\right)}$

$$\sigma_b = \frac{1.44 \times 10^6 \times 70/2}{\left(\dfrac{\pi}{64} 70^4\right)} = 42.76 \text{ MPa} = \sigma_x$$

(ii) Shear stress due to torque, $M_t = F.\ R$

where R = radius at which F is acting = 150 mm

$M_t = 12000 \times 150 = 1.8 \times 10^6$ Nm

$$r = \frac{d}{2} = \frac{70}{2}$$

$$J = \frac{\pi d^4}{32} = \frac{\pi 70^4}{32}$$

$\therefore$

$$\tau = \frac{1.8 \times 10^6 \times 70/2}{\dfrac{\pi}{32} 70^4} = 26.73 \text{ MPa} = \tau_{xy}$$

(iii) Combined or principal stress, $\sigma_{max} = \frac{1}{2}\left[\sigma_x + \sqrt{\sigma_x^2 + 4\tau_{xy}^2}\right]$

$$\sigma_{max} = \frac{1}{2}\left[42.76 + \sqrt{42.76^2 + 4 \times 26.73^2}\right] = 55.61 \text{ MPa}$$

Max shear stress, $\tau_{max} = \frac{1}{2}\sqrt{\sigma_x^2 + 4\tau_{xy}^2}$

$$\tau_{max} = \frac{1}{2}\sqrt{42.76^2 + 4 \times 26.73^2} = 34.23 \text{ MPa.}$$

Problem 35: Determine the suitable diameter for the main bearing of a side crank subjected to a load of 15 kN as shown in the Fig. 1.38. The material has an ultimate tensile stress of 540 MPa. Take factor of safety as 5.

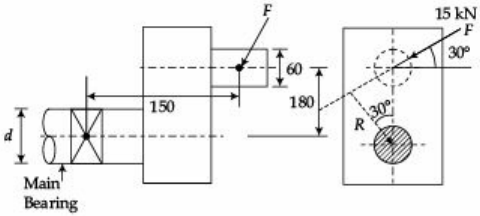


Fig. 1.38

Given data: Load, $F = 15 \text{ kN} = 15 \times 10^3 \text{ N}$

Ultimate stress, $\sigma_u = 540 \text{ MPa}$

Bending arm, $l = 150 \text{ mm}$

$\therefore$ Bending moment, $M_b = F \cdot l$

$$M_b = 15 \times 10^3 \times 150 = 2.25\ 10^6 \text{ N mm}$$

$\therefore$ Bending stress, $\sigma_b = \sigma_x = \frac{M_b \cdot C}{I}$

$$\sigma_b = \frac{2.25 \times 10^6 \times d/2}{\frac{\pi}{64}d^4} = \frac{22.92 \times 10^6}{d^3} = \sigma_x$$

Torque, $M_t = F \cdot R$

when $R = \perp lr$ distance from axis to force = 150 cos 30

$R = 129.904 \text{ mm}$

$\therefore$ $M_t = 15 \times 10^3 \times 129.904 = 1.949 \times 10^6 \text{ N-mm}$

$$\therefore \quad \text{Shear stress, } \tau = \frac{M_t r}{J} = \frac{1.949 \times 10^6 \times d/2}{\left(\frac{\pi}{32} d^4\right)} = \frac{9.924 \times 10^6}{d^3} = \tau_{xy}$$

$$\therefore \quad \text{Given ultimate stress, } \sigma_u = 540 \text{ MPa and FOS} = 5$$

$$\therefore \quad \text{Allowable tensile stress, } \sigma = \frac{\sigma_u}{\text{FOS}} = \frac{540}{5} = 108 \text{ MPa}$$

Assume allowable shear stress, $\tau_{max} = 0.5\sigma = 0.5 \times 108 = 54$ MPa

$$\text{Combined stress, } \sigma_{max} = \frac{1}{2}\left[\sigma_x + \sqrt{\sigma_x^2 + 4\tau_{xy}^2}\right]$$

$$108 = \frac{1}{2}\left[\frac{22.92 \times 10^6}{d^3} + \sqrt{\left(\frac{22.92 \times 10^6}{d^3}\right)^2 + 4\left(\frac{9.924 \times 10^6}{d^3}\right)^2}\right]$$

$$\therefore \quad d = 62.699 \simeq 63 \text{ mm}$$

$$\text{Max shear stress, } \tau_{max} = \frac{1}{2}\sqrt{\sigma_x^2 + 4\tau_{xy}^2}$$

$$54 = \frac{1}{2}\sqrt{\left(\frac{22.92 \times 10^6}{d^3}\right)^2 + 4\left(\frac{9.924 \times 10^6}{d^3}\right)^2}$$

$$d = 65.48 \text{ mm}$$

Recommended dia, $d = 65.48 = 66$ mm.

1.12 ECCENTRIC LOADING

The loads acting along c.g. axis are known as axial loads (tensile or compressive). If the load acts away from c.g. axis and parallel to it is known as eccentric loading.

Figure 1.39 shows some examples of eccentric loading.

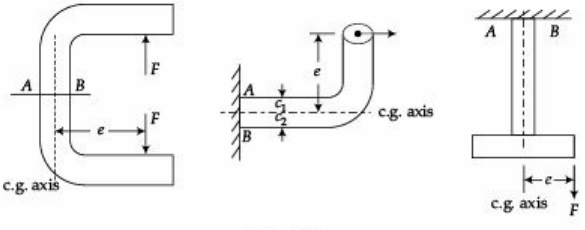


Fig. 1.39

The distance from c.g. axis to load is called eccentricity.
Any eccentric load has two effects as shown in Fig. 1.40.

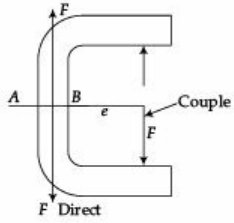
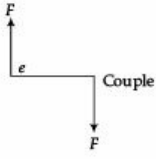


Fig. 1.40

Introduce equal and opposite load F along c.g. axis as shown in Fig. 1.40.

(i) Direct load F induces tensile stress at both A and B

$$\sigma_d = \frac{F}{A} \text{ (tensile at both } A \text{ and } B)$$



(ii) Bending stress due to couple

$$\sigma_b = \frac{M_b \cdot C}{I}$$

where M_b = bending moment = couple = $F \times e$

C = distance from c.g. axis to outer fibre

I = moment of Inertia of c/s

This induces tensile stress at A and compressive stress at B.

$\therefore$ Combined stress at A $\sigma_A = \sigma_d + \sigma_b$

$$\sigma_A = \frac{F}{A} + \frac{M_b \cdot C}{I} = \frac{F}{A} + \frac{M_b}{I/C} \text{ (Tensile)}$$

or $$\sigma_A = \frac{F}{A} + \frac{M_b}{Z}$$

where $$Z = \frac{I}{C} = \text{section modulus}$$

(iii) By combined stress at B $\sigma_B = \sigma_d - \sigma_b$

$$= \frac{F}{A} - \frac{M_b C}{I} = \frac{F}{A} - \frac{M_b}{(I/C)} = \frac{F}{A} - \frac{M_b}{Z} \text{ (compressive)}$$

For symmetric sections, distance from c.g. to outer fibre is same i.e., $C_1 = C_2 = C$.
For unsymmetric sections, C will be different for tensile side to compressive side.

Problems on Eccentric Loading

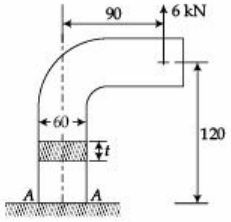


Fig. 1.41

Problem 36: Determine the thickness of the steel bracket loaded as shown in Fig. 1.41 taking the allowable stress as 90 MPa.

Given data: Load, $F = 6$ kN $= 6000$ N

Eccentricity = distance from c.g. to force, $e = 90$ mm

Allowable stress, $\sigma = 90$ MPa

This is an eccentric load that acts parallel to and away from c.g. axis. This has two effects: direct tensile stress and bending stress due to couple

Direct tensile stress, $\sigma_d = \dfrac{F}{A}$

where A = area of $c/s = 60t$

$$\sigma_d = \frac{6000}{60t} = \frac{100}{t}.$$ This is tensile at both sides.

Bending stress due to couple, $\sigma_b = \dfrac{M_b \cdot C}{I}$

where $M_b = F \cdot e = 6000 \times 90 = 540 \times 10^3$ N mm

C = distance from c.g. to outer edge $= \dfrac{60}{2} = 30$ mm

I = moment of tnertia $= \dfrac{tb^3}{12} = \dfrac{t \times 60^3}{12} = 18000t$

$\therefore \quad \sigma_b = \dfrac{540 \times 10^3 \times 30}{(18000t)} = \dfrac{900}{t}$

$\therefore$ Combined stress, $\sigma = \sigma_d + \sigma_b$

$$90 = \frac{100}{t} + \frac{900}{t} \Rightarrow t = \frac{1000}{90} = 11.11 \text{ mm}$$

$\therefore$ Thickness, $t = 12$ mm.

Problem 37: Determine the stresses at A and for a clamp loaded as shown in Fig. 1.42.

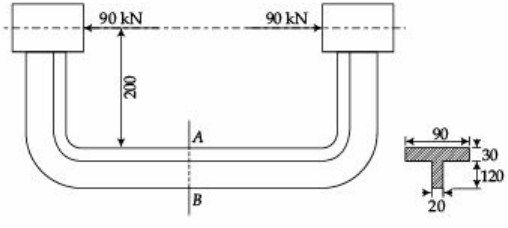


Fig. 1.42

Given data: Load $F = 90\ \text{kN} = 90 \times 10^3\ \text{N}$

This is an eccentric load that acts parallel to and away from c.g. axis:

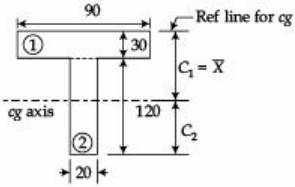
This has two effects: Direct tensile stress and bending stress due to couple which is tensile at A and compressive at B.

(i) Direct tensile stress, $\sigma_d = \dfrac{F}{A}$

A = area of c/s = $A_1 + A_2 = 90 \times 30 + 120 \times 20$

$\therefore \quad \sigma_d = \dfrac{90000}{5100} = 17.65$ MPa Tensile both A & B

(ii) Bending stress due to couple, $\sigma_b = \dfrac{M_b \cdot C}{I}$



C = distance from c.g. axis to outer fibre

= C_1 for tensile stress at 'A'

= C_2 for compressive stress at 'B'

I = moment of inertia = $I_1 + I_2$ about c.g. axis

First find c.g.:

$$C_1 = \bar{X} = \frac{A_1X_1 + A_2X_2}{A_1 + A_2} = \frac{(90 \times 30) \times \dfrac{30}{2} + (120 \times 20)\left(30 + \dfrac{120}{2}\right)}{90 \times 30 + 120 \times 20}$$

$C_1 = \bar{X} = 50.294$ mm

$\therefore \quad C_2 = (30 + 120) - C_1 = 99.706$ mm

Moment of inertia of the section about c.g. axis

$I = I_1 + I_2$

where I_1 = MI of Fig. ① about c.g. axis

$= I_{cg} + A_1 K_1^2$ (Using parallel axis theorem)

$$= \frac{bd^3}{12} + bd(\bar{X} - X_1)^2$$

$$= \frac{90 \times 30^3}{12} + 90 \times 30\left(50.294 - \frac{30}{2}\right)^2$$

$I_1 = 3.566 \times 10^6\ \text{mm}^4$

(iii) By I_2 = MI of Fig. ② about c.g. axis

$$= I_{cg} + A_2K_2^2 = \frac{bd^3}{12} + bd(\bar{X} - X_2)^2$$

$$I_2 = \frac{20 \times 120^3}{12} + 20 \times 120\ (50.294 - 90)^2 = 6.664 \times 10^6\ \text{mm}^4$$

('b' is the side parallel to c.g. axis and 'd' is the side of rectangle $\perp lr$ to c.g. axis; $x_2 = 30 + 120/2 = 90$ mm)

$\therefore$ $$I = I_1 + I_2 = 3.566 \times 10^6 + 6.664 \times 10^6 = 10.23 \times 10^6\ \text{mm}^4$$

Bending moment, $M_b = F.e$

e = eccentricity = distance from c.g. to load

$e = C_1 + 200$ (from Fig.)

$e = 50.294 + 200 = 250.294$ mm

$\therefore$ $$M_b = 90 \times 10^3 \times 250.294 = 22.53 \times 10^6\ \text{N-mm}$$

$\therefore$ Bending stress at A (tensile) $\sigma_{bA} = +\dfrac{M_b C_1}{I}$

$$\sigma_{bA} = +\frac{22.53 \times 10^6 \times 50.294}{10.23 \times 10^6} = +110.76\ \text{MPa}$$

and bending stress at B (compressive) $\sigma_{bB} = -\dfrac{M_b C_2}{I}$

$$\sigma_{bB} = -\frac{22.53 \times 10^6 \times 99.706}{10.23 \times 10^6} = -219.59\ \text{MPa}$$

$\therefore$ Combined stress at A, $\sigma_A = \sigma_d + \sigma_{bA}$

$\sigma_A = 17.65 + 110.76 = 128.41$ MPa (tensile)

Combined stress at B, $\sigma_B = \sigma_d + \sigma_{bB}$

$\sigma_B = 17.65 - 219.59 = -201.94$ MPa (compressive)

Problem 38: Determine the dimensions of the rectangular cross section whose depth is twice the width for a C-clamp frame made of C.I. and loaded as shown in the Figure 1.43. The ultimate stress for C.I. used for frame is 125.6 MPa. Take factor of safety as 4.

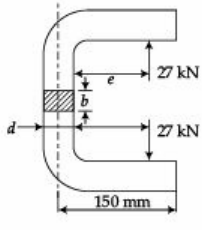


Fig. 1.43

Given data: Load, $F = 27$ kN $= 27 \times 10^3$ N

Ultimate stress, $\sigma_u = 125.6$ MPa

FOS = 4

$\therefore$ Allowable stress, $\sigma = \dfrac{\sigma_u}{\text{FOS}} = \dfrac{125.6}{4} = 31.4$ MPa

depth, $d = 2b$.

This is an eccentric load that acts away from c.g. axis and parallel to it. This has two effects, direct tensile stress and bending stress due to couple.

(i) Direct tensile stress, $\sigma_d = \frac{F}{A} = \frac{27 \times 10^3}{b \times d}$

$$\sigma_d = \frac{27 \times 10^3}{(b \times 2b)} = \frac{13.5 \times 10^3}{b^2} \quad ...(1)$$

(ii) Bending stress due to couple, $\sigma_b = \frac{M_b C}{I}$

where M_b = bending moment = couple = $F \cdot e$

$M_b = 27 \times 10^3 \times 150 = 4.05 \times 10^6$ N-mm

$$C = \frac{d}{2} = \frac{2b}{2} = b$$

$$I = \frac{bd^3}{12} \quad \{b = \text{side is parallel to } cg \text{ axis}, \quad d = \text{side} \perp lr \text{ to c.g. axis}\}$$

$$= \frac{b(2b)^3}{12} = \frac{b \cdot (8b^3)}{12} = 0.667b^4$$

$\therefore$ $$\sigma_b = \frac{4.05 \times 10^6 \times b}{0.667b^4} = \frac{6.075 \times 10^6}{b^3} \quad ...(2)$$

$\therefore$ Combined stress, $\sigma = \sigma_d + \sigma_b$

$$31.4 = \frac{13.5 \times 10^3}{b^2} + \frac{6.075 \times 10^6}{b^3}$$

$$31.4\, b^3 - 13.5 \times 10^3 b - 6.075 \times 10^6 = 0$$

Solving $b = 60.31$ mm $\simeq 61$ mm

$\therefore$ $d = 2b = 2 \times 61 = 122$ mm.

Problem 39: A 45 mm dia steal rod carries a 12 kN load and a twisting moment of 180 Nm as shown in Fig. 1.44. Determine the maximum tensile and maximum shear stress induced in it.

Given data: Load, $F = 12$ kN $= 12 \times 10^3$ N

Torque, $M_t = 180$ Nm $= 180 \times 10^3$ N mm

Dia of rod $= d = 45$ mm

Eccentricity, $e = 36$ mm

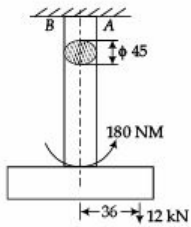


Fig. 1.44

(i) Shear stress due to torque, $\tau = \frac{M_t \cdot r}{J}$

$$\tau = \frac{180 \times 10^3 \times 45/2}{(\pi/32 \times 45^4)}$$

$$\tau = \tau_{xy} = 10.06 \text{ MPa}$$

(ii) The load F is eccentric load that acts away from c.g. axis and parallel to it. This has two effects.

(a) Direct tensile stress, $\sigma_d = \frac{F}{A} = \frac{12 \times 10^3}{\pi/4 \times 45^2} = 7.55$ MPa, and

(b) Bending stress due to couple, $\sigma_b = \frac{M_b C}{I}$

where M_b = bending moment = couple = $F \cdot e$

$M_b = 12 \times 10^3 \times 36 = 432 \times 10^3$ N-mm

$$c = \frac{d}{2} = \frac{45}{2} \quad \text{and} \quad I = \frac{\pi d^4}{64} = \frac{\pi \times 45^4}{64}$$

$$\therefore \quad \sigma_b = \frac{432 \times 10^3 \times (45/2)}{(\pi/64) \times 45^4} = 48.29 \text{ MPa}$$

This is tensile at A and compressive at B.

$\therefore$ Combined or maximum stress at A due to load F

$$\sigma = \sigma_d + \sigma_b = 7.55 + 48.29 = 55.84 \text{ MPa}$$

$$\sigma = \sigma_x = 55.84 \text{ MPa}$$

$\therefore$ Max. tensile stress = principal stress at A

$$\sigma_1 = \frac{1}{2}\left[\sigma_x + \sqrt{\sigma_x^2 + 4\tau_{xy}^2}\right]$$

$$= \frac{1}{2}\left[55.84 + \sqrt{55.84^2 + 4 \times 10.06^2}\right] = 57.6 \text{ MPa}$$

Maximum shear stress, $\tau_{max} = \frac{1}{2}\sqrt{\sigma_x^2 + 4\tau_{xy}^2}$

$$\tau_{max} = \frac{1}{2}\sqrt{55.84^2 + 4 \times 10.06^2} = 29.68 \text{ MPa.}$$

Problem 40: Determine the maximum tensile and compressive stresses induced in the link loaded as shown in Fig. 1.45.

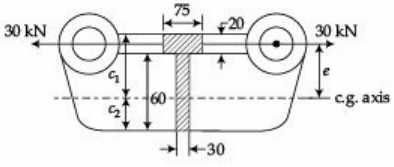


Fig. 1.45

Given data:

The given load is eccentric in nature that acts away from c.g. axis and parallel to it. This has two effects, direct tensile stress and bending stress due to couple.

(i) Direct tensile stress, $\sigma_d = \frac{F}{A}$, $F = 30 \text{ kN} = 30 \times 10^3 \text{ N}$

A = total area of T c/s = $A_1 + A_2$

$A = 75 \times 20 + 60 \times 30 = 3300 \text{ mm}^2$

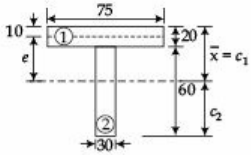
$$\therefore \quad \sigma_d = \frac{30 \times 10^3}{3300} = 9.09 \text{ MPa}$$

(ii) Bending stress due to couple, $\sigma_b = \frac{M_b \cdot C}{I}$

where M_b = bending moment = couple = $F \cdot e$

e = distance from c.g. to load.

$$= \bar{X} - \frac{20}{2}$$



where $\bar{X} = c_1 = \frac{A_1X_1 + A_2X_2}{A_1 + A_2}$

$$= \frac{(75 \times 20)\frac{20}{2} + (60 \times 30)\left(20 + \frac{60}{2}\right)}{75 \times 20 + 60 \times 30} = 31.818 \text{ mm}$$

$$\therefore \quad c_2 = (20 + 60) - c_1 = 48.812 \text{ mm}$$

$$\therefore \quad e = 31.818 - 10 = 21.818 \text{ mm}$$

$$\therefore \quad M_b = 30 \times 10^3 \times 21.818 = 654.55 \times 10^3 \text{ N-mm}$$

Moment of inertia $I = I_1 + I_2$

$$I_1 = I_{cg} + A_1k_1^2 = \frac{75 \times 20^3}{12} + (75 \times 20)\,(31.818 - 10)^2$$

$(k_1 = \bar{X} - X_1 = 31.818 - 20/2 = 21.818)$

$$I_1 = 764.04 \times 10^3 \text{ mm}^4$$

$$I_2 = I_{cg} + A_2k_2^2 \text{ where } k_2 = X_2 - \bar{X} = (31.818 - 50)$$

$$= \frac{30 \times 60^3}{12} + 30 \times 60\,(31.818 - 50)^2 = 1.135 \times 10^6 \text{ mm}^4$$

$$\therefore \quad I = I_1 + I_2 = 1.9 \times 10^6 \text{ mm}^4$$

$C = C_1$ for tensile stress and $C = C_2$ for compressive stress.

$$\therefore \quad \sigma_b \text{ tensile} = \frac{M_b \cdot C_1}{I} = \frac{654.55 \times 10^3 \times 31.818}{1.9 \times 10^6} = 10.96 \text{ MPa}$$

$$\sigma_b \text{ compressive} = \frac{-M_b \cdot C_2}{I} = \frac{-654.55 \times 10^3 \times 48.812}{1.9 \times 10^6} = -16.6 \text{ MPa}$$

$\therefore$ Combined stress at inside, $\sigma_i = \sigma_d + \sigma_{bt}$

$$\sigma_i = 9.09 + 10.96 = 20.05 \text{ MPa (tensile)}$$

Combined stress at outside, $\sigma_0 = \sigma_d + \sigma_{bc}$

$$\sigma_0 = 9.09 - 16.6 = -7.51 \text{ MPa (compressive)}$$

Problem 41: Figure 1.46 shows a wall bracket subjected to a load of 36 kN. Find the width and depth of the rectangular cross section, taking depth as thrice the width. Take $\sigma_y = 328.6$ MPa and adopt a factor of safety of 2.5.

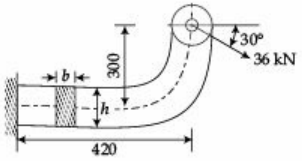
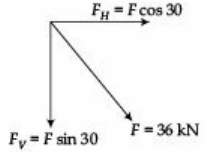


Fig. 1.46

Given data: Yield stress, $\sigma_y = 328.6$ mpa

FOS = 2.5



$\therefore$ Allowable stress, $\sigma = \dfrac{\sigma_y}{\text{FOS}}$

$$\sigma = \frac{328.6}{2.5} = 131.44 \text{ MPa}$$

Depth, $h = 3b$

Resolve the inclined load into components, horizontal component $F_H = F \cos 30$

$$F_H = 36 \times 10^3 \times \cos 30 = 31.177 \times 10^3 \text{ N}$$

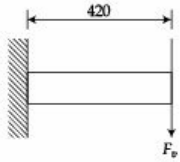
Vertical component $F_v = 30 \times 10^3 \times \sin 30 = 18 \times 10^3$ N

(a) Consider the vertical component.

The beam is like a cantilever subjected to bending. WKT, bending stress $\sigma_b = \sigma_1 = \dfrac{M_b \cdot C}{I}$

where, M_b = bending moment = $F_v \times 420 = 18 \times 10^3 \times 420$

$M_b = 7.56 \times 10^6$ N-mm.



$$c = \frac{h}{2} = \frac{3b}{2} = 1.5b$$

$$I = \frac{bh^3}{12} = \frac{b(3b)^3}{12} = \frac{27b^4}{12} = 2.25b^4$$

$$\therefore \quad \sigma_1 = \frac{7.56 \times 10^6 \times 1.5b}{2.25b^4} = \frac{5.04 \times 10^6}{b^3} \text{ MPa} \qquad ...(1)$$

(b) Consider horizontal component. This is an eccentric load that acts away from c.g. axis and parallel to it. This has two effects,

(i) Direct tensile stress, $\sigma_2 = \frac{F_H}{A} = \frac{31.177 \times 10^3}{bh}$

$$\sigma_2 = \frac{31.177 \times 10^3}{b \times 3b} = \frac{10.392 \times 10^3}{b^2} \text{ MPa} \qquad ...(2)$$

(ii) Bending stress due to couple, $\sigma_3 = \frac{M_b \cdot C}{I}$

where M_b = bending moment = couple due to F_H

$= F_H \times 300 = 31.177 \times 10^3 \times 300 = 9.35 \times 10^6$ N-mm

$$\therefore \quad \sigma_3 = \frac{9.35 \times 10^6 \times 1.5b}{2.25b^4} = \frac{6.235 \times 10^6}{b^3} \text{ MPa} \qquad ...(3)$$

$\therefore$ Combined stress, $\sigma = \sigma_1 + \sigma_2 + \sigma_3$

Equating this to allowable stress

$$131.44 = \frac{5.04 \times 10^6}{b^3} + \frac{10.392 \times 10^3}{b^2} + \frac{6.235 \times 10^6}{b^3}$$

$$131.44b^3 - 10.392 \times 10^3 b - 11.275 \times 10^6 = 0$$

Solving, $b = 44.7$ mm $\simeq 45$ mm

$$h = 3b = 3 \times 45 \text{ mm}$$

EXERCISE

1. Figure 1.47 shows a crank. Determine the maximum shear stress induced at *AA* and also in parts 1, 2 and 3.

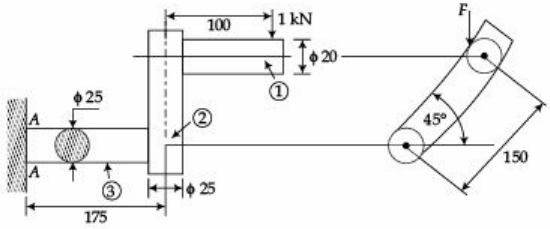


Fig. 1.47

2. A cantilever with rectangular c/s of 150 mm × 100 mm is loaded as shown in Fig. 1.48. Determine the maximum principal and shear stresses at point *A*.

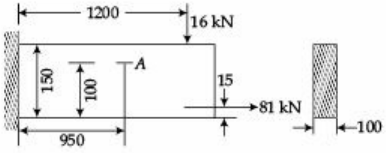


Fig. 1.48

3. A steel member is loaded as shown in Fig. 1.49. Determine the cross section dimensions of the rectangle, if the allowable stress is 120 MPa. The depth may be taken as thrice the width.

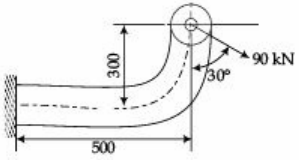


Fig. 1.49

4. Determine the dia of rod for the state of loads shown in Fig. 1.50. The allowable normal and shear stress are 120 MPa and 90 MPa respectively.

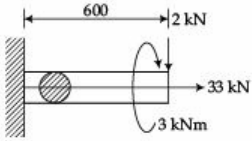


Fig. 1.50

5. Figure 1.51 shows an element subjected to stresses. Determine the following:
   (i) Normal and shear stresses along a plane inclined at 30°.
   (ii) Principal stresses and their location
   (iii) Maximum shear stress and its location.

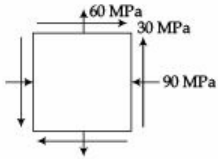


Fig. 1.51

CHAPTER

2 Theories of Failure

In the previous chapter, we have seen that a member is subjected to any of the simple stresses—tensile, compressive, shear or bending stress—then it is easy to predict the failure of the member. But in practice machine members will be subjected to more than one type of stress simultaneously and hence it will be difficult to predict the failure of such machine members using these simple stress theories.

In order to predict the failure of such members subjected to combined stresses, the following theories of failure are being suggested by different people:

(i) Rankine's theory or maximum normal stress theory.
(ii) Guest's theory or maximum shear stress theory.
(iii) Hencky-Von-Mises theory or distortion energy theory or shear energy theory.
(iv) Saint Venant theory or maximum strain theory.

BIAXIAL STRESSES WITH SHEAR STRESS

2.1 RANKINE'S THEORY OR MAXIMUM NORMAL STRESS THEORY

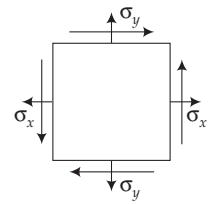


Fig. 2.1

Figure 2.1 shows an element subjected to stresses, σ_x action along x-direction (tensile or compressive), σ_y acting along y-direction $\perp lr$ to x (tensile or compressive) combined with shear stress, τ_{xy}.

According to the maximum normal stress theory or Rankine's theory of failure, equivalent stress

$$\sigma_e = \frac{1}{2}\left[(\sigma_x + \sigma_y) + \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}\right] \quad \ldots\left(\frac{5-20}{\text{P } 5.8}\right)$$

If $\sigma_e \le \sigma_{allowable}$, then the member is safe.

$\sigma_e \le \dfrac{\sigma_{ys}}{n}$, where n = factor of safety and σ_{ys} = yield stress in tension.

If $\sigma_e > \sigma_{ys}$, then the member yields
If $\sigma_e > \sigma_u$, then the member fails in fracture.

2.2 GUEST'S THEORY OR MAXIMUM SHEAR STRESS THEORY OF FAILURE

According to this theory, the member fails if the equivalent shear stress (τ_e) exceeds the torsional yield stress

$$\text{Equivalent shear stress, } \tau_e = \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

(or) $$\sigma_e = \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \quad \left(\text{considering } \tau_e = \frac{\sigma_e}{2}\right) \quad \text{...}\left(\frac{5-21}{P\,5.8}\right)$$

$$\text{For safety, } \tau_e \le \frac{\tau_{ys}}{n}$$

where τ_y = torsional yield stress $\simeq \dfrac{\sigma_y}{2}$

$\therefore$ If $\tau_e \le \dfrac{\sigma_{ys}}{2n}$, then the member is safe or $\sigma_e \le \dfrac{\sigma_y}{n}$

If $\tau_e > \dfrac{\sigma_{ys}}{2}$, then the member yields and if $\tau_e > \tau_u$, then the member fails in fracture.

2.3 HENCKY-VON-MISES THEORY OR DISTORTION ENERGY THEORY OR SHEAR ENERGY THEORY

According to this theory, the member fails if the equivalent stress (σ_e) exceeds the normal yield stress.

$$\text{Equivalent stress, } \sigma_e = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2} \quad \text{...(5-22)}$$

For safety $$\sigma_e \le \frac{\sigma_{ys}}{n}$$

If $\sigma_e > \sigma_{ys}$, then the member yields and
if $\sigma_e > \sigma_u$, then the member fails in fracture.

2.4 SAINT VENANT THEORY OR MAXIMUM STRAIN THEORY OF FAILURE

According to this theory, the member fails it the equivalent stress (σ_e) exceeds the normal yield stress.

$$\text{Equivalent stress, } \sigma_e = \frac{1}{2}\left[(1-v)(\sigma_x+\sigma_y)+(1+v)\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}\right]$$

v = Poisson ratio ($\simeq$ 0.3 for steels) ...$\left(\frac{5-23}{P5.8}\right)$

If $\sigma_e \le \frac{\sigma_{ys}}{n}$, then the member is safe.

If $\sigma_e > \sigma_{ys}$, then the member yields.

If $\sigma_e > \sigma_u$, then the member fails in fracture.

2.5 THEORIES OF FAILURE FOR TRIAXIAL STRESSES

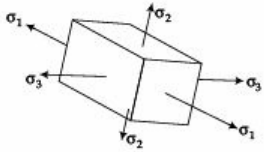


Fig. 2.2

Figure 2.2 shows an element subjected to triaxial stresses. Arrange them in descending order, $\sigma_1 > \sigma_2 > \sigma_3$

(a) Maximum normal stress theory states that the failure occurs

if $\sigma_1 > \sigma_{ys}$ or for safety, $\sigma_1 = \frac{\sigma_{ys}}{FOS}$ ...$\left(\frac{2-43}{P2.7}\right)$

(b) Maximum shear stress theory:

Shear stress, $\tau_{12} = \frac{\sigma_1-\sigma_2}{2}$

$$\tau_{23} = \frac{\sigma_2-\sigma_3}{2} \quad \text{and} \quad \tau_{13} = \frac{\sigma_1-\sigma_3}{2} \qquad ...(2\text{-}44)$$

then, τ_{max} = maximum of τ_{12}, τ_{23} or τ_{13}.

for safety, $\tau_{max} = \frac{\tau_y}{FOS} = \frac{\sigma_{ys}}{2FOS}$

(c) Shear energy theory or Hencky-Von-Mises theory

$$\sigma_e = \sqrt{\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}}$$

for safety, $\sigma_e = \frac{\sigma_{ys}}{FOS}$

(d) Octahedral shear stress theory

$$\tau_e = \frac{1}{3}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2} \qquad ...(2\text{-}46a)$$

for safety, $\tau_e = \frac{\tau_y}{FOS} = \frac{\sigma_{ys}}{2FOS}$

PROBLEMS

Problem 1: A machine element is subjected to the following stresses σ_x = 60 MPa, σ_y = 45 MPa, τ_{xy} = 30 MPa. Find the factor of safety if it is made of C45 steal having yield stress as 353 MPa, using the following theories of failure.

(i) Maximum principal stress theory,
(ii) Maximum shear stress theory,
(iii) Shear energy theory, and
(iv) Maximum strain theory taking Poisson ratio as 0.3

Given data: σ_x = 60 MPa, σ_y = 45 MPa, τ_{xy} = 30 MPa yield stress, σ_{ys} = 353 MPa
Poisson ratio v = 0.3.

(i) According to maximum principal stress or Rankine's theory of equivalent stress

$$\sigma_e = \frac{1}{2} = \left[(\sigma_x + \sigma_y) + \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}\right] \quad ...(5\text{-}20)$$

$$\sigma_e = \frac{1}{2} = \left[(60 + 45) + \sqrt{(60 - 45)^2 + 4(30)^2}\right] = 83.42 \text{ MPa}$$

$$\therefore \quad \text{FOS} = \frac{\sigma_{ys}}{\sigma_e} = \frac{353}{83.42} = 4.23$$

(ii) According to max. shear stress theory or Guest's theory equivalent stress

$$\tau_e = \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \quad ...(5\text{-}21)$$

or
$$\sigma_e = \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \qquad \left(\because \ \tau_e = \frac{\sigma_e}{2}\right)$$

$$= \sqrt{(60 - 45)^2 + 4(30)^2} = 61.85 \text{ MPa}$$

$$\therefore \quad \text{FOS} = \frac{\sigma_{ys}}{\sigma_e} = 353/61.85 = 5.71$$

(iii) According to shear energy theory or Hencky-Von-Mises theory, equivalent stress

$$\sigma_e = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2} \quad ...(5\text{-}22)$$

$$\sigma_e = \sqrt{60^2 + 45^2 - 60 \times 45 + 3 \times 30^2} = 75 \text{ MPa}$$

$$\therefore \quad \text{FOS} = \frac{\sigma_{ys}}{\sigma_e} = \frac{353}{75} = 4.71$$

(iv) According to Max-Strain theory or Saint-Venant theory. Equivalent stress

$$\sigma_e = \frac{1}{2}\left[(1-v)(\sigma_x+\sigma_y) + (1+v)\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}\right] \quad \text{...(5-23)}$$

$$\sigma_e = \frac{1}{2}\left[(1-0.3)(60+45) + (1+0.3)\sqrt{(60-45)^2+4(30)^2}\right]$$

$$= 76.95 \text{ MPa}$$

$$\therefore \quad \text{FOS} = \frac{\sigma_{ys}}{\sigma_e} = \frac{353}{76.95} = 4.59.$$

Problem 2: A M.S. shaft having yield stress as 232 MPa is subjected to the following stresses. σ_x = 120 MPa, σ_y = –60 MPa and τ_{xy} = 36 MPa. Find the factor of safety using:

(i) Rankine's theory of failure,

(ii) Guest's theory of failure and

(iii) Von-Mises theory of failure.

Given data: Yield stress, σ_{ys} = 232 MPa

σ_x = 120 MPa, σ_y = –60 MPa and τ_{xy} = 36 MPa.

According to Rankine's theory or maximum normal stress theory of failure

$$\sigma_e = \frac{1}{2}\left[(\sigma_x+\sigma_y) + \sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}\right]$$

$$\sigma_e = \frac{1}{2}\left[(120-60) + \sqrt{[120-(-60)]^2+4(36)^2}\right] = 126.93 \text{ MPa}$$

$$\text{FOS} = \frac{\sigma_{ys}}{\sigma_e} = \frac{232}{126.93} = 1.828$$

(ii) According to Guest's theory or max shear stress theory of failure

$$\tau_e = \frac{1}{2}\sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2}$$

or $$\sigma_e = \sqrt{(\sigma_x-\sigma_y)^2+4\tau_{xy}^2} = \sqrt{[120-(-60)]^4+4(36)^2}$$

$$\sigma_e = 193.87 \text{ MPa}$$

$$\therefore \quad \text{FOS} = \frac{\sigma_{ys}}{\sigma_e} = \frac{232}{193.87} = 1.197$$

(ii) According to Hencky-Von-Mises theory or shear energy theory of failure

$$\sigma_e = \sqrt{\sigma_x^2+\sigma_y^2-\sigma_x\sigma_y+3\tau_{xy}^2}$$

$$= \sqrt{120^2+(-60)^2-120\times(-60)+3(36)^2} = 170.55 \text{ MPa}$$

$$\therefore \quad \text{FOS} = \frac{\sigma_{ys}}{\sigma_e} = \frac{232}{170.55} = 1.36$$

Problem 3: A machine member is subjected to the following stresses σ_x = 150 MPa, τ_{xy} = 24 MPa. Find the equivalent stress as per the following theories of failure.

(i) Shear stress theory,
(ii) Normal stress theory,
(iii) Von-Mises theory.

Given data: σ_x = 150 MPa, τ_{xy} = 24 MPa

(σ_y = Not given) (σ_y = 0, Not given)

(i) According to maximum shear stress theory, equivalent stress $s_e = \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$

$$\sigma_e = \sqrt{150^2 + 4 \times 24^2} = 157.49 \text{ MPa}$$

(ii) According to maximum normal stress theory, equivalent stress

$$\sigma_e = \frac{1}{2}\left[(\sigma_x + \sigma_y) + \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}\right]$$

$$\sigma_e = \frac{1}{2}\left[150 + \sqrt{150^2 + 4(24)^2}\right] = 153.75 \text{ MPa}$$

(iii) According to Von-Mises theory, equivalent stress

$$\sigma_e = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 4\tau_{xy}^2}$$

$$\sigma_e = \sqrt{150^2 + 3(24)^2} = 155.65 \text{ MPa.}$$

Problem 4: Find the diameter of a rod subjected to a bending moment of 3 kNm and a twisting moment of 1.8 kNm according to the following theories of failure, taking normal yield stress as 420 MPa and factor of safety as 3.

(i) Normal stress theory, (ii) Shear stress theory.

Given data: Bending moment, M_b = 3 kNm = 3×10^6 N-mm

Twisting moment, M_t = 1.8 kNm = 1.8×10^6 N-mm

Yield stress, σ_{ys} = 420 MPa FOS = 3

$\therefore$ Allowable stress, $\sigma = \sigma_e = \dfrac{\sigma_{ys}}{\text{FOS}} = \dfrac{420}{3} = 140$ MPa

$$\text{Bending stress, } \sigma = \frac{M_b \cdot C}{I} = \frac{3 \times 10^6 \times d/2}{(\pi d^4/64)} = \frac{30.56 \times 10^6}{d^3}$$

$$\sigma = \frac{30.56 \times 10^6}{d^3} = \sigma_x$$

$$\text{Shear stress, } \tau = \frac{M_t r}{J} = \frac{1.8 \times 10^6 \times d/2}{(\pi d^4/32)} = \frac{9.167 \times 10^6}{d^3} = \tau_{xy}$$

(i) According to maximum normal stress theory,

$$\sigma_e = \frac{1}{2}\left[(\sigma_x + \sigma_y + \sqrt{(\sigma_x - \sigma_y)^2} + 4\tau_{xy}^2\right]$$

(Here $\sigma_y = 0$, no stress in $\perp lr$ direction)

$$140 = \frac{1}{2}\left[\frac{30.56 \times 10^6}{d^3} + \sqrt{\left(\frac{30.56 \times 10^6}{d^3}\right)^2 + 4\left(\frac{9.167 \times 10^6}{d^3}\right)^2}\right]$$

$\therefore$ $d = 61.834$ mm

(ii) According to maximum shear stress theory

$$\sigma_e = \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$140 = \sqrt{\left(\frac{30.56 \times 10^6}{d^3}\right)^2 + 4\left(\frac{9.167 \times 10^6}{d^3}\right)^2}$$

$\therefore$ $d = 63.376$ mm

$\therefore$ Recommended diameter, $d = 63.376 \simeq 64$ mm. (Take bigger one always).

Problem 5: A bolt is subjected to a tensile load of 18 kN and a shear load of 12 kN. The material has an yield stress of 328.6 MPa. Taking factor of safety as 2.5, determine the core diameter of bolt according to the following theories of failure.

(i) Rankine's theory,
(ii) Shear stress theory,
(iii) Shear energy theory and
(iv) Saint Venant's theory. Take Possion ratio = 0.298

Given data: Tensile load, $F_T = 18$ kN $= 18 \times 10^3$ N

Shear load, $F_s = 12$ kN $= 12 \times 10^3$ N

Yield stress, $\sigma_{ys} = 328.6$ MPa FOS = 2.5

$\therefore$ Allowable stress, $\sigma_e = \frac{\sigma_{ys}}{\text{FOS}} = \frac{328.6}{2.5} = 131.44$ MPa.

$$\text{Tensile stress, } \sigma = \frac{F_T}{A} = \frac{18 \times 10^3}{A} = \sigma_x$$

$$\text{Shear stress, } \tau = \frac{F_s}{A} = \frac{12 \times 10^3}{A} = \tau_{xy}$$

($\sigma_y = 0$, not given)

(i) According to Rankine's theory of failure

$$\sigma_e = \frac{1}{2}\left[\sigma_x + \sqrt{\sigma_x^2 + 4\tau_{xy}^2}\right]$$

$$131.44 = \frac{1}{2}\left[\frac{18\times10^3}{A} + \sqrt{\left(\frac{18\times10^3}{A}\right)^2 + 4\left(\frac{12\times10^3}{A}\right)^2}\right]$$

$$A = 182.59 = \frac{\pi d_c^2}{4}$$

$\therefore$ Core dia, $dc = 15.25$ mm

(ii) According to maximum shear stress theory,

$$\sigma_e = \sqrt{\sigma_x^2 + 4\tau_{xy}^2}$$

$$131.44 = \sqrt{\left(\frac{18\times10^3}{A}\right)^2 + 4\left(\frac{12\times10^3}{A}\right)^2}$$

$\therefore$ $$A = 228.24 = \frac{\pi d_c^2}{4}$$

$\therefore$ Core dia, $dc = 17.05$ mm

(iii) According to Von-Mises theory of failure

$$\sigma_e = \sqrt{\sigma_x^2 + 3\tau_{xy}^2}$$

$$131.44 = \sqrt{\left(\frac{18\times10^3}{A}\right)^2 + 3\left(\frac{12\times10^3}{A}\right)^2}$$

$$A = 209.19 = \frac{\pi d_c^2}{4}$$

$\therefore$ Core dia, $dc = 16.32$ mm

(iv) According to Saint Venant's theory of failure

$$\sigma_e = \frac{1}{2}\left[(1-v)(\sigma_x) + (1+v)\sqrt{\sigma_x^2 + 4\tau_{xy}^2}\right]$$

$$131.44 = \frac{1}{2}\left[(1-0.298)\frac{18\times10^3}{A} + (1+0.298)\sqrt{\left(\frac{18\times10^3}{A}\right)^2 + 4\left(\frac{12\times10^3}{A}\right)^2}\right]$$

$$A = 196.196 = \frac{\pi d_c^2}{4}$$

$\therefore$ Core dia, $dc = 15.81$ mm.

Problem 6: A SAE 1045 steel rod (σ_{ys} = 309.9 MPa) of 80 mm diameter is subjected to a bending moment of 3 kNm and torque T. Taking Factor of safety as 2.5, find the maximum value of torque T that can be safely carried by rod according to:

(i) Maximum normal stress theory,

(ii) Maximum shear stress theory.

Given data: Material SAE 1045.

Yield stress, σ_{ys} = 309.9 MPa

FOS = 2.5 diameter d = 80 mm

$$\therefore \quad \text{Allowable stress, } \sigma_e = \frac{\sigma_{ys}}{\text{FOS}} = \frac{309.9}{2.5} = 123.96 \text{ MPa}$$

Bending moment, M_b = 3 kNm = 3×10^6 N-mm.

$$\therefore \quad \text{Bending stress, } \sigma = \frac{M_b \cdot C}{I} = \frac{3 \times 10^6 (80/2)}{(\pi/64 \times 80^4)} = 59.68 \text{ MPa} = \sigma_x$$

Torque, $M_t = T$

$$\therefore \quad \text{Shear stress, } \tau = \frac{M_t \cdot r}{J} = \frac{T \cdot (80/2)}{(\pi/32 \times 80)^4} = (9.95 \times 10^{-6}) \text{ MPa}$$

$$\therefore \quad \tau = \tau_{xy} = (9.95\ 10^{-6})\ T$$

(σ_y = 0, not given)

(i) According to maximum normal stress theory

$$\sigma_e = \frac{1}{2}\left[\sigma_x + \sqrt{\sigma_x^2 + 4\tau_{xy}^2}\right]$$

$$123.96 = \frac{1}{2}\left[59.68 + \sqrt{59.68^2 + 4(9.95 \times 10^{-6} T)^2}\right]$$

$$\therefore \quad \text{Torque, } T = 8.971 \times 10^6 \text{ N-mm} = 8.971 \text{ kNm}$$

(ii) According to maximum shear stress theory

$$\tau_e = \frac{1}{2}\sqrt{\sigma_x^2 + 4\tau_{xy}^2}$$

Assuming, $\tau_e = 0.5\ \sigma_e = 0.5 \times 123.96 = 61.98$ MPa

$$61.98 = \frac{1}{2}\sqrt{59.68^2 + 4(9.95 \times 10^{-6} T)^2}$$

Torque, $T = 5.46 \times 10^6$ N-mm = 5.46 kNm.

Problem 7: A stressed element is loaded as shown in Fig. 2.3. Determine the following:

(i) Von-Mises stress,
(ii) Maximum shear stress,
(iii) Maximum normal stress,
(iv) Octahedral shear stress.

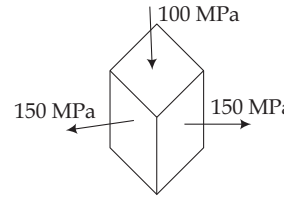


Fig. 2.3

Given data: Arranging in descending order $150 \geq 150 > -100$

$\therefore$ $\sigma_1 = 150$ MPa,

$\sigma_2 = 150$ MPa and $\sigma_3 = -100$ MPa (compressive)

(i) Von-Mises stress

$$\tau_e = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}}$$

$$= \sqrt{\frac{(150 - 150)^2 + (150 + 100)^2 + (-100 - 150)^2}{2}} = 250 \text{ MPa}$$

(ii) Maximum shear stress

$$\tau_{12} = \frac{\sigma_1 - \sigma_2}{2} = \frac{150 - 150}{2} = 0$$

$$\tau_{23} = \frac{\sigma_2 - \sigma_3}{2} = \frac{150 - (-100)}{2} = 125 \text{ MPa}$$

$$\tau_{13} = \frac{\sigma_1 - \sigma_3}{2} = \frac{150 - (-100)}{2} = 125 \text{ MPa}$$

$\therefore$ $\tau_{max} = 125$ MPa (max of these 3 values)

(iii) Maximum normal stress

$\sigma_1 > \sigma_2 > \sigma_3$

then $\sigma_{max} = \sigma_1 = 150$ MPa.

(iv) Octahedral shear stress

$$\tau_e = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

$$= \frac{1}{3}\sqrt{(150 - 150)^2 + (150 - 100)^2 + (-100 - 150)^2} = 117.85 \text{ MPa.}$$

Problem 8: A material has a yield strength of 600 MPa. Compute the factor of safety for each of the failure theories for the each of the following stresses:

(i) $\sigma_1 = 420$ MPa, $\sigma_2 = 410$ MPa, $\sigma_3 = 0$,

(ii) $\sigma_1 = 420$ MPa, $\sigma_2 = 180$ MPa, $\sigma_3 = 0$,

(iii) $\sigma_1 = 420$ MPa, $\sigma_2 = 0$, $\sigma_3 = -180$ MPa,

(iv) $\sigma_1 = 0$, $\sigma_2 = -180$ MPa, $\sigma_3 = -420$ MPa

(a) Von-Mises theory,

(b) Max. normal stress theory and

(c) Max. shear stress theory.

Given data: $\sigma_{ys} = 600$ MPa

(i) $\sigma_1 = 420$ MPa, $\sigma_2 = 410$ MPa, $\sigma_3 = 0$

$\sigma_1 > \sigma_2 > \sigma_3$

(a) Von-Mises theory

$$\sigma_e = \sqrt{\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}} = \sqrt{\frac{(420-410)^2+(410-0)^2+(420-0)^2}{2}}$$

$$\sigma_e = 415.09 \text{ MPa}$$

$$\therefore \quad \text{FOS} = \frac{\sigma_{ys}}{\sigma_e} = \frac{600}{415.09} = 1.445.$$

(b) Max. normal stress theory, $\sigma_e = \sigma_1 = 420$ MPa

$$\therefore \quad \text{FOS} = \frac{600}{420} = 1.4286$$

(c) Max. shear stress theory

$$\tau_e = \tau_{max} \text{ of } \tau_{12} \text{ or } \tau_{23} \text{ or } \tau_{13}$$

$$\tau_{12} = \frac{\sigma_1-\sigma_2}{2} = \frac{420-410}{2} \ 5 \text{ MPa}$$

$$\tau_{23} = \frac{\sigma_2-\sigma_3}{2} = \frac{410}{2} = 205 \text{ MPa}$$

$$\tau_{13} = \frac{\sigma_1-\sigma_3}{2} = \frac{420}{2} = 210 \text{ MPa}$$

$$\therefore \quad \tau_{max} = 210 \text{ MPa} = \tau_e$$

$$\therefore \quad \text{FOS} = \frac{\tau_{ys}}{\tau_e} = \frac{\tau_{ys}}{2\tau_e} = \frac{600}{2\times 210} = 1.4286$$

(ii) $\sigma_1 = 420$, $\sigma_2 = 180$ and $\sigma_3 = 0$ MPa

$\sigma_1 > \sigma_2 > \sigma_3$

(a) Von-mises theory, $\sigma_e = \sqrt{\frac{(\sigma_1 - \sigma_2)^{-2} + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}}$

$$\sigma_e = \sqrt{\frac{(420-180)^2 + (180-0)^2 + (420-0)^2}{2}} = 364.97 \text{ MPa}$$

$$\therefore \quad \text{FOS} = \frac{\tau_{ys}}{\tau_e} = \frac{600}{364.97} = 1.644$$

(b) Max. normal stress theory, $\sigma_e = \sigma_1 = 420$ MPa

$$\therefore \quad \text{FOS} = \frac{\sigma_{ys}}{\sigma_e} = \frac{600}{420} = 1.4286$$

(c) Max. shear stress theory

$$\tau_{12} = \frac{\sigma_1 - \sigma_2}{2} = \frac{420 - 180}{2} = 120 \text{ MPa}$$

$$\tau_{23} = \frac{\sigma_2 - \sigma_3}{2} = \frac{180}{2} = 90 \text{ MPa}$$

$$\tau_{13} = \frac{\sigma_1 - \sigma_3}{2} = \frac{420}{2} = 210 \text{ MPa}$$

$$\therefore \quad \tau_{max} = 210 \text{ MPa} = \tau_e$$

$$\therefore \quad \text{FOS} = \frac{\sigma_{ys}}{2\tau_{max}} = \frac{600}{2 \times 210} = 1.4286$$

(iii) $\sigma_1 = 0, \quad \sigma_2 = -180$ MPa, $\quad \sigma_3 = -420$ MPa

(a) Von-Mises theory, $\sigma_e = \sqrt{\frac{(0+180)^2 + (-180+420)^2 + (0+420)^2}{2}} = 364.96$

$$\therefore \quad \text{FOS} = \frac{\sigma_{ys}}{\sigma_e} = \frac{600}{364.93} = 1.644$$

(b) Max. normal stress theory, $\sigma_e = \sigma_1 = 0$

$$\therefore \quad \text{FOS} = \frac{600}{0} = \infty$$

But, in compression $\quad \text{FOS} = \frac{600}{420} = 1.4286$

(c) Max. shear stress theory

$$\tau_{12} = \frac{\sigma_1 - \sigma_2}{2} = \frac{0 + 180}{2} = 90 \text{ MPa}$$

$$\tau_{23} = \frac{\sigma_2 - \sigma_3}{2} = \frac{-180 + 4.20}{2} = +120 \text{ MPa}$$

$$\tau_{13} = \frac{\sigma_1 - \sigma_3}{2} = \frac{0 + 420}{2} = 210 \text{ MPa} \quad \therefore \ \tau_{max} = 210 \text{ MPa}$$

$$\text{FOS} = \frac{\sigma_{ys}}{2\tau_{max}} = \frac{600}{2 \times 210} = 1.4286.$$

Problem 9: A hot rolled bar has yield stress of 390 MPa. Compute the factor of safety for the following theories of failure:

(i) Maximum normal stress theory,

(ii) Maximum shear stress theory and

(iii) Distortion energy theory for the following states of stress.

(a) $\sigma_1 = 225$ MPa, $\sigma_2 = 225$ MPa, $\sigma_3 = 0$

(b) $\sigma_1 = 225$ MPa, $\sigma_2 = 120$ MPa, $\sigma_3 = 0$

(c) $\sigma_1 = 225$ MPa, $\sigma_2 = 0$, $\sigma_3 = -120$ MPa.

Given data: Yield stress, $\sigma_{ys} = 390$ MPa

$$\text{FOS} = \frac{\sigma_{ys}}{\sigma_e}$$

(a) $\sigma_1 = 225$ MPa, $\sigma_2 = 225$ MPa, $\sigma_3 = 0$

$\sigma_1 > \sigma_2 > \sigma_3$

(i) Maximum normal stress theory, $\sigma_e = \sigma_1 = 225$ MPa

$$\therefore \quad \text{FOS} = \frac{390}{225} = 1.733$$

(ii) Maximum shear stress theory

$$\tau_{12} = \frac{\sigma_1 - \sigma_2}{2} = \frac{225 - 225}{2} = 0$$

$$\tau_{23} = \frac{\sigma_2 - \sigma_3}{2} = \frac{225 - 0}{2} = 112.5 \text{ MPa}$$

$$\tau_{13} = \frac{\sigma_1 - \sigma_3}{2} = \frac{225 - 0}{2} = 112.5 \text{ MPa}$$

$$\therefore \quad \tau_e = \tau_{max} = 112.5 \text{ MPa}$$

and

$$\text{FOS} = \frac{\sigma_{ys}}{2\tau_{max}} = \frac{390}{2 \times 112.5} = 1.733$$

(iii) Distortion energy theory

$$\sigma_e = \sqrt{\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}}$$

$$= \sqrt{\frac{(225-225)^2+(225-0)^2+(0-225)^2}{2}} = 225 \text{ MPa}$$

$$\therefore \quad \text{FOS} = \frac{\sigma_{ys}}{\sigma_e} = \frac{390}{225} = 1.733$$

(b) $\sigma_1 = 225$ MPa, $\sigma_2 = 120$ MPa, $\sigma_3 = 0 \Rightarrow \sigma_1 > \sigma_2 > \sigma_3$

(i) Maximum normal stress theory, $\sigma_e = \sigma_1 = 225$ MPa

$$\therefore \quad \text{FOS} = \frac{\sigma_{ys}}{\sigma_e} = \frac{390}{225} = 1.733$$

(ii) Maximum shear stress theory

$$\tau_{12} = \frac{\sigma_1-\sigma_2}{2} = \frac{225-120}{2} = 52.5 \text{ MPa}$$

$$\tau_{23} = \frac{\sigma_2-\sigma_3}{2} = \frac{120-0}{2} = 60 \text{ MPa}$$

$$\tau_{13} = \frac{\sigma_1-\sigma_3}{2} = \frac{225-0}{2} = 112.5 \text{ MPa}$$

$$\therefore \quad \tau_{max} = \tau_e = 112.5 \text{ MPa} \quad \therefore \quad \text{FOS} = \frac{\sigma_{ys}}{2\tau_{max}} = \frac{390}{2\times 112.5} = 1.733$$

(iii) Distortion energy theory

$$\sigma_e = \sqrt{\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2}{2}}$$

$$= \sqrt{\frac{(225-120)^2+(120-0)^2+(225-0)^2}{2}} = 195 \text{ MPa}$$

$$\therefore \quad \text{FOS} = \frac{\sigma_y}{\sigma_e} = \frac{390}{195} = 2$$

(c) $\sigma_1 = 225$, $\sigma_2 = 0$, $\sigma_3 = -120$, $\sigma_1 > \sigma_2 > \sigma_3$

(i) Maximum normal stress theory, $\sigma_e = \sigma_1 = 225$ MPa.

$$\text{FOS} = \frac{\sigma_{ys}}{\sigma_e} = \frac{390}{225} = 1.733$$

(ii) Maximum shear stress theory

$$\tau_{12} = \frac{\sigma_1-\sigma_2}{2} = \frac{225-0}{2} = 112.5 \qquad \tau_{23} = \frac{\sigma_2-\sigma_3}{2} = \frac{0-(-120)}{2} = 60$$

$$\tau_{13} = \frac{\sigma_1 - \sigma_3}{2} = \frac{225 - (-120)}{2} = 172.5 \qquad FOS = \frac{390}{2 \times 172.5} = 1.13$$

(iii) Distortion energy theory

$$\sigma_e = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}{2}}$$

$$\sigma_e = \sqrt{\frac{(225 - 0)^2 + (0 + 120)^2 + (225 + 120)^2}{2}}$$

$$= 303.36 \text{ MPa}$$

$$FOS = \frac{390}{303.36} = 1.286$$

EXERCISE

1. A rod of circular cross section is to sustain a torsional moment of 3 kN-M and a bending moment of 2 kN-M. Selecting C45 steal (σ_{yt} = 353 MPa) and assuming factor of safety as 3, determine the diameter of the rod as per the following theories of failure:
   (i) Maximum normal stress theory, (ii) Distorsion energy theory and
   (iii) Maximum shear stress theory.
2. A critical section in a shaft is subjected to bending and twisting moment simultaneously. The bending moment causes a stress of 50 MPa and twisting moment causes a shear stress of 31.5 MPa. Determine the factor of safety according to: (i) Maximum normal stress theory (ii) Maximum shear stress theory given proportionality limit in tension is 284 MPa.
3. A rod of 50 mm diameter is subjected to a compressive load of 20 kN together with a twisting moment of 1.5 kN-M. It is make of C40 steal (σ_{yt} = 328.6 MPa). Determine the factor of safety according to:
   (i) Maximum normal stress theory, (ii) Maximum shear stress theory.
4. Explain the following theories of failure:
   (i) Maximum normal stress theory, (ii) Maximum shear stress theory,
   (iii) Distortion energy theory.
5. A material has an yield strength of 630 MPa. Find the factor of safety for each of the states of stress using:
   (i) Maximum normal stress theory, (ii) Maximum shear stress theory,
   (iii) Distortion energy theory.

(a) σ_1 = 200 MPa, σ_2 = 0, σ_3 = –90 MPa
(b) σ_1 = 200 MPa, σ_2 = 90 MPa, σ_3 = 0
(c) σ_1 = 200 MPa, σ_2 = 200 MPa, σ_3 = 200 MPa
(d) σ_1 = 90 MPa, σ_2 = –90 MPa σ_3 = –200 MPa.

CHAPTER

3 Stress Concentration

3.1 DEFINITION

Stress concentration is defined as the concentration of stress in a machine member either due to change in cross section or due to irregularities in the member. It is not practically possible to have machine elements without change in the cross section. Some examples are key ways in shafts, step in shaft to accommodate bearings, gears etc.

Due to the irregularities or due to change in cross section, the stress induced in the member is much higher than the one that is calculated using the known theories. The ratio of the maximum stress induced to the theoretical or nominal stress calculated is defined as stress concentration factor.

$$\text{Stress concentration factor, } k_\sigma = \frac{\sigma_{max}}{\sigma_{nom}} \quad \text{or} \quad k_\tau = \frac{\tau_{max}}{\tau_{nom}}$$

k_σ = is used for axial and bending stresses

k_τ = is used for shear stresses.

where σ_{max} or τ_{max} is maximum stress induced and

σ_{nom} or τ_{nom} = nominal or theoretical stress.

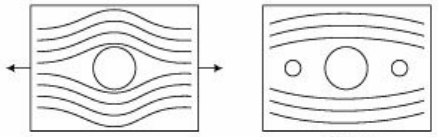
For axial loads, $$\sigma_{nom} = \frac{F}{A}$$

For bending loads, $$\sigma_{nom} = \frac{M_b C}{I} = \frac{M_b}{(I/C)}, \quad \text{and}$$

For torsional load, $$\tau_{nom} = \frac{M_t . r}{J} = \frac{M_t}{(J/r)} = \frac{M_t}{(J/C)},$$

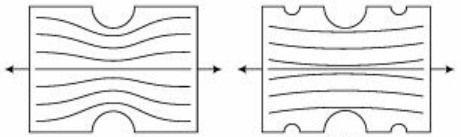
3.2 STRESS CONCENTRATION AND METHODS TO REDUCE STRESS CONCENTRATION

Figures below show the stress concentration and some methods to reduce stress concentration.



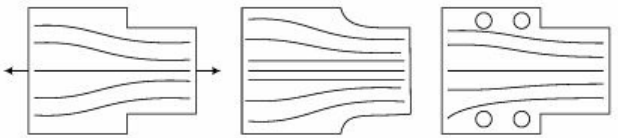
Better
By adding two small holes
on either side of big hole.

(a) Plate with hole



Better
Small notches are provided
on both sides of big notch.

(b) Notched plate

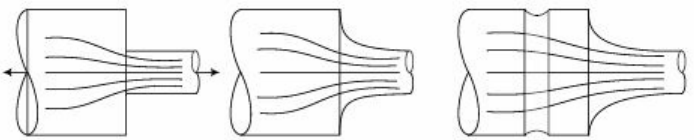


Better
By providing a fillet

Better
By drilling holes

(c) Stepped plate

Fig. 3.1 Plates



Better
By providing a fillet
Stepped Shaft

Better
By providing a grove
Grooved shaft

Fig. 3.2 Shafts

3.3 GRAPHS FOR STRESS CONCENTRATION FACTOR

Published data in the form of graphs is available in various books for determination of stress concentration factor for different types of specimens under various loading conditions.

Some of the important graphs are given below:

(i) plate with hole subjected to tension and bending Figs. 3.3 and 3.4.

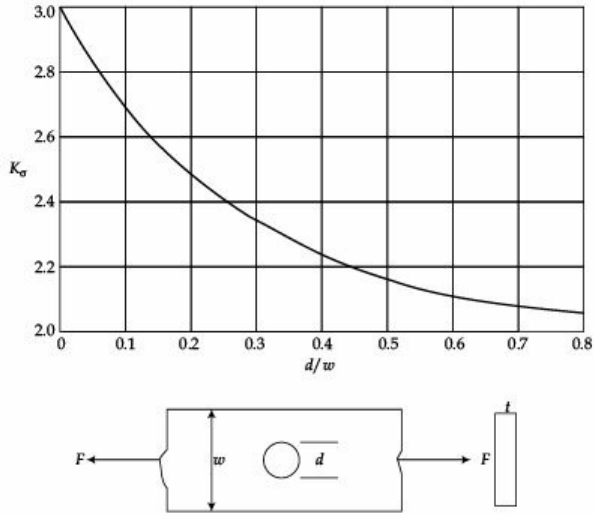


Fig 3.3 Theoretical stress concentration factor (k_σ) for finite width plate with a transverse hole.

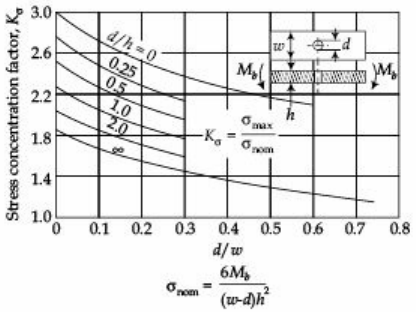
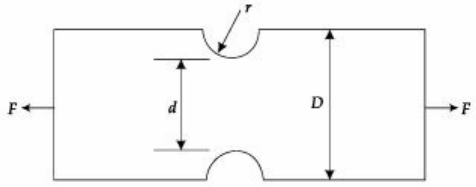


Fig. 3.4

(ii) Notched plate subjected to tension and bending (Figs. 3.5 and 3.6).



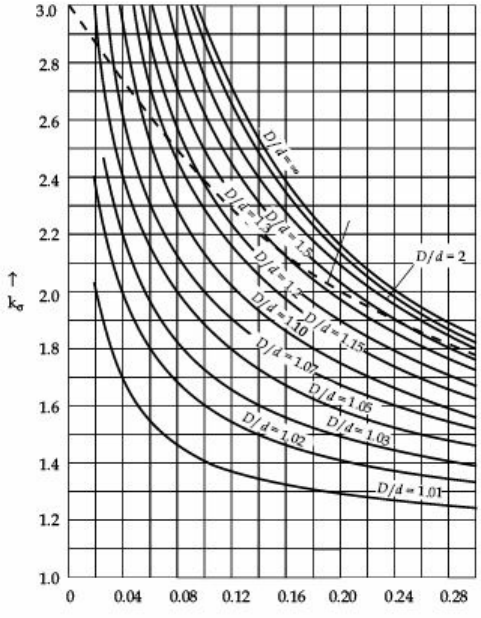
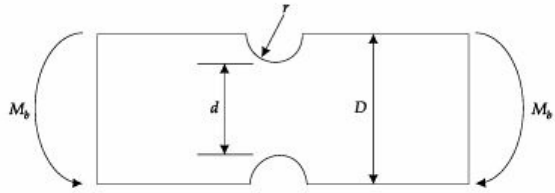


Fig. 3.5 Theoretical stress concentration factor (k_σ) for a notched flat bar in tension.



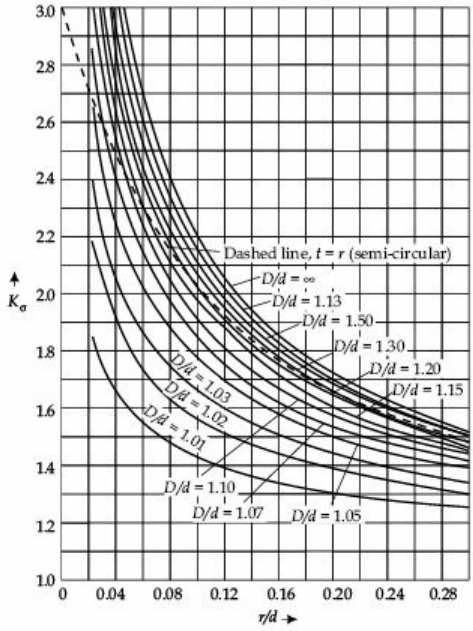
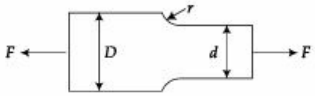


Fig. 3.6 Theoretical stress concentration factor (k_σ) for a notched flat bar in bending.

(iii) Stepped plate subjected to tension and bending (Figs. 3.7 and 3.8).



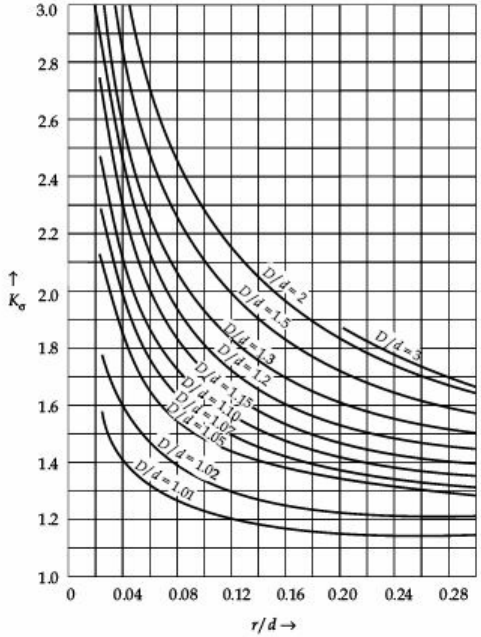
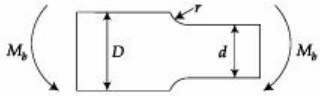


Fig. 3.7 Theoretical stress concentration factor (k_σ) for a flat bar with shoulder fillet in tension.



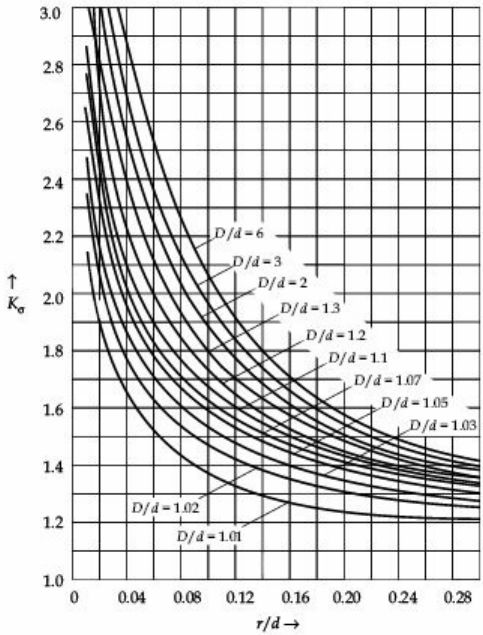


Fig. 3.8 Theoretical stress concentration factor (k_σ) for a flat bar with shoulder fillet in bending.

(iv) Stepped shaft subjected to tension, bending and torsion (Figs. 3.9, 3.10 and 3.11).

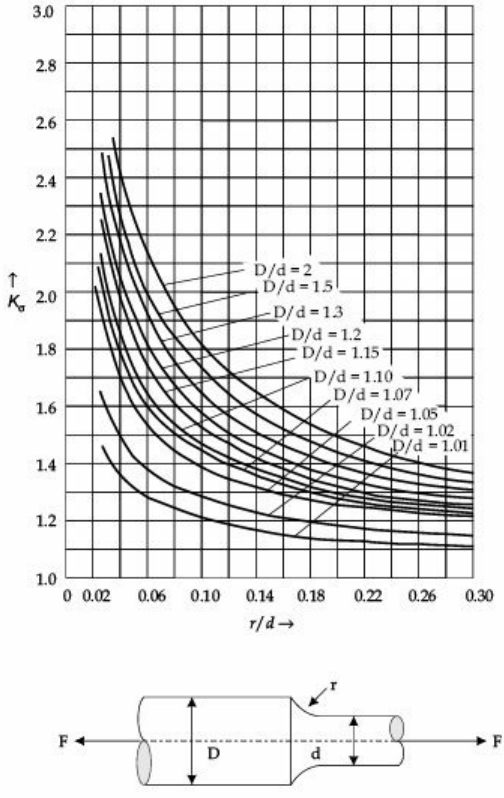


Fig. 3.9 Theoretical stress concentration factor (k_a) for shaft with a shoulder fillet in tension.

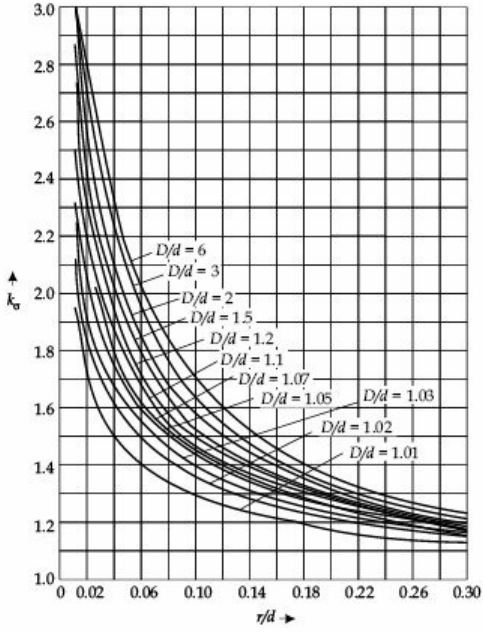


Fig. 3.10 Theoretical stress concentration factor (k_a) for shaft with a shoulder fillet in bending.

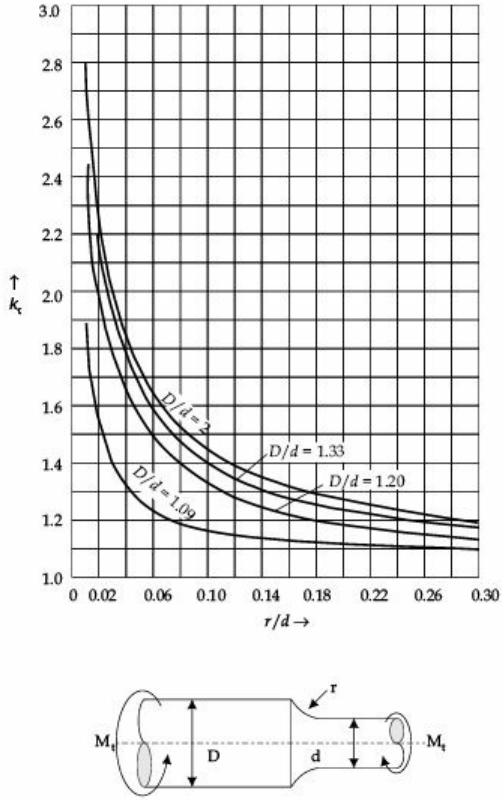


Fig. 3.11 Theoretical stress concentration factor (k_σ) for shaft with a shoulder filled in torsion.

(v) Grooved shaft subjected to tension, bending and torsion (Figs. 3.12, 3.13 and 3.14).

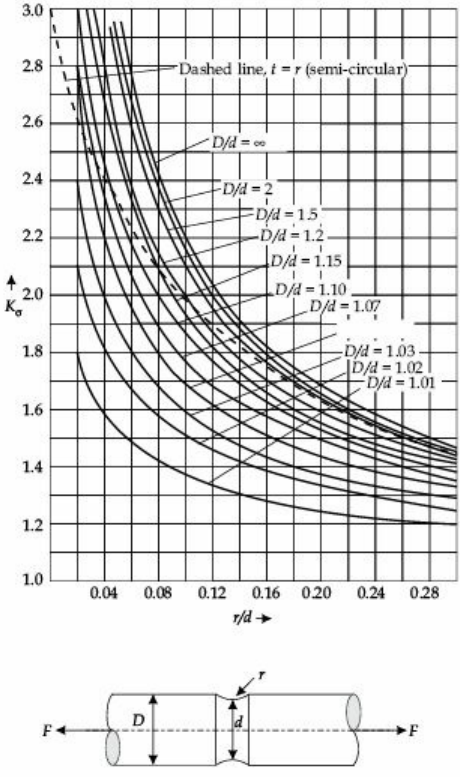


Fig. 3.12 Theoretical stress concentration factor (k_σ) for a grooved shaft in tension.

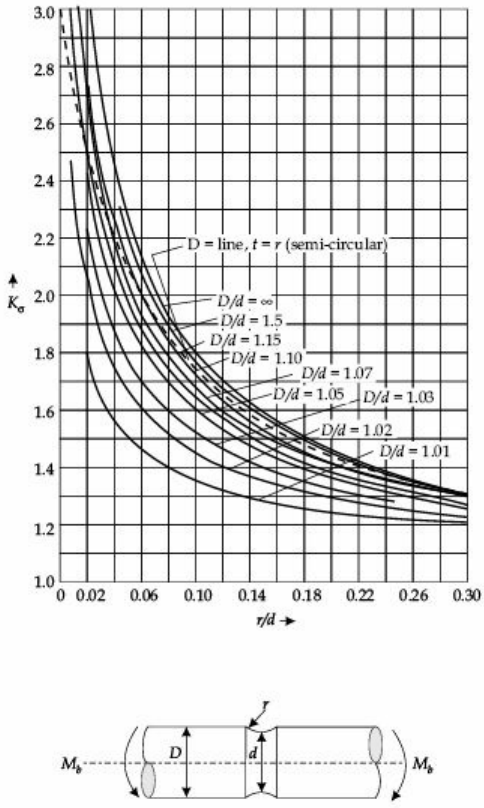
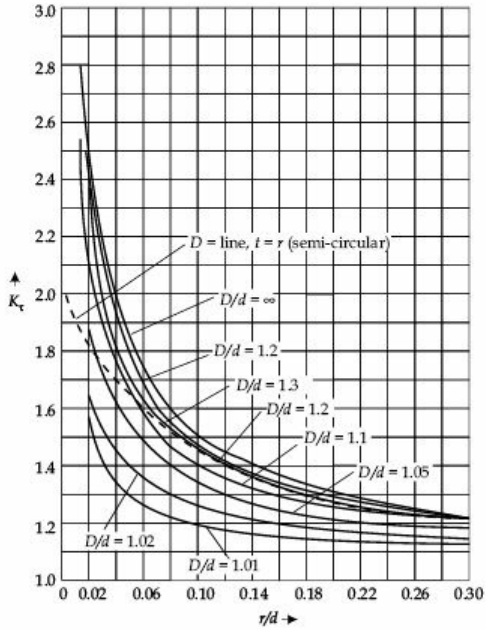


Fig. 3.13 Theoretical stress concentration factor (k_σ) for a grooved shaft in bending.



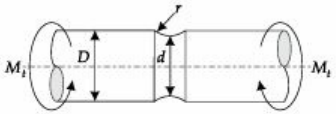


Fig. 3.14 Theoretical stress concentration factor (k_t) for a grooved shaft in torsion.

PROBLEMS

Problem 1: Determine the maximum stress induced in the following cases taking stress concentration into account.

(i) A rectangular plate 80 mm wide, 12 mm thick with a central hole of dia 16 mm subjected to a tensile load of 30 kN.

(ii) A stepped shaft stepped down from 60 mm diameter to 40 mm diameter with a fillet radius of 8 mm subjected to a twisting moment of 120 kNm.

Given data:

(i) Plate with hole subjected to tensile force refer to Graph 3.3,

$d = 16$ mm

$w = 80$ mm

$h = 12$ mm

$F = 30\ \text{kN} = 30 \times 10^3$ N

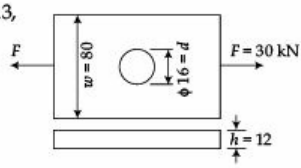


Fig. 3.15

$$x\text{-axis of graph } \frac{d}{w} = \frac{16}{80} = 0.2$$

$\therefore$ Stress concentration factor, $k_\sigma = 2.5$ from graph (at $d/w = 0.2$) ... $\left(\frac{\text{Fig. 4.5}}{\text{P 4.5}}\right)$

$$\text{For axial load, } \sigma_{nom} = \frac{F}{A} = \frac{F}{(w-d)h}$$

$$\sigma_{nom} = \frac{30 \times 10^3}{(80-16)^{12}} = 39.06 \text{ MPa}$$

$$\text{WKT, } k_\sigma = \left(\frac{\sigma_{max}}{\sigma_{nom}}\right)$$

$\therefore$ Maximum stress induced, $\sigma_{max} = k_\sigma . \sigma_{nom}$

$$\sigma_{max} = 2.5 \times 39.06 = 97.65 \text{ MPa}$$

(ii) Stepped shaft subjected to torque (refer graph 3.11)

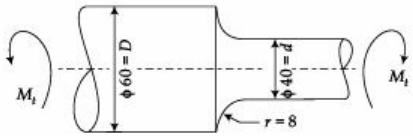


Fig. 3.16

$$D = 60 \qquad \left(\text{Fig. } \frac{4.19\,A}{\text{P } 4.18}\right)$$

$d = 40$

$r = 8$

$M_t = 120\ \text{kNm} = 120 \times 10^3$ N-mm.

$$x\text{-axis } \frac{r}{d} = \frac{8}{40} = 0.2$$

$$\frac{D}{d} = \frac{60}{40} = 1.5$$

$\therefore$ Stress concentration factor, $k_\tau = 1.25$ (approx) from graph (at $r/d = 0.2$ and $D/d = 1.5$)

For torsion, $$\tau_{nom} = \frac{M_t}{\left(\frac{J}{C}\right)} = \frac{M_t}{\left(\frac{\pi d^3}{16}\right)}$$

$$= \frac{120 \times 10^3}{(\pi \times 40^3/16)} = 9.55 \text{ MPa}$$

but $$k_\tau = \frac{\tau_{max}}{\tau_{nom}}$$

$\therefore$ Max. shear stress induced, $\tau_{max} = k_\tau \cdot \tau_{nom}$

$$\tau_{max} = 1.25 \times 9.55 = 11.94 \text{ MPa.}$$

Problem 2: A plate of rectangular cross-section 60 mm wide carries a tensile load of 54 kN. For some reason, a circular hole of 10 mm diameter is to be drilled exactly at the centre of the plate. Determine the suitable thickness of the plate it is made of C40 steel ($\sigma_y = 328.6$ MPa).

Given data: $\sigma_y = 328.6$ MPa

Assume factor of safety FOS = 2.5

$\therefore$ Allowable stress $$\sigma_{max} = \frac{\sigma_y}{FOS} = \frac{328.6}{2.5} = 131.44 \text{ MPa}$$

Ref. Fig. 3.3, $w = 60$, $d = 10$

$\therefore$ $$x\text{-axis } \frac{d}{w} = \frac{10}{60} = 0.167$$

From graph, stress concentration factor, $k_\sigma = 2.56$

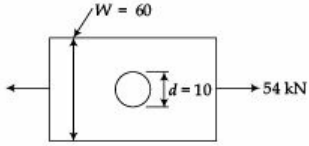


Fig. 3.16a

WKT $$k_\sigma = \frac{\sigma_{max}}{\sigma_{nom}}$$

$\therefore$ $$\sigma_{nom} = \frac{\sigma_{max}}{K_\sigma} = \frac{131.44}{2.56} = 51.34 \text{ MPa.} \quad \ldots \left(\frac{\text{Fig. 4.5}}{\text{P 4.5}}\right)$$

But, $$\sigma_{nom} = \frac{F}{(W-d)h}$$

$$51.34 = \frac{54 \times 10^3}{(60-10)h} \Rightarrow h = 21.03$$

say thickness, $h = 21$ mm

Problem 3: Determine the load carrying capacity of a plate of rectangular cross-section 90 mm wide, 15 mm thick with a central hole of 9 mm dia limiting the stress to 90 MPa.

Given data:

$w = 90$ mm

$d = 9$ mm

$h = 15$ mm

$\sigma_{max} = 90$ MPa

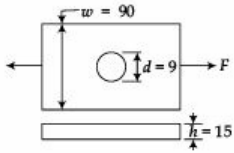


Fig. 3.17

Ref Fig. 3.3, x-axis $\dfrac{d}{w} = \dfrac{9}{90} = 0.1$

$\therefore$ Stress concentration factor, $k_\sigma = 2.7$

But $k_\sigma = \dfrac{\sigma_{max}}{\sigma_{nom}}$ ...(Fig. 4.5 / P 4.5)

$\therefore$ $\sigma_{nom} = \dfrac{\sigma_{max}}{K_\sigma} = \dfrac{90}{2.7} = 33.33$ MPa

But $\sigma_{nom} = (w - d) \cdot h$

$F = \sigma_{nom} (w-d) \cdot h$

$= 33.33\ (90 - 9)\ 15 = 40{,}500$ N

Load capacity, $F = 40.5 \times 10^3$ N.

Problem 4: A rectangular plate 70 mm wide with a semicircular groove of 12 mm radius is subjected to:

(i) A tensile load of 10 kN,

(ii) A bending moment of 15 N-m.

Determine the thickness in each case separately taking allowable stress as 120 MPa.

Given data:

$D = 70$ mm

$r = 12$ mm

$\therefore$ $d = D - 2r = 70 - 2 \times 12 = 46$ m

$\sigma_{max} = 120$ MPa

(i) Plate subjected to tension :

$F = 10 \times 10^3$ N

Ref. Fig. 3.5 x-axis $\dfrac{r}{d} = \dfrac{12}{46} = 0.261$

$\dfrac{D}{d} = \dfrac{70}{46} = 1.522$

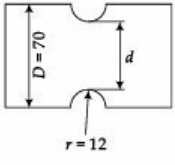


Fig. 3.18

$\therefore$ Stress concentration factor, $k_\sigma = 1.87$

$k_\sigma = \dfrac{\sigma_{max}}{\sigma_{nom}} = \dfrac{120}{\sigma_{nom}} = 1.87$ ...(Fig. 4.22 A / P 4.21)

$\therefore \quad \sigma_{nom} = \dfrac{120}{1.87} = 64.17 \text{ MPa}$

For tensile load, $\sigma_{nom} = \dfrac{F}{A} = \dfrac{F}{hd}$

$$64.17 = \frac{10 \times 10^3}{h \times 46} \Rightarrow \text{thickness } h = 3.39 \text{ mm}$$

(ii) Plate subjected to bending moment,

$$M_b = 15 \text{ N-m} = 15 \times 10^3 \text{ N-mm.}$$

Ref. Fig. 3.6 x-axis $\dfrac{r}{d} = 0.261$

$$\frac{D}{d} = \frac{70}{46} = 1.522 \qquad \ldots\left(\frac{\text{Fig. 4.23 A}}{\text{P 4.21}}\right)$$

$\therefore$ Stress concentration factor, $k_\sigma = 1.55$

$$k_\sigma = \frac{\sigma_{max}}{\sigma_{nom}} \Rightarrow \sigma_{nom} = \frac{\sigma_{max}}{K_\sigma} = \frac{120}{1.55} = 77.42 \text{ MPa}$$

For bending load, $\sigma_{nom} = \dfrac{Mb}{(I/C)}$

But $\dfrac{I}{C} = \dfrac{hd^2}{6}$

$\therefore \quad 77.42 = \dfrac{15 \times 10^3}{(h \times 46^2/6)}$

Thickness, $h = 0.549$ mm.

Problem 5: Determine the safe load that can be carried by a bar of rectangular cross section shown in Fig. 3.19 limited the maximum stress to 130 MPa. Take stress concentration into account.

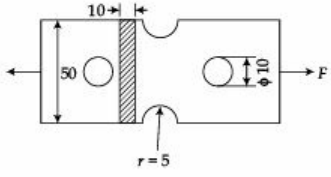


Fig. 3.19

Given data: $\sigma_{max} = 130$ MPa

(i) Consider the plate with hole Fig. (a) under tension

$$w = 50, \quad d = 10, \quad h = 10 \text{ mm}$$

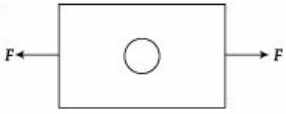


Fig. (a)

Ref. Fig. 3.3, x-axis $\dfrac{d}{w} = \dfrac{10}{50} = 0.2$

$\therefore$ Stress concentration factor

$$k_\sigma = 2.5$$

But $$k_\sigma = \frac{\sigma_{max}}{\sigma_{nom}} \quad \ldots\left(\frac{\text{Fig. 4.5}}{\text{P 4.5}}\right)$$

$$\therefore \quad \sigma_{nom} = \frac{\sigma_{max}}{K_\sigma} = \frac{130}{2.5} = 52 \text{ MPa}$$

For tensile load, $$\sigma_{nom} = \frac{F}{A} = \frac{F}{(w-d)h}$$

$$\therefore \quad \text{load, } F = \sigma_{nom} = (w-d)h = 52 \times (55 - 10) \times 10$$

$$F = 20{,}800 \text{ N}$$

(ii) Consider notched plate under tension (Fig. (b))

$D = 50$ mm

$r = 5$ mm

$h = 10$ mm

$$\therefore \quad d = D - 2r = 50 - 2 \times 5 = 40 \text{ mm}$$

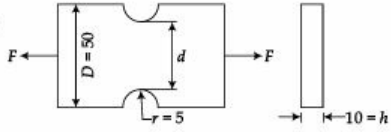


Fig. (b)

Refer Fig. 3.5, x-axis $\dfrac{r}{d} = \dfrac{5}{40} = 0.125$

$$\frac{D}{d} = \frac{50}{40} = 1.25 \quad \ldots\left(\frac{\text{Fig. 4.22 A}}{\text{P 4.21}}\right)$$

$\therefore$ Stress concentration factor from graph $k_\sigma = 2.25$

$$k_\sigma = \frac{\sigma_{max}}{\sigma_{nom}} \Rightarrow \sigma_{nom} = \frac{130}{2.25} = 57.78 \text{ MPa}$$

For tension loads $$\sigma_{nom} = \frac{F}{A} = \frac{F}{hd}$$

$$57.78 = \frac{F}{10 \times 40} \Rightarrow F = 23111.11 \text{ N}$$

$\therefore$ The maximum capacity of the plate or safe load

$F = 20{,}800$ N (smaller of these two values)

Problem 6: Find the thickness of a flat plate shown in the Fig. 3.20 subjected to a tensile load of 90 kN. The allowable stress for the plate material is 120 MPa.

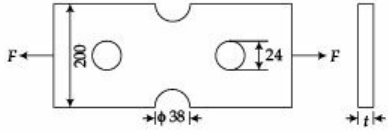


Fig. 3.20

Given data: $F = 90 \text{ kN} = 90 \times 10^3$ N

Allowable stress $\sigma_{max} = 120$ MPa.

The plate has three critical sections where the failure could take place, along left side hole, at notch or at right side hole. Since both holes of the same dia consider one of them and the notch.

(i) Consider plate with central hole (Fig. (a)). Ref. Fig. 3.3

$F = 90 \times 10^3$ N

$w = 200$ mm $\quad d = 24$ mm

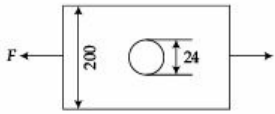


Fig. (a)

x-axis $\dfrac{d}{w} = \dfrac{24}{200} = 0.12$

$\therefore$ Stress concentration factor, $k_\sigma = 2.65$

and $\quad k_\sigma = \dfrac{\sigma_{max}}{\sigma_{nom}}$ $\quad \ldots\left(\dfrac{\text{Fig. 4.5}}{\text{P 4.5}}\right)$

$\therefore \quad \sigma_{nom} = \dfrac{\sigma_{max}}{K_\sigma} = \dfrac{120}{2.65} = 45.28$ MPa

But $\sigma_{nom} = \dfrac{F}{A} = \dfrac{F}{(w-d)h}$

$$45.58 = \frac{90 \times 10^3}{(200-24)h}$$

$\therefore$ Thickness required, $h = 11.29 \simeq 12$ mm

(ii) Consider notched plate, Fig. (b)

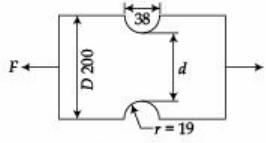


Fig. (b)

$D = 200$ mm

$r = \dfrac{38}{2} = 19$ mm

$\therefore \quad d = D - 2r = 200 - 2 \times 19 = 162$ mm $\quad \ldots\left(\dfrac{\text{Fig. 4.22 A}}{\text{P 4.21}}\right)$

Ref Fig. 3.5, x-axis $\dfrac{r}{d} = \dfrac{19}{162} = 0.117$

$$\frac{D}{d} = \frac{200}{162} = 1.235$$

$\therefore$ Stress concentration factor, $k_\sigma = 2.28$ from graph

But $$k_\sigma = \frac{\sigma_{max}}{\sigma_{nom}} \Rightarrow \sigma_{nom} = \frac{120}{2.28} = 52.63 \text{ MPa}$$

$$\sigma_{nom} = \frac{F}{A} = \frac{F}{hd} = \frac{90 \times 10^3}{h \times 162}$$

$\therefore$ $$h = \frac{90 \times 10^3}{52.63 \times 162} = 10.55 \simeq 11 \text{ mm}$$

$\therefore$ Required thickness, $h = 12$ mm (bigger of the above two)

Problem 7: A SAE 1025 water quenched steel plate ($\sigma_y = 400$ MPa) of rectangular cross section shown in Fig. 3.21 is subjected to a tensile load of 45 kN. Taking factor of safety as 2.5, find the required thickness.

Given data:

$F = 45 \text{ kN} = 45 \times 10^3 \text{ N}$

$\sigma_y = 400 \text{ MPa}$

$\text{FOS} = 2.5$

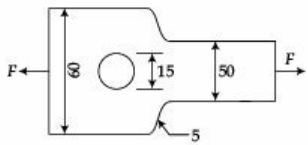


Fig. 3.21

$\therefore$ Allowable tensile stress, $\sigma = \dfrac{\sigma_y}{\text{FOS}}$

$$\sigma = \sigma_{max} = \frac{400}{2.5} = 160 \text{ MPa}.$$

There are two critical sectors where stress concentration is present – plate with hole of dia 15 mm, and stepped plate.

(i) Consider plate with hole subjected to tensile load (Fig. (a))

Ref. Fig. 3.3, $w = 60$, $d = 15$ mm

$$x\text{-axis } \frac{d}{w} = \frac{15}{60} = 0.25$$

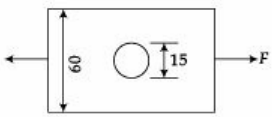


Fig. (a)

$\therefore$ Stress concentration factor from graph

$$k_\sigma = 2.45 = \frac{\sigma_{max}}{\sigma_{nom}} = \frac{160}{\sigma_{nom}} \quad \text{...(Fig. 4.5/P 4.5)}$$

$\therefore$ $$\sigma_{nom} = \frac{160}{2.45} = 65.31 \text{ MPa}.$$

But $$\sigma_{nom} = \frac{F}{A} = \frac{F}{(w-a)h}$$

$$65.31 = \frac{45\times10^3}{(60-15)h} \Rightarrow \therefore \text{Thickness } h = 15.31 \simeq 16 \text{ mm}$$

(ii) Consider stepped plate subjected to tension (Fig. (b))

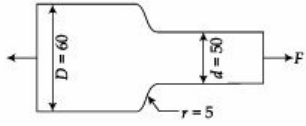


Fig. (b)

Ref Fig. 3.7 x-axis $\frac{r}{d} = \frac{5}{50} = 0.1$

$$\frac{D}{d} = \frac{60}{50} = 1.2$$

$\therefore$ Stress concentration factor, $k_\sigma = 1.8$ ...(Fig. 4.24 A/P 4.22)

$$\text{But } k_\sigma = \frac{\sigma_{max}}{\sigma_{nom}} \Rightarrow \sigma_{nom} = \frac{\sigma_{max}}{K_\sigma} = \frac{160}{1.8} = 88.89 \text{ MPa}$$

$$\text{But } \sigma_{nom} = \frac{F}{A} = \frac{F}{hd} = \frac{45\times10^3}{h\times50}$$

$\therefore$ $$h = \frac{45\times10^3}{50\times88.89} = 10.125 \simeq 11 \text{ mm}$$

$\therefore$ Recommended thickness, $h = 16$ mm (bigger value).

Problem 8: What diameter of maximum hole that can be derived in a flat plate shown in Fig. 3.22, if the stress concentration at step is same as that at the hole.

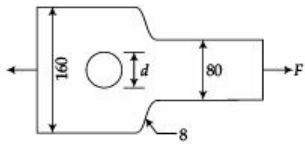


Fig. 3.22

Given data: Type of load = Tensile

(i) Consider stepped plate (Fig. (a))

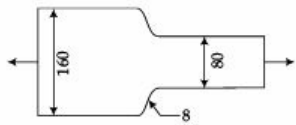


Fig. (a)

Ref Fig. 3.7 x-axis $\frac{r}{d} = \frac{8}{80} = 0.1$

$$\frac{D}{d} = \frac{160}{80} = 2.0$$

$\therefore$ Stress concentration factor, $k_\sigma = 2.28$ ...(Fig. 4.24 A/P 4.22)

(ii) Consider plate with hole (Fig. (b)). Ref Fig. 3.3
For same stress concentration factor as (i) $k_\sigma = 2.28$
From graph at $k_\sigma = 2.28$ on y-axis, the x-axis $\frac{d}{w} = 0.38$

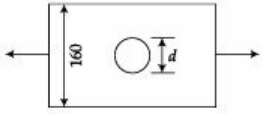


Fig. (b)

$\therefore$ Max. hole diameter $d = 0.38\ w$
$d = 0.38 \times 160 = 60.8$ mm. ...(Fig. 4.5/P 4.5)

Problem 9: Figure 3.23 shows a flat plate with step ratio 1.5 subjected to a bending moment of 3 kN-m. The allowable stress in the plate is 120 MPa. Fillet radius is 10% of smaller width. Determine the dimensions of the plate taking thickness as 18 mm.

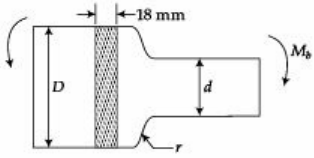


Fig. 3.23

Given data: $M_b = 3$ kNm $= 3 \times 10^6$ N-mm

$$\text{Step ratio} = 1.5 = \frac{D}{d}$$

$r = 10\%$ of $d = 0.1\ d$. ...(Fig. 4.25 A/P 4.22)

$\therefore$ $\frac{r}{d} = 0.1$

Ref. Fig. 3.8 for $\frac{r}{d} = 0.1$ on x-axis and $\frac{D}{d} = 1.5$

Stress concentration factor, $k_\sigma = 1.82$

$\sigma_{max} = 120$ MPa

$$k_\sigma = \frac{\sigma_{max}}{\sigma_{nom}} \Rightarrow \sigma_{nom} = \frac{\sigma_{max}}{k_\sigma} = \frac{120}{1.82} = 65.93 \text{ MPa}$$

For bending, $\sigma_{nom} = \frac{M_b}{(I/C)} = \frac{M_b}{(hd^2/6)}$

$$65.93 = \frac{3 \times 10^6}{18 \times d^2/6} \Rightarrow d = 123.15 \text{ nm} \simeq 124 \text{ mm}$$

$\therefore$ $d = 124$ mm
$D = 1.5d = 186$ mm
$r = 0.1d = 12.4$ mm.

Problem 10: A stepped shaft of diameter $1.5d$ is stepped down to diameter d with a fillet radius of 1/8 of smaller diameter, and carries a transverse load of 90 kN as shown in Fig. 3.24. Determine the diameter of the shaft taking factor safety of 3 based on yield stress of 480 MPa.

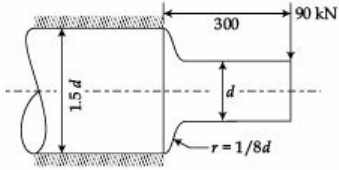


Fig. 3.24

Given data: $D = 1.5d$, $d = d$, $r = \frac{d}{8}$ ...(Fig. 4.21 A/P 4.20)

Bending moment, $M_b = Fl$

$$M_b = 90 \times 10^3 \times 300 = 27 \times 10^6 \text{ N-mm}$$

$$\sigma_y = 480 \text{ MPa.} \quad FOS = 3$$

$\therefore$ Allowable stress, $\sigma_{max} = \dfrac{\sigma_y}{FOS}$

$$\sigma_{max} = \frac{480}{3} = 160 \text{ MPa.}$$

Ref Fig. 3.10, for $\dfrac{r}{d} = \left(\dfrac{d}{8}\right)\Big/d = 0.125$

$$\frac{D}{d} = \frac{1.5d}{d} = 1.5$$

Stress concentration factor, $k_\sigma = 1.45 = \dfrac{\sigma_{max}}{\sigma_{nom}}$ ...$\left(\dfrac{\text{Fig. 4 – 21 A}}{\text{P 4.20}}\right)$

$\therefore$ $$\sigma_{nom} = \frac{\sigma_{max}}{K_\sigma} = \frac{160}{1.45} = 110.35 \text{ MPa}$$

For bending loads, $$\sigma_{nom} = \frac{M_b}{(I/C)} = \frac{M_b}{(\pi d^3/32)}$$

$$110.35 = \frac{27 \times 10^6}{(\pi d^3/32)} \Rightarrow \therefore d = 135.58 \simeq 136 \text{ mm}$$

$\therefore$ $D = 1.5d = 204$ mm

$$r = \frac{d}{8} = \frac{136}{8} = 13.6 \text{ mm.}$$

Problem 11: A stepped shaft with diameter ratio as 1.2 has a fillet radius of $0.1d$ is required to transmit 60 kW at 1200 rpm. It is made of C30 steel ($\sigma_y = 294.2$ MPa). Taking FOS as 2.5, determine the diameter of the shaft.

Given data: Diameters ratio $\dfrac{D}{d} = 1.2$

$r = 0.1\ d$

Power, $P = 60$ kW

Speed, $n = 1200$ rpm.

$\sigma_y = 294.2$ MPa and FOS = 2.5

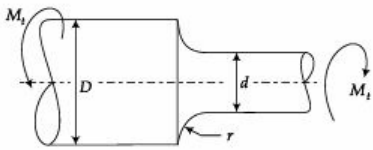


Fig. 3.25

$\therefore$ Allowable normal stress, $\sigma = \dfrac{\sigma_y}{FOS} = \dfrac{294.2}{2.5} = 117.68$ MPa ...(Fig. 4.19 A/P 4.18)

Assume allowable shear stress, $\tau = \tau_{max} = 0.5\sigma = 58.84$ MPa

Power transmission is torque transmission.

$$\therefore \quad \text{Torque, } M_t = \frac{9.55 \times 10^6 P}{n} = 477.5 \times 10^3 \text{ N-mm}$$

Ref Fig. 3.11, for $\frac{r}{d} = 0.1$ and $\frac{D}{d} = 1.2$ ...$\left(\frac{\text{Fig. 4 – 19 A}}{\text{P 4.18}}\right)$

$\therefore$ Stress concentration factor, $k_\tau = 1.33$

$$\text{WKT, } k_\tau = \frac{\sigma_{max}}{\sigma_{nom}} = \frac{58.84}{\tau_{nom}}$$

$$\therefore \quad \tau_{nom} = \frac{58.84}{1.33} = 44.24 \text{ MPa}$$

For torsion, $$\tau_{nom} = \frac{M_t}{(J/C)} = \frac{M_t}{(\pi d^3/16)}$$

$$44.24 = \frac{477.5 \times 10^3}{(\pi d^3/16)} \Rightarrow d = 38.02 \text{ mm}$$

$$d = 38 \text{ mm}$$

$$\therefore \quad D = 1.2d = 45.6 \text{ mm}$$

$$r = 0.1d = 3.8 \text{ mm.}$$

Problem 12: A stepped shaft stepped down from 60 mm dia to 40 mm dia with a fillet radius of 8 mm rotates at 2000 rpm. Find the safe power that can be transmitted by this shaft limiting the shear stress to 60 MPa.

Given data: Power transmission is torque transmission.

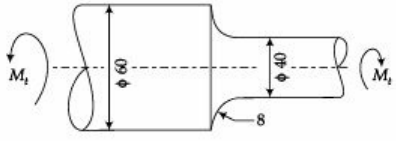


Fig. 3.26

$D = 60$ mm, $d = 40$ mm, $r = 8$ mm,

$n = 2000$ rpm. $\tau_{max} = 60$ MPa.

Ref Fig. 3.11, $\frac{r}{d} = \frac{8}{40} = 0.2$, $\frac{D}{d} = \frac{60}{40} = 1.5$ ...(Fig. 4.19 A/4.18)

$\therefore$ Stress concentration factor, $k_t = 1.25$

But $$k_t = \frac{\sigma_{max}}{\sigma_{nom}} \Rightarrow \tau_{nom} = \frac{60}{1.25} = 48 \text{ MPa}$$

$$\tau_{nom} = \frac{M_t}{(J/C)} = \frac{M_t}{(\pi d^3/16)}$$

$$48 = \frac{M_t}{(\pi \times 40^3/16)} \Rightarrow \text{torque } M_t = 603.186 \times 10^3 \text{ N-mm}$$

$$M_t = 603.186 \text{ N-m}$$

But $$P = \frac{2\pi n M_t}{60000} \text{ kW with } M_t \text{ in N-m}$$

$$\text{Power, } P = \frac{2\pi \times 2000 \times 603.186}{60000} = 126.33 \text{ kW.}$$

Problem 13: A stepped shaft is subjected to a transverse load of 8kN as shown in Fig. 3.27. The shaft is made of steal with ultimate tensile strength of 400 MPa. Determine the diameter of the shaft based on factor of safety of 2.

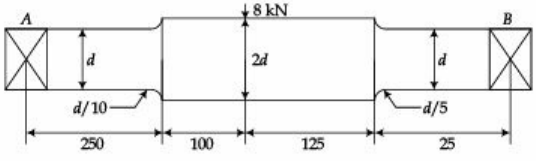


Fig. 3.27

Given data: $\sigma_u = 400$ MPa FOS = 2

$\therefore$ Allowable stress, $\sigma_{max} = \frac{\sigma_u}{FOS} = \frac{400}{2} = 200$ MPa

First find reactions at A and B

$$R_A + R_B = 8000$$

Taking moments about A, $8000 \times 350 = R_B \times 500$

$\therefore$ $R_B = 5600$ N and $R_A = 8000 - R_B = 2400$ N

(i) Consider stress concentration towards left fillet due to bending under R_A. (Fig. (a))

$$M_b = 2400 \times 250$$
$$= 600 \times 10^3 \text{ N-mm}$$

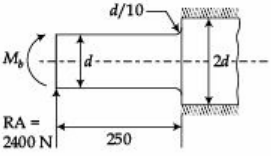


Fig. (a)

$$\frac{r}{d} = 0.1$$

$$\frac{D}{d} = 2$$

Ref Fig. 3.10, stress concentration factor, $k_\sigma = 1.58$ ...(Fig. 4.21 A/P 4.20)

But $$k_\sigma = \frac{\sigma_{max}}{\sigma_{nom}} \Rightarrow \sigma_{nom} = \frac{\sigma_{max}}{K_\sigma} = \frac{200}{1.58} = 126.58 \text{ MPa}$$

For bending $$\sigma_{nom} = \frac{M_b}{(I/C)} = \frac{M_b}{(\pi d^3/32)}$$

$$126.58 = \frac{600 \times 10^3}{(\pi d^3/32)} \Rightarrow d = 36.41 \text{ mm}$$

(ii) Consider stress concentration towards right fillet due to bending under R_B (Fig. (b))

$$r = \frac{d}{5}, \qquad d = d, \qquad D = 2d$$

$\therefore$ $$\frac{r}{d} = \frac{1}{5} = 0.2$$

$$\frac{D}{d} = 2$$

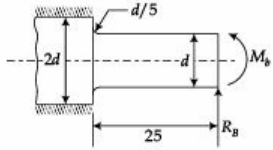


Fig. (b)

Form Fig. 3.10, stress concentration factor, $k_\sigma = 1.32$

$$k_\sigma = \frac{\sigma_{max}}{\sigma_{nom}} \Rightarrow \sigma_{nom} = \frac{\sigma_{max}}{K_\sigma} = \frac{200}{1.32} = 151.52 \text{ MPa}$$

Bending moment, $M_b = R_B \times 25$

$$M_b = 5600 \times 25 = 140 \times 10^3 \text{ N-mm}$$

$$\sigma_{nom} = \frac{M_b}{(I/C)} = \frac{M_b}{(\pi d^3/32)}$$

$$151.52 = \frac{140 \times 10^3}{(\pi d^3/32)} \Rightarrow d = 21.11 \text{ mm}$$

Recommended diameter, $d = 36.41 \simeq 37$ mm (Bigger value)

Problem 14: A grooved shaft of diameter D has a semicircular groove of radius $0.1D$. It is required to transmit 60 kW at 3000 rpm. Determine the suitable diameter of shaft taking allowable shear stress as 75 MPa.

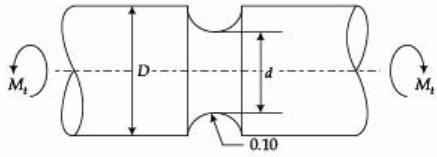


Fig. 3.28

Given data: $P = 60$ kW, $n = 3000$ rpm

$D = D$ $r = 0.1D$

$d = D - 2r = D - 2 \times 0.1D = 0.8D$

$$\frac{r}{d} = \frac{0.1D}{0.8D} = 0.125 \quad \text{and} \quad \frac{D}{d} = \frac{D}{0.8D} = 1.25$$

Ref Fig. 3.14, stress concentration factor, $k_\tau = 1.42$ ...(Fig. 4.18 A/P 4.18)

$\tau_{max} = 75$ MPa

$$k_\tau = \frac{\tau_{max}}{\tau_{nom}}, \quad \therefore \quad \tau_{nom} = \frac{\tau_{max}}{K_\tau} = \frac{75}{1.42} = 52.82 \text{ MPa}$$

$$\tau_{nom} = \frac{M_t}{(J/C)} = \frac{M_t}{(\pi d^3/16)}$$

where $$M_t = \frac{9.55 \times 10^6 P}{N} = \frac{9.55 \times 10^6 \times 60}{3000}$$

$M_t = 191000$ N-mm

$\therefore$ $$52.82 = \frac{191000}{(\pi d^3/16)} \Rightarrow d = 26.41 \text{ mm}$$

But $d = 0.8D \Rightarrow D = \frac{d}{0.8} = \frac{26.41}{0.8} = 33$ mm

$\therefore$ $D = 33$ mm.

Problem 15: A grooved shaft 50 mm with a groove radius of 5 mm is made of steel having allowable stress of 90 MPa. Find the safe axial load that can be applied on this shaft.

Given data: $D = 50$ mm

$r = 5$ mm

$d = D - 2r = 40$ mm

$F = ?$ $\sigma_{max} = 90$ MPa

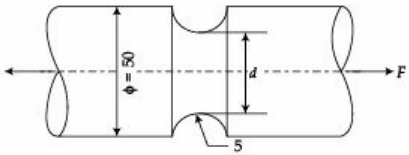


Fig. 3.29

Ref. Fig 3.12,

$$\frac{r}{d} = \frac{5}{40} = 0.125, \quad \frac{D}{d} = \frac{50}{40} = 1.25$$

$\therefore$ Stress concentration factor, $k_\sigma = 1.85$ ...(Fig. 4.14 A/P 4.16)

But $$k_\sigma = \frac{\sigma_{max}}{\sigma_{nom}}$$

$\therefore$ $$\sigma_{nom} = \frac{\sigma_{max}}{K_\sigma} = \frac{90}{1.85} = 48.65 \text{ MPa}$$

But $$\sigma_{nom} = \frac{F}{A} = \frac{F}{(\pi d^2/4)}$$

$\therefore$ $$F = \sigma_{nom} \times \frac{\pi}{4} d^2 = 48.65 \times \frac{\pi}{4} 40^2 = 61.13 \times 10^3 \text{ N}$$

Load capacity, $F = 61.13 \times 10^3$ N.

EXERCISE

1. Determine the maximum stress induced in the following cases taking stress concentration into account:
   (a) A grooved shaft 60 mm diameter with a semicircular groove of radius 6 mm subjected to a bending moment of 2 kNM.
   (b) A notched plate 100 mm wide 10 mm thick with a semicircular notch of radius 10 mm subjected to a tensile load of 30 kN.
2. Determine the safe power that can be transmitted by a grooved shaft 50 mm dia with a semicircular groove of radius 4 mm limiting the shear stress to 60 MPa. The shaft runs at 300 rpm.
3. Determine the diameter of a shaft made of SAE 1045 steel (σ_y = 309.9 MPa) loaded as shown in the Fig. 3.30. Take FOS = 2.

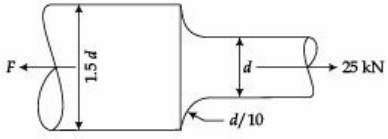


Fig. 3.30

4. A stepped shaft of circular cross section shown in Fig. 3.31 is made of 20 M_n 2 steel (σ_y = 431.5 MPa). Determine the value of d and radius r, so that the maximum stress will be limited to a value corresponding to a FOS = 2.5. Take stress concentration into account.

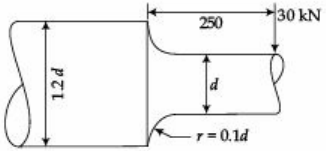


Fig. 3.31

5. A rectangular plate 15 mm thick made of brittle material is shown in Fig. 3.32. Calculate the stress at each hole.

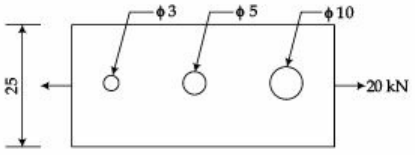


Fig. 3.32

CHAPTER

4 Fluctuating or Variable Loads

4.1 FLUCTUATING STRESS

Variable stresses may be classified as: (a) fluctuating stress, (b) repeated stress, (c) reversed stress and (d) alternating stress, as shown in Fig. 4.1.

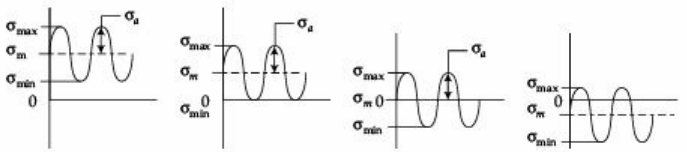


Fig. 4.1

σ_{max} = maximum stress, σ_{min} = minimum stress

σ_m = mean stress = $\frac{\sigma_{max} + \sigma_{min}}{2}$, σ_a = amplitude stress = $\frac{\sigma_{max} - \sigma_{min}}{2}$

(Amplitude stress is also known as variable stress).

4.2 NOTCH SENSITIVITY (q)

Notch sensitivity 'q' of a material can be used to determine the actual stress concentration factor $k_{\sigma a}$ as a function of theoretical stress concentration factor k_σ for the case of fluctuating loads

$$q = \frac{k_{\sigma a} - 1}{k_\sigma - 1} \qquad \ldots\left(\frac{5-5}{P\,5-2}\right)$$

or actual or real stress concentration factor

$$k_{\sigma a} = 1 + q(k_\sigma - 1) \qquad \ldots(5\text{-}6)$$

$$\text{or } k_{\tau a} = 1 + q(k_\tau - 1)$$

where q = notch sensitivity factor ($q = 0$ to 1.0)

k_σ = theoretical stress concentration factor from graphs (refer chapter 3).

4.3 ENDURANCE LIMIT

Endurance limit of a material is frequently determined experimentally by rotating a test specimen while it is loaded in bending. A graph is drawn for stress (S) versus no. of cycles (N) called S–N curved. A typical S–N curve for steels is shown in Fig. 4.2.

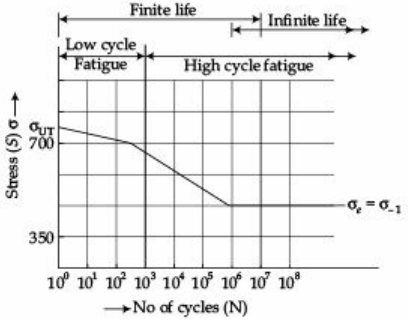


Fig. 4.2 Typical S–N diagram for steels.

For steel and iron, the S–N diagram becomes horizontal at some point. The strength at this point is called endurance limit σ_e or σ_{-1}. This occurs close to 10^5 to 10^6 cycles for typical steels. No. of cycles 1 to 1000 is called low-cycles fatigue and high-cycles fatigue is concerned with the failure corresponding to cycles greater than 10^3.

There are two equations popularly used for fluctuating loads.

(i) Soderberg's relation

$$k_{\sigma a}\frac{\sigma_a}{\sigma_{-1d}}+\frac{\sigma_m}{\sigma_{yd}}=1 \text{ and} \qquad \text{...(5-52/P 5.21)}$$

$$k_{\tau a}\frac{\tau a}{\tau_{-1d}}+\frac{\tau_m}{\tau_{yd}}=1 \qquad \text{...(5-64/P 5-12)}$$

(ii) Goodman's relation

$$k_{\sigma a}\frac{\sigma_a}{\sigma_{-1d}}+\frac{\sigma_m}{\sigma_{ud}}=1 \text{ and} \qquad \text{...(5-51/P 5.11)}$$

$$k_{\tau a}\frac{\tau_a}{\tau_{-1d}}+\frac{\tau_m}{\tau_{ud}}=1 \qquad \text{...(5-63/P 5-12)}$$

where

$k_{\sigma a}$ = actual stress concentration factor in tension and bending

$k_{\tau a}$ = actual stress concentration factor in torsion

$k_{\sigma a} = 1 + q\,(k_\sigma - 1)$

$k_{\tau a} = 1 + q\,(k_\tau - 1)$

k_σ or k_τ = theoretical stress concentration factor from graphs

q = notch sensitivity index.

σ_a = stress amplitude or variable stress = $\dfrac{\sigma_{max}-\sigma_{min}}{2}$

σ_m = mean stress = $\dfrac{\sigma_{max}+\sigma_{min}}{2}$

σ_{max} = maximum stress

σ_{min} = minimum stress

$$\left(\text{Similarly } \tau_a=\frac{\tau_{max}-\tau_{min}}{2} \text{ and } \tau_m=\frac{\tau_{max}+\tau_{min}}{2}\right)$$

σ_{-1d} = design endurance stress = $\dfrac{\sigma_{-1actual}}{FOS}=\dfrac{k_l k_s k_z \sigma_{-1}}{FOS}$

$\sigma_{-1\ actual}$ = actual endurance stress = $k_l\,k_s\,k_z\sigma_{-1}$

$\sigma_{-1} = \sigma_{en}$ = endurance stress in reverse bending $\approx \dfrac{\sigma_u}{2}$ for steels ...(T 5-5/P 5.14)

$k_l = A$ = load factor = 1 for bending loads

= 0.7 for axial loading

= 0.6 for torsional loading ...(5-19e/P 5.5)

$k_z = B$ = size factor = 0.85 for size ranging from 12.5 mm to 50 mm

k_s = C = surface correction factor = 1 for polished specimen

= 0.8 to 0.9 depending on endurance strength and type of surface ...(5-19g/P 5.5)
($\simeq$ 0.85)

σ_{yd} = design stress based on yield stress = $\dfrac{\sigma_y}{FOS}$

σ_y = yield stress

σ_{ud} = design stress based on ultimate stress = $\dfrac{\sigma_y}{FOS}$

σ_u = ultimate stress.

N = factor of safety = FOS (1.25 to 3)

Substituting, the Soderberg's relations are:

$$k_{\sigma a}\frac{\sigma_a}{\sigma_{-1}k_l k_s k_z}+\frac{\sigma_m}{\sigma_y}=\frac{1}{N}$$

$$k_{\tau a}\frac{\tau_a}{\sigma_{-1}k_l k_s k_z}+\frac{\tau_m}{\tau_y}=\frac{1}{N}$$

Goodman's relations are:

$$k_{\sigma a}\frac{\sigma_a}{\sigma_{-1}k_l k_s k_z}+\frac{\sigma_m}{\sigma_u}=\frac{1}{N} \text{ and}$$

$$k_{\tau a}\frac{\tau_a}{\sigma_{-1}k_l k_s k_z}+\frac{\tau_m}{\tau_u}=\frac{1}{N}$$

Note:

σ_{-1} = endurance stress in reverse bending.

If is used for axial load, $\sigma_{-1\,Axial} = \sigma_{-1} \times k_l$ and $k_l = 0.7$ for axial loads.

If is used for torsional load

$\tau_{-1} = \sigma_{-1}.k_l$, and $k_l = 0.6$ for torsional loads.

4.4 DERIVATION OF SODERBERG'S RELATION

Figure 4.3 shows a graph drawn for σ_a V_s σ_m. Line AB is Soderberg's failure line. Line CD is safe Soderberg's line. Consider any point $P(\sigma_m, k_\sigma\sigma_a)$ on line CD. Drop a perpendicular PE from P.

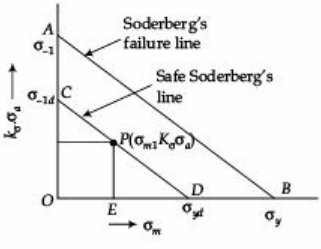


Fig. 4.3

Δs COD and PED are similar

$$\frac{PE}{OC}=\frac{DE}{OD}$$

But $DE = OD - OE$

$$\therefore \quad \frac{PE}{OC} = \frac{OD - OE}{OD} = 1 - \frac{OE}{OD}$$

$$\frac{PE}{OC} + \frac{OE}{OD} = 1$$

where $PE = y$ coordinate of $P = k_\sigma \sigma_a$

$OC = \sigma_{-1d}$

$OE = x$-coordinate of $E = \sigma_m$

$OD = \sigma_{yd}$

Substituting

$$\frac{k_\sigma \sigma_a}{\sigma_{-1d}} + \frac{\sigma_m}{\sigma_{yd}} = 1$$

where, $\sigma_a = \dfrac{\sigma_{max} - \sigma_{min}}{2}$, $\quad \sigma_m = \dfrac{\sigma_{max} + \sigma_{min}}{2}$

$$\sigma_{-1d} = \frac{\sigma_{-1} k_l k_s k_z}{FOS} \qquad \sigma_{yd} = \frac{\sigma_y}{FOS}$$

FOS = Factor of safety.

4.5 DERIVATION GOODMAN'S RELATION

Figure 4.4 shows a graph drawn for σ_a V_s σ_m. Line AB is Goodman's failure line. Line CD is safe AB is Goodman's line. Consider any point $P(\sigma_m k_\sigma \sigma_a)$ on line CD. Drop a perpendicular PE

Δls COD and PED are similar.

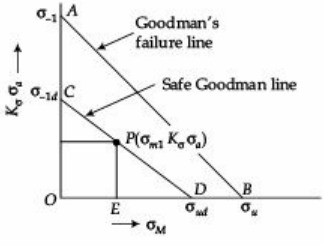


Fig. 4.4

$$\therefore \quad \frac{PE}{OC} = \frac{DE}{OD}$$

But $DE = OD - OE$

$$\frac{PE}{OC} = \frac{OD - OE}{OD} = 1 - \frac{OE}{OD}$$

$$\frac{PE}{OC} + \frac{OE}{OD} = 1$$

$$\frac{k_\sigma \sigma_a}{\sigma_{-1d}} + \frac{\sigma_m}{\sigma_{ud}} = 1$$

where $\sigma_{ud} = \dfrac{\sigma_u}{FOS}$, $\quad \sigma_{-1d} = \dfrac{\sigma_{-1} k_l k_s k_z}{FOS}$

$\sigma_a = \dfrac{\sigma_{max} - \sigma_{min}}{2}$, $\quad \sigma_m = \dfrac{\sigma_{max} + \sigma_{min}}{2}$

FOS = Factor of safety

(By using τ in place of σ, similar equations can be derived for shear loads)

PROBLEMS

Problem 1: A SAE 1045 steel rod is subjected to a tensile load that varies from 60 to 120 kN. The material has ultimate stress of 586 MPa, yield stress of 309.9 MPa and endurance stress of 290 MPa. Find the diameter of the rod taking factor of safety as 2.

Given data:

$F_{max} = 120 \text{ kN} = 120 \times 10^3 \text{ N}$

$F_{min} = 60 \text{ kN} = 60 \times 10^3 \text{ N}$

$\sigma_u = 586 \text{ MPa}$

$\sigma_y = 309.9 \text{ MPa}$

$\sigma_{-1} = 290 \text{ MPa}$

$FOS = 2$

For axial loads, load factor, $k_l = 0.7$

Assume size factor, $k_z = 0.85$ and

Surface factor, $k_s = 0.85$

$$\therefore \quad \sigma_{ud} = \frac{\sigma_u}{FOS} = \frac{586}{2} = 293 \text{ MPa}$$

$$\sigma_{yd} = \frac{\sigma_y}{FOS} = \frac{309.9}{2} = 154.95 \text{ MPa}$$

$$\sigma_{-1d} = \frac{\sigma_{-1} k_l k_s k_z}{FOS} = \frac{290 \times 0.7 \times 0.85 \times 0.85}{2} = 73.33 \text{ MPa}$$

Since there is no change in the area of c/s, (Stress concentration factor, $k_\sigma = k_{\sigma a} = 1$)

For axial loads stress, $\sigma = \frac{F}{A}$

$$\therefore \quad \sigma_{max} = \frac{F_{max}}{A} = \frac{120 \times 10^3}{A} \text{ MPa}$$

$$\sigma_{min} = \frac{F_{min}}{A} = \frac{60 \times 10^3}{A} \text{ MPa}$$

$$\text{Mean stress, } \sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{\frac{120 \times 10^3}{A} + \frac{60 \times 10^3}{A}}{2}$$

$$\sigma_m = \frac{90 \times 10^3}{A} \text{ MPa}$$

$$\text{Amplitude stress, } \sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

$$\sigma_a = \frac{\frac{120 \times 10^3}{A} - \frac{60 \times 10^3}{A}}{2} = \frac{30 \times 10^3}{A} \text{ MPa.}$$

(i) From Goodman's relation

$$\frac{k_{\sigma a}\sigma_a}{\sigma_{-1d}} + \frac{\sigma_m}{\sigma_{ud}} = 1$$

$$\frac{1 \times 30 \times 10^3}{A \times 73.33} + \frac{90 \times 10^3}{A \times 293} = 1$$

$$\therefore \qquad A = 716.28 = \frac{\pi d^2}{4} \Rightarrow d = 30.2 \simeq 31 \text{ mm.}$$

(ii) From Soderberg's relation

$$\frac{k_{\sigma a}\sigma_a}{\sigma_{-1d}} + \frac{\sigma_m}{\sigma_{yd}} = 1$$

$$\frac{1 \times 30 \times 10^3}{A \times 73.33} + \frac{90 \times 10^3}{A \times 154.93} = 1$$

$$A = 989.94 = \frac{\pi d^2}{4} \Rightarrow d = 35.5 \simeq 36 \text{ mm.}$$

Problem 2: A steel connecting rod of rectangular cross section having sides ratio as 2, is subject to a completely reversed axial load 18 kW. The normal endurance stress for the material of connecting rod is 300 MPa and yield stress is 420 MPa. Determine suitable cross-sectional dimensions of the connecting rod. Taking factor of safety as 1.8, the size factor may be taken as 0.9 stress concentration factor is 1.5. Neglect column effect.

Given data: Rectangular cross-section $d = 2\,b$

Completely reversed load of 18 kN = 18000 N

$\therefore$ $F_{max} = +18{,}000$ N

$F_{min} = -18{,}000$ N

$\sigma_{-1} = 300$ MPa $\qquad \sigma_y = 420$ MPa

FOS = 1.8 $\qquad$ Size factor, $k_z = 0.9$

For axial loads, load factor, $k_l = 0.7$

Assume surface factor, $k_s = 0.85$

Stress concentration factor, $k_\sigma = 1.5$

Since notch sensitivity is not given, $k_{\sigma s} = k_\sigma = 1.5$

$$\sigma_{yd} = \frac{\sigma_y}{FOS} = \frac{420}{1.8} = 233.33 \text{ MPa}$$

$$\sigma_{-1d} = \frac{\sigma_{-1} k_l k_s k_z}{FOS} = \frac{300 \times 0.7 \times 0.85 \times 0.9}{1.8} = 89.25 \text{ MPa}$$

For axial load, $\sigma = \frac{F}{A}$

$$\sigma_{max} = \frac{F_{max}}{A} = \frac{+18 \times 10^3}{A}$$

$$\sigma_{min} = \frac{F_{min}}{A} = \frac{-18 \times 10^3}{A}$$

$\therefore$ Mean stress, $\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = 0$

$$\text{Amplitude stress, } \sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{\frac{18 \times 10^3}{A} - \left(\frac{-18 \times 10^3}{A}\right)}{2}$$

$$= \frac{18 \times 10^3}{A}$$

Applying Soderberg's relation

$$\frac{k_{as}\sigma_a}{\sigma_{-1d}} + \frac{\sigma_m}{\sigma_{yd}} = 1$$

$$1.8 \times \frac{18 \times 10^3}{A \times 89.25} + 0 = 1$$

$$A = 363.025 = bd = b \times 2b$$

$\therefore$ width, $b = 13.47 \simeq 14$ mm

depth, $d = 2b = 28$ mm.

Problem 3: Figure 4.5 shows a cantilever beam made of C40 steel (σ_u = 620 MPa and σ_y = 328.6 MPa) is subjected to a transverse load that varies from 60 kN to 120 kN. The material has a notch sensitivity of 0.9. The size factor may be taken as 0.83 and the specimen is polished. Determine the diameter taking factor of safety as 2.

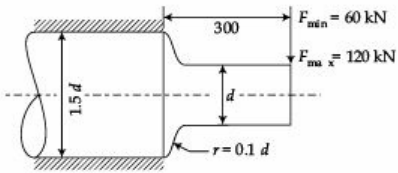


Fig. 4.5

Given data:

$F_{max} = 120$ kN $= 120 \times 10^3$ N

$F_{min} = 60$ kN 60×10^3 N

$\sigma_u = 620$ MPa

$\sigma_y = 328.6$ MPa

Assume, $\sigma_{en} = \sigma_{-1} = \frac{1}{2}\sigma_u = \frac{1}{2} \times 620 = 310$ MPa

Notch sensitivity factor, $q = 0.9$

size factor, $k_z = 0.83$

For polished specimen, surface factor $k_s = 1$

For bending loads, load factor $k_l = 1$

Factor of safety = 2

$$\therefore \quad \sigma_{ud} = \frac{\sigma_u}{FOS} = \frac{620}{2} = 310 \text{ MPa}$$

$$\sigma_{yd} = \frac{\sigma_y}{FOS} = \frac{328.6}{2} = 164.3 \text{ MPa}$$

$$\sigma_{-1d} = \frac{\sigma_{-1} k_l k_s k_z}{FOS} = \frac{310 \times 1 \times 1 \times 0.83}{2} = 128.65 \text{ MPa}$$

For bending loads, $\sigma = \dfrac{M_b \cdot C}{I}$

$$\sigma_{max} = \frac{M_{b\,max} C}{I}$$

$$M_{b\,max} = F_{max} \cdot l = 120 \times 10^3 \times 300 = 36 \times 10^6 \text{ N mm}$$

$$\therefore \quad \sigma_{max} = \frac{36 \times 10^6 \times d/2}{\left(\frac{\pi d^4}{64}\right)} = \frac{366.69 \times 10^6}{d^3}$$

Similarly, $\sigma_{min} = \dfrac{M_{b\,min} \cdot C}{I}$

$$M_{b\,min} = F_{min} \times l = 60 \times 10^3 \times 300 = 18 \times 10^6 \text{ N-mm}$$

$$\therefore \quad \sigma_{min} = \frac{18 \times 10^6 \times d/2}{\left(\frac{\pi d^4}{64}\right)} = \frac{183.35 \times 10^6}{d^3}$$

$$\therefore \quad \text{Mean stress, } \sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{275.02 \times 10^6}{d^3}$$

$$\text{Amplitude stress, } \sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{91.67 \times 10^6}{d^3}$$

From Fig. 3.13 for stress concentration factor. ...(Fig. 4-21 A/P 4.20)

$$x\text{-axis } \frac{r}{d} = \frac{0.1d}{d} = 0.1$$

$$\frac{D}{d} = \frac{1.5d}{d} = 1.5$$

$\therefore$ Stress concentration factor, $k_\sigma = 1.77$

With notch sensitivity, actual stress concentration factor,

$$k_{\sigma a} = 1 + q(k_\sigma - 1)$$
$$= 1 + 0.9(1.77 - 1) = 1.693$$

From Goodman's relation

$$\frac{k_{\sigma a}\sigma_a}{\sigma_{-1}d} + \frac{\sigma_m}{\sigma_{ud}} = 1$$

$$1.693 \times \frac{91.67 \times 10^6}{d^3 \times 128.65} + \frac{275.02 \times 10^6}{d^3 \times 310} = 1$$

$\therefore$ $$d = 127.93 \simeq 128 \text{ mm}$$

From Soderberg's relation $\dfrac{k_{\sigma a}\sigma_a}{\sigma_{-1d}} + \dfrac{\sigma_m}{\sigma_{yd}} = 1$

$$1.693 \times \frac{91.67 \times 10^6}{d^3 \times 128.65} + \frac{275.02 \times 10^6}{d^3 \times 164.3} = 1$$

$$d = 142.28 \simeq 143 \text{ mm}.$$

Problem 4: A SAE 1025 water quenched steel rod ($\sigma_u = 620.8$ MPa, $\sigma_y = 400.1$ MPa and $\sigma_{-1} = 345.2$ MPa) of circular cross section shown in Fig. 4.6, is subjected to load varying from P to $3P$. Determine the value of P. Analyse at change of cross section use factor of safety of 3.

Given data:

$F_{max} = +3P$

$F_{min} = -P$ (opposite direction)

$l = 200$ mm (upto change of c/s)

$D = 60$ mm, $d = 30$ mm $r = 6$ mm

$\sigma_u = 620.8$ MPa, $\sigma_y = 400.1$ MPa, $\sigma_{-1} = 345.2$ MPa

FOS = 3

For bending load,

Load factor, $k_l = 1$

Assume size factor, $k_z = 0.85$

Surface factor, $k_s = 0.85$

$\therefore$ $$\sigma_{ud} = \frac{\sigma_u}{FOS} = \frac{620.8}{3} = 206.93 \text{ MPa}$$

$\therefore$ $$\sigma_{yd} = \frac{\sigma_y}{FOS} = \frac{400.1}{3} = 133.37 \text{ MPa}$$

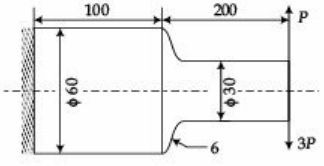


Fig. 4.6

$$\sigma_{-1d} = \frac{\sigma_{-1} k_l k_s k_z}{FOS} = \frac{345.2 \times 1 \times 0.85 \times 0.85}{3} = 83.14 \text{ MPa}$$

$$M_{bmax} = F_{max} \cdot l = 3P \times 200 = 600P$$

$$M_{bmin} = F_{min}\, l = P \times 200 = 200P$$

$$C = d/2 \text{ and } I = \pi d^4/64$$

For bending loads stress, $\sigma = \dfrac{M_b C}{I}$

$$\sigma_{max} = \frac{M_{b\,max} C}{I} = \frac{600p \times \dfrac{30}{2}}{\left(\dfrac{\pi 30^4}{64}\right)} = 0.2264P$$

$$\sigma_{min} = \frac{M_{b\,min} C}{I} = \frac{-200p \times \dfrac{30}{2}}{\left(\dfrac{\pi \times 30^4}{64}\right)} = -0.0755\,P$$

$\therefore$ Mean stress, $\sigma_m = \dfrac{\sigma_{max} + \sigma_{min}}{2} = \dfrac{(0.2264 - 0.0755)P}{2}$

$$\sigma_m = 0.0755P$$

Amplitude stress, $\sigma_a = \dfrac{\sigma_{max} - \sigma_{min}}{2} = \dfrac{[0.2264 - (-0.0755)]P}{2}$

$$\sigma_a = 0.1509P$$

From Fig. 3.13 stress concentration factor ...(Fig. 4-21 A/P 4.20)

$$\frac{r}{d} = \frac{6}{30} = 0.2, \quad \frac{D}{d} = \frac{60}{30} = 2, \quad k_\sigma = 1.43$$

Since notch sensitivity factor is not given, $q = 1$

$\therefore$ $k_{\sigma a} = k_\sigma = 1.43$

From Goodman's relation

$$\frac{k_{\sigma a}\sigma_a}{\sigma_{-1d}} + \frac{\sigma_m}{\sigma_{ud}} = 1$$

$$\frac{1.43 \times 0.0755P}{83.14} + \frac{0.1509P}{206.93} = 1$$

$\therefore$ $P = 493.14$ N

From Soderberg's relation

$$\frac{k_{\sigma a}\sigma_a}{\sigma_{-1d}} + \frac{\sigma_m}{\sigma_{yd}} = 1$$

$$\frac{1.43 \times 0.0755P}{83.14} + \frac{0.1509P}{133.37} = 1$$

$\therefore$ $$P = 411.52 \text{ N}.$$

Problem 5: A cantilever beam of rectangular cross section has a span of 800 mm. The depth is 200 mm. The free end of the beam is subjected to a transverse load that fluctuates 80 kN upward to 50 kN downward. It is made of steel having $\sigma_u = 550$ MPa and $\sigma_y = 400$ MPa. Find the width of section taking factor of safety of 2.5. The size and surface factors are 0.8 and 0.85 respectively.

Given data: Cantilever of rectangular as cross section

$$l = 800 \text{ mm}$$

$$F_{max} = 80 \text{ kN} = 80 \times 10^3 \text{ N}$$

$$F_{min} = -50 \text{ kN} = -50 \times 10^3 \text{ N}$$

(-ve because of opposite direction)

$$\sigma_u = 550 \text{ MPa}, \quad \sigma_y = 400 \text{ MPa}$$

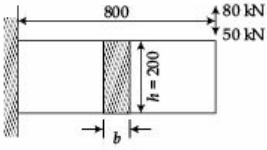


Fig. 4.7

Assume endurance stress, $\sigma_{-1} = \frac{1}{2}\sigma_u = \frac{1}{2} \times 550 = 275$ MPa

Factor of safety, FOS = 2.5

Size factor, $k_z = 0.8$

Surface factor, $k_s = 0.85$

For bending loads, load factor, $k_l = 1$

$\therefore$ $$\sigma_{ud} = \frac{\sigma_u}{FOS} = \frac{550}{2.5} = 220 \text{ MPa}$$

$$\sigma_{yd} = \frac{\sigma_y}{FOS} = \frac{400}{2.5} = 160 \text{ MPa}$$

$$\sigma_{-1d} = \frac{\sigma_{-1} k_l k_s k_z}{FOS} = \frac{275 \times 1 \times 0.85 \times 0.8}{2.5} = 74.8 \text{ MPa}$$

Maximum bending moment, $M_{b\,max} = F_{max} \cdot l$

$$M_{b\,max} = 80 \times 10^3 \times 800 = 64 \times 10^6 \text{ N-mm}$$

Minimum bending moment, $M_{b\,min} = F_{min} l$

$$M_{b\,min} = -50 \times 10^3 \times 800 = -40 \times 10^6 \text{ N-mm}$$

For bending loads stress, $\sigma = \frac{M_b.C}{I}$

For rectangular c/s, $= C = \frac{d}{2} = \frac{200}{2} = 100$

$$I = \frac{bd^3}{12} = \frac{b(200)^3}{12} = 666666.67(b)$$

$$\therefore \quad \sigma_{max} = \frac{M_{b_{max}} C}{I} = \frac{64 \times 10^6 \times 100}{666666.67(b)} = \frac{9600}{b}$$

$$\sigma_{min} = \frac{M_{b_{min}} C}{I} = \frac{-40 \times 10^6 \times 100}{666666.67(b)} = \frac{-6000}{b}$$

$$\therefore \quad \text{Mean stress, } \sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{\frac{9600}{b} + \left(\frac{-6000}{b}\right)}{2} = \frac{1800}{b}$$

$$\text{Amplitude stress, } \sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{\frac{9600}{b} - \left(\frac{-6000}{b}\right)}{2} = \frac{7800}{b}$$

From Goodman's relation

$$\frac{k_{\sigma a}\sigma_a}{\sigma_{-1d}} + \frac{\sigma_m}{\sigma_{ud}} = 1$$

$(k_{\sigma a} = k_\sigma = 1$ since not given)

$$\frac{1 \times 7800}{b \times 74.8} + \frac{1800}{b \times 220} = 1$$

$$b = 112.46 \simeq 113 \text{ mm}$$

From Soderberg's relation $\frac{k_\sigma \sigma_a}{\sigma_{-1d}} + \frac{\sigma_m}{\sigma_{yd}} = 1$

$$\frac{1 \times 7800}{b \times 74.8} + \frac{1800}{b \times 160} = 1$$

$$b = 115.53 \simeq 116 \text{ mm}.$$

Problem 6: Determine the maximum load for the simply supported beam cyclically loaded as shown in Fig. 4.8. The ultimate strength = 690 MPa, yield point in tension = 520 MPa, endurance strength in reverse bending is 345 MPa. Design factor = 1.3. Take size factor = 0.85 and surface factor

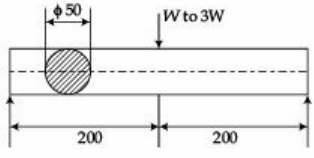


Fig. 4.8

= 0.9.

Given data: Max. load, $F_{max} = 3\,W$

Min. load, $F_{min} = W$

Distance between supports, $l = 400$

Load acting in the middle.

Diameter, $d = 50$ mm

$\sigma_u = 690$ MPa, $\sigma_y = 520$ MPa, $\sigma_{-1} = 345$ MPa

Design factor = FOS = 1.3

Size factor, $k_z = 0.85$, surface factor, $k_s = 0.9$

For bending, load factor, $k_l = 1$

Stress concentration factor, $k_\sigma = k_{\sigma u} = 1$

(No change in cross section)

$\therefore$ $$\sigma_{ud} = \frac{\sigma_u}{FOS} = \frac{690}{1.3} = 530.77 \text{ MPa}$$

$$\sigma_{yd} = \frac{\sigma_y}{FOS} = \frac{520}{1.3} = 400 \text{ MPa}$$

$$\sigma_{-1d} = \frac{\sigma_{-1} k_l k_s k_z}{FOS} = \frac{345 \times 1 \times 0.9 \times 0.85}{1.3} = 203.02 \text{ MPa}$$

For simply supported beam with central load,

Bending moment, $M_b = \frac{Fl}{4}$

$\therefore$ $$M_{bmax} = \frac{F_{max}\, l}{4} = \frac{3W \times 400}{4} = 300\,W$$

$$M_{bmin} = \frac{F_{min}\, l}{4} = \frac{W \times 400}{4} = 100\,W$$

Bending stress, $\sigma = \frac{M_b . C}{I}$

$$C = \frac{d}{2} = \frac{50}{2} = 25 \text{ mm}$$

$$I = \frac{\pi d^4}{64} = \frac{\pi \times 50^4}{64} = 306.8 \times 10^3 \text{ mm}^4$$

$\therefore$ $$\sigma_{max} = \frac{M_{b_{max}} \cdot C}{I} = \frac{300W \times 25}{306.8 \times 10^3} = 0.02445\,W$$

$$\sigma_{min} = \frac{M_{b_{min}} \cdot C}{I} = \frac{100W \times 25}{306.8 \times 10^3} = 0.008149\,W$$

$$\therefore \quad \text{Mean stress, } \sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = 0.0163\ W$$

$$\text{Amplitude stress, } \sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = 0.008149\ W$$

From Goodman's relation. $\dfrac{k_{ca}\sigma_a}{\sigma_{-1d}} + \dfrac{\sigma_m}{\sigma_{ud}} = 1$

$$\frac{1 \times 0.008149\,W}{203.02} + \frac{0.0163\,W}{530.77} = 1$$

$$W = 14.115 \times 10^3\ \text{N} \quad \text{and} \quad 3W = 42.345 \times 10^3\ \text{N}$$

From Soderberg's relation $\dfrac{k_{ca}\sigma_a}{\sigma_{-1d}} + \dfrac{\sigma_m}{\sigma_{yd}} = 1$

$$\frac{1 \times 0.008149\,W}{203.02} + \frac{0.0163\,W}{400} = 1$$

$$W = 12.363 \times 10^3\ \text{N} \quad \text{and} \quad 3W = 37.09 \times 10^3\ \text{N}.$$

Problem 7: A section of a shaft of diameter 'd' is joined to a section of diameter 1.5 d with a fillet which produces an actual stress concentration factor of 1.22 for the shaft in torsion. The material has an yield point in shear as 336 MPa and endurance stress in reverse bending as 375 MPa. Determine the size of the shaft required for a torque which varies from 0 to 2.26 kNm Use design factor = 2.

Given data: $d = d$

$D = 1.5\ \text{d}$

Actual stress concentration factor, $k_{ca} = 1.22$

Torsional yield stress, $\tau_y = 336$ MPa

Endurance stress in reverse bending, $\sigma_{-1} = 375$ MPa.

FOS = 2

Max. torque, $M_{t\max} = 2.26$ kNm $= 2.26 \times 10^6$ N-mm

Min. torque, $M_{t\min} = 0$

For torsional loads, load factor, $k_l = 0.6$

Assume surface factor, $k_s = 0.85$

Size factor, $k_z = 0.85$

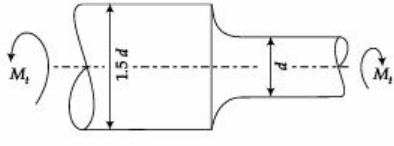


Fig. 4.9

$$\tau_{yd} = \frac{\tau_y}{\text{FOS}} = \frac{336}{2} = 168\ \text{MPa}$$

$$\tau_{-1d} = \frac{\tau_{-1\,\text{actual}}}{\text{FOS}} = \frac{\tau_{-1} k_s z_z}{\text{FOS}}$$

But $\tau_{-1} = k_l \sigma_{-1}$

$$\therefore \quad \tau_{-1d} = \frac{\sigma_{-1} k_l k_s k_z}{FOS} = \frac{375 \times 0.6 \times 0.85 \times 0.85}{2} = 81.82 \text{ MPa}$$

For torque, shear stress, $\tau = \frac{M_t \cdot r}{J}$

$$r = \frac{d}{2} \quad \text{and} \quad J = \frac{\pi d^4}{32}$$

$$\tau_{max} = \frac{M_{t\max} \cdot r}{J} = \frac{2.26 \times 10^6 \times \frac{d}{2}}{\left(\frac{\pi d^4}{32}\right)} = \frac{11.51 \times 10^6}{d^3}$$

$$\tau_{min} = \frac{M_{\tau min} \cdot r}{J} = 0$$

$$\therefore \quad \text{Mean stress, } \tau_m = \frac{\tau_{max} + \tau_{min}}{2} = \frac{11.51 \times 10^6 + 0}{2d^3} = \frac{5.755 \times 10^6}{d^3}$$

$$\text{Amplitude stress, } \tau_a = \frac{\tau_{max} - \tau_{min}}{2} = \frac{11.51 \times 10^6 - 0}{2d^3} = \frac{5.755 \times 10^6}{d^3}$$

From Soderberg's relation $= \frac{k_{\tau_e} \cdot \tau_a}{\tau_{-1d}} + \frac{\tau_m}{\tau_{yd}} = 1$

$$1.22 \times \frac{5.755 \times 10^6}{d^3 \times 81.28} + \frac{5.755 \times 10^6}{d^3 \times 168} = 1$$

$$d = 49.41 \text{ mm} \simeq 50 \text{ mm}.$$

Problem 8: A grooved shaft of diameter D has a semicircular groove of radius $0.1D$ is made of steel having torsional yield stress of 300 MPa and torsional endurance stress of 210 MPa. The size and surface factors are 0.85 and 0.87 respectively. The notch sensitivity factor = 0.95. The shaft is to sustain a twisting that moment fluctuates between 900 Nm to –300 Nm. Find the diameter of the shaft taking factor of safety as 3.

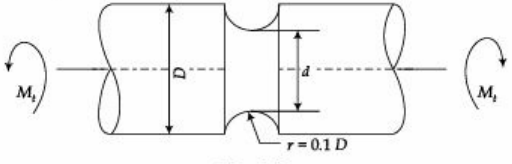


Fig. 4.10

Given data: Grooved shaft subjected to torque

$$D = D$$

$$r = 0.1D$$

$$d = D - 2r = 0.8D$$

Torsional yield stress, $\tau_y = 300$ MPa

Torsional endurance stress, $\tau_{-1} = 210$ MPa

$$FOS = 3, \quad k_z = 0.85, \quad k_s = 0.87$$

(Since torsional endurance stress is given directly, no need to use load factor. If endurance stress in reverse bending is given, then, $k_l = 0.6$, $\tau_{-1} = 0.6\,\sigma_{-1}$)

Notch sensitivity factor $q = 0.95$ ...(Fig. 4-18 A/P4.18)

Ref. Figure 3 -11,

$$\frac{r}{d} = \frac{0.1D}{0.8D} = 0.125$$

$$\frac{D}{d} = \frac{D}{0.8D} = 1.25$$

$\therefore$ Stress concentration factor, $k_\tau = 1.38$

with notch sensitivity, actual stress concentration factor

$$k_{\tau a} = 1 + q\,(k_\tau - 1)$$

$$k_{\tau a} = 1 + 0.95\,(1.38 - 1) = 1.361$$

$$\tau_{yd} = \frac{\tau_y}{FOS} = \frac{300}{3} = 100 \text{ MPa}$$

$$\tau_{-1d} = \frac{(\sigma_{-1} k_l) k_s k_z}{FOS} = \frac{\tau_{-1} k_s k_z}{FOS} = \frac{210 \times 0.85 \times 0.87}{3}$$

$$\tau_{-1d} = 51.77 \text{ MPa}$$

Maximum torque, $M_{t\max} = 900$ N-m $= 900 \times 10^3$ N-mm

Minimum torque, $M_{t\min} = -300$ N-m $= -300 \times 10^3$ N-mm

For torsional loads, $\tau = \dfrac{M_t r}{J}$

$$r = \frac{d}{2} \quad \text{and} \quad J = \frac{\pi d^4}{32}$$

$\therefore$

$$\tau_{\max} = \frac{M_{t\max} r}{J} = \frac{900 \times 10^3 \cdot \frac{d}{2}}{\left(\frac{\pi d^4}{32}\right)} = \frac{4.584 \times 10^6}{d^3}$$

$$\tau_{min} = \frac{M_{t\,min} \cdot r}{J} = \frac{-300 \times 10^3 \times \frac{d}{2}}{\left(\frac{\pi d^4}{32}\right)} = \frac{-1.529 \times 10^6}{d^3}$$

$$\therefore \quad \text{Mean stress, } \tau_m = \frac{\tau_{max} + \tau_{min}}{2} = \frac{\frac{4.584 \times 10^6}{d^3} - \frac{1.529 \times 10^6}{d^3}}{2}$$

$$\tau_m = \frac{1.529 \times 10^6}{d^3}$$

$$\text{Amplitude stress, } \tau_a = \frac{\tau_{max} - \tau_{min}}{2} = \frac{\frac{4.584 \times 10^6}{d^3} - \left(\frac{-1.529 \times 10^6}{d^3}\right)}{2}$$

$$\tau_a = \frac{3.056 \times 10^6}{d^3}$$

From Soderberg's equation for shear stress.

$$\frac{k_{ta} \cdot \tau a}{\tau_{-1d}} + \frac{\tau_m}{\tau_{yd}} = 1$$

$$1.361 \times \frac{3.056 \times 10^6}{d^3 \times 51.77} + \frac{1.529 \times 10^6}{d^3 \times 100} = 1$$

$$d = 45.73 \simeq 46 \text{ mm}$$

But $$d = 0.8D \Rightarrow D = \frac{d}{0.8} = \frac{46}{0.8} = 57.5 \text{ m} \simeq 58 \text{ m}$$

Dia of shaft, $D = 58$ mm.

Problem 9: A thin walled cylindrical pressure vessel of diameter 600 mm is subjected to internal pressure varying between 3 MPa and 6 MPa. Find the thickness of pressure vessel based on an yield strength of 360 MPa and endurance strength in reverse bending of 240 MPa. Assume factor of safety as 2.1. Use Soderberg's criterion.

Given data: Dia = d = 600 mm

Max. pressure, p_{max} = 6 MPa

Min. pressure, p_{min} = 3 MPa

Yield stress, σ_y = 360 MPa

Endurance stress, σ_{-1} = 240 MPa

Factor of safety, FOS = 2.1

Due to internal pressure, the pressure vessel wall is subjected to tensile stress (Hoop stress)

Hence, take load factor, $k_l = 0.7$

Assume size factor, $k_z = 0.85$ and

Surface factor, $k_s = 0.85$

$$\therefore \quad \sigma_{yd} = \frac{\sigma_y}{FOS} = \frac{360}{2.1} = 171.43 \text{ MPa}$$

$$\sigma_{-1d} = \frac{\sigma_{-1} k_l k_s k_z}{FOS} = \frac{240 \times 0.7 \times 0.85 \times 0.85}{2.1} = 57.8 \text{ MPa}$$

For thin cylinders, thickness of plate, $h = \frac{pd}{2\sigma}$

$$\therefore \quad \text{Stress, } \sigma = \frac{pd}{2h}$$

$$\sigma_{max} = \frac{p_{max} d}{2h} = \frac{6 \times 600}{2h} = \frac{1800}{h}$$

$$\sigma_{min} = \frac{p_{min} d}{2h} = \frac{3 \times 600}{2h} = \frac{900}{h}$$

$$\therefore \quad \text{Mean stress, } \sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{\frac{1800}{h} + \frac{900}{h}}{2} = \frac{1350}{h}$$

$$\text{Amplitude stress, } \sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{\frac{1800}{h} - \frac{900}{h}}{2} = \frac{450}{h}$$

Neglect stress concentration, $k_\sigma = k_{\sigma a} = 1$

From Soderberg's criterion, $\frac{k_{\sigma a}\sigma_a}{\sigma_{-1d}} + \frac{\sigma_m}{\sigma_{yd}} = 1$

$$\frac{1 \times 450}{h \times 57.8} + \frac{1800}{h \times 171.43} = 1$$

Thickness of wall, $h = 18.28 \simeq 19$ mm.

4.6 COMBINED STRESS

(i) When a member carries variable bending moment and variable axial load together.

Equivalent maximum normal stress = $\sigma_{eq\,max} = \frac{\sigma_y}{FOS}$

where $\sigma_{eq\,max} = \sigma_{eqA} + \sigma_{eqB}$

σ_{eqA} = Equivalent stress due to axial load

$$\sigma_{eqA} = \sigma_{mA} + \frac{\sigma_y \cdot k_{\sigma A} \cdot \sigma_{aA}}{\sigma_{-1} k_l k_s k_z}$$

Take $k_l = 0.7$ for axial laods

Similarly, σ_{eqB} = Equivalent stress due to bending

$$\sigma_{eqB} = \sigma_{mB} + \frac{\sigma_y k_{\sigma B} \cdot \sigma_{aB}}{\sigma_{-1} k_l k_s k_z}$$

Take $k_l = 1$ for bending loads

σ_{mA} = mean axial stress

σ_{aA} = amplitude axial stress

$k_{\sigma A}$ = stress concentration factor in axial load

σ_{-1} = endurance stress in reverse bending

k_s = surface factor

k_z = size factor (k_s and k_z are same for both)

σ_{mB} = mean bending stress

σ_{aB} = Amplitude bending stress

$k_{\sigma B}$ = stress concentration factor in bending

σ_y = yield stress.

(ii) Variable bending moment or axial load combined with variable torque.

Equivalent maximum normal stress, $\sigma_{eq\max} = \dfrac{\sigma_y}{\text{FOS}}$

where $\sigma_{eq\max} = \dfrac{1}{2}\left[\sigma_{eq} + \sqrt{\sigma_{eq}^2 + 4\tau_{eq}^2}\right]$.

and equivalent maximum shear stress, $\tau_{eq\max} = \dfrac{\tau_y}{\text{FOS}}$

or $\tau_{eq\max} = \dfrac{\sigma_y}{2\,\text{FOS}} \quad \because \tau_y = \dfrac{\sigma_y}{2}$

$$\tau_{eq\,\max} = \frac{1}{2}\sqrt{\sigma_{eq}^2 + 4\tau_{eq}^2}$$

where σ_{eq} = equivalent normal stress

$$\sigma_{eq} = \sigma_m + \frac{\sigma_y}{\sigma_{-1}} \frac{k_\sigma \cdot \sigma_a}{k_l k_s k_z}$$

$k_l = 0.7$ for axial load

= 1.0 for bending load.

σ_m = mean stress (axial or bending)

σ_a = amplitude stress (axial or bending)

k_σ = stress concentration factor (either in axial or bending)

τ_{eq} = equivalent shear stress

$$\tau_{eq} = \tau_m + \frac{\tau_y}{\sigma_{-1}} \frac{k_\tau \cdot \tau_a}{k_l k_s k_z}$$

k_l = 0.6 for torsional loads

$$\tau_y = \text{torsional yield stress} = \frac{\sigma_y}{2}$$

τ_m = mean shear stress

τ_a = amplitude shear stress

σ_{-1} = endurance stress in reverse bending (with load factor = 0.6, $\sigma_{-1} \times k_l = \tau_{-1}$)

k_τ = stress concentration factor in torsion.

Generally, equivalent maximum normal stress relation is used for brittle materials and equivalent maximum shear stress relation is used for ductile materials.

Problem 10: A cold drawn steel rod of circular cross section is subjected to a variable bending moment of 540 N-m to 1080 N-m as the axial load varies from 4kN to 10 kN. The maximum axial load and maximum bending moment occur at the same instant. The material has an ultimate stress of 570 MPa and yield stress of 420 MPa. Taking factor of safety as 2, determine the diameter of the rod.

Given data

Maximum bending moment, $M_{b\max} = 1080$ N-m $= 1080 \times 10^3$ N-mm

Minimum bending moment, $M_{b\min} = 540$ N-m $\times 540 \times 10^3$ N-mm

Minimum axial load, $F_{\max} = 10$ kN $= 10 \times 10^3$ N

Minimum axial load, $F_{\min} = 4$ kN $= 4 \times 10^3$ N

$\sigma_u = 570$ MPa, $\sigma_y = 420$ MPa

FOS = 2

Assume $$\sigma_{-1} = \frac{\sigma_u}{2} = \frac{570}{2} = 285 \text{ MPa}$$

Load factor, k_l = 0.7 for axial load

= 1.0 for bending

Assume surface factor, k_s = 0.85

Size factor, k_z = 0.85

Stress concentration factor, k_σ = 1 for both axial and bending (not given)

(i) Consider bending

$$\sigma_{b\max} = \frac{M_{b\max} \cdot C}{I} = \frac{1080 \times 10^3 \times \frac{d}{2}}{\left(\frac{\pi d^4}{64}\right)} = \frac{11 \times 10^6}{d^3}$$

$$\sigma_{b\min} = \frac{M_{b\min} \cdot C}{I} = \frac{540 \times 10^3 \times \frac{d}{2}}{\left(\frac{\pi d^4}{64}\right)} = \frac{5.5 \times 10^6}{d^3}$$

$\therefore$ Mean bending stress, $\sigma_{bm} = \dfrac{\sigma_{b\max} + \sigma_{b\min}}{2} = \dfrac{8.25 \times 10^6}{d^3}$

Amplitude bending stress, $\sigma_{ba} = \dfrac{\sigma_{b\max} - \sigma_{b\min}}{2} = \dfrac{2.75 \times 10^6}{d^3}$

Equivalent stress due to bending

$$\sigma_{eqB} = \sigma_{bm} + \left(\frac{\sigma_y}{\sigma_{-1}}\right)\frac{k_{\sigma B}\sigma_{ba}}{k_l k_s k_z}$$

$k_l = 1$ for bending

$$\sigma_{eqB} = \frac{8.25 \times 10^6}{d^3} + \frac{420}{285}\cdot\frac{1 \times 2.75 \times 10^6}{d^3 \times 1 \times 0.85 \times 0.85} = \frac{13.86 \times 10^6}{d^3}$$

(ii) Consider axial load

$$\sigma_{A\max} = \frac{F_{\max}}{A} = \frac{10000}{\left(\frac{\pi d^2}{4}\right)} = \frac{12.73 \times 10^3}{d^2}$$

$$\sigma_{A\min} = \frac{F_{\min}}{A} = \frac{4000}{\left(\frac{\pi}{4}d^2\right)} = \frac{5.09 \times 10^3}{d^2}$$

Mean axial stress, $\sigma_{Am} = \dfrac{\sigma_{A\max} + \sigma_{A\min}}{2} = \dfrac{8911.48}{d^2}$

Amplitude axial stress, $\sigma_{Aa} = \dfrac{\sigma_{A\max} - \sigma_{A\min}}{2} = \dfrac{3820}{d^2}$

∴ Equivalent stress due to axial load

$$\sigma_{eqA} = \sigma_{Am} + \left(\frac{\sigma_y}{\sigma_{-1}}\right)\frac{k_{\sigma A}\sigma_{Aa}}{k_l k_s k_z} \quad (\text{Here } k_l = 0.7)$$

$$= \frac{8911.48}{d^2} + \left(\frac{420}{285}\right)\frac{1 \times 3820}{d^2 \times 0.7 \times 0.85 \times 0.85}$$

$$\sigma_{eqA} = \frac{20.13 \times 10^3}{d^2}$$

(iii) ∴ Maximum Equivalent normal stress

$$\sigma_{eq\,max} = \sigma_{eqB} + \sigma_{eqA}$$

$$\sigma_{eq\,max} = \frac{13.86 \times 10^6}{d^3} + \frac{20.13 \times 10^3}{d^2}$$

Equate this to $\frac{\sigma_y}{FOS}$

$$\frac{13.86 \times 10^6}{d^3} + \frac{20.13 \times 10^3}{d^2} = \frac{420}{2}$$

$$210d^3 - 20.13 \times 10^3 d - 13.80 \times 10^6 = 0$$

Solving, $d = 41.2 \simeq 42$ mm.

Problem 11: A solid steel shaft is subject to a bending moment that fluctuates between 600 N-m and –200 N-m and at the same time the torque on the shaft varies between 420 N-m clockwise to 210 N-m counter clockwise. Take σ_u = 500 MPa, σ_y = 360 MPa design factor = 2. Neglect stress concentration effect.

Given data:

$M_{bmax} = 600$ N-m $= 600 \times 10^3$ N-mm

$M_{bmin} = -200$ N-m $= -200 \times 10^3$ N-mm

$M_{tmax} = 420$ N-m $= 420 \times 10^3$ N-mm

$M_{tmin} = -210$ N-m $= -210 \times 10^3$ N-mm

$\sigma_u = 500$ MPa $\quad \sigma_y = 360$ mpa

Assume $\sigma_{-1} = \frac{\sigma_u}{2} = 250$ MPa

Factor for safety = 2. Neglecting stress concentration, $k_\sigma = 1$

Load factor for bending, $k_l = 1.0$

For torsion, $k_l = 0.7$

Assume size factor, $k_z = 0.85$

Surface factor, $k_s = 0.85$

(i) Consider bending

$$\sigma_{max} = \frac{M_{b\,max} \cdot C}{I} = \frac{600 \times 10^3 \cdot \frac{d}{2}}{\left(\frac{\pi d^4}{64}\right)} = \frac{6.112 \times 10^6}{d^3}$$

$$\sigma_{min} = \frac{M_{b\,min} \cdot C}{I} = \frac{-200 \times 10^3 \times \frac{d}{2}}{\left(\frac{\pi d^4}{64}\right)} = \frac{-2.037 \times 10^6}{d^3}$$

$\therefore$ Mean bending stress, $\sigma_m = \dfrac{\sigma_{max} + \sigma_{min}}{2} = \dfrac{2.037 \times 10^6}{d^3}$

Amplitude bending stress, $\sigma_a = \dfrac{\sigma_{max} - \sigma_{min}}{2} = \dfrac{4.075 \times 10^6}{d^3}$

Equivalent normal stress due to bending

$$\sigma_{eq} = \sigma_m + \frac{\sigma_y}{\sigma_{-1}} \frac{k_{\sigma B} \cdot \sigma_a}{k_l k_s k_z}$$

$$= \frac{2.037 \times 10^6}{d^3} + \left(\frac{360}{250}\right) \frac{1 \times 4.075 \times 10^6}{d^3 \times 1 \times 0.85 \times 0.85} = \frac{10.16 \times 10^6}{d^3}$$

(For bending $k_l = 1.0$)

(ii) Consider torque

$$\tau_{max} = \frac{M_{t\,max} \cdot r}{J} = \frac{420 \times 10^3 \times \frac{d}{2}}{\left(\frac{\pi d^4}{32}\right)} = \frac{2.139 \times 10^6}{d^3}$$

$$\tau_{min} = \frac{M_{t\,min} \cdot r}{J} = \frac{-210 \times 10^3 \times \frac{d}{2}}{\left(\frac{\pi d^4}{32}\right)} = \frac{-1.07 \times 10^6}{d^3}$$

$\therefore$ Mean shear stress, $\tau_m = \dfrac{\tau_{max} + \tau_{min}}{2} = \dfrac{0.535 \times 10^6}{d^3}$

Amplitude shear stress, $\tau_a = \dfrac{\tau_{max} - \tau_{min}}{2} = \dfrac{1.604 \times 10^6}{d^3}$

Equivalent shear stress due to torque

$$\tau_{eq} = \tau_m + \left(\frac{\tau_y}{\sigma_{-1}}\right)\frac{k_t \cdot \tau_a}{k_l k_s k_z}$$

For torsion, $k_l = 0.6$

$$\tau_y = 0.5\,\sigma_y = 0.5 \times 360 = 180 \text{ MPa}$$

$$\tau_{eq} = \frac{0.535 \times 10^6}{d^3} + \left(\frac{180}{250}\right)\frac{1 \times 1.604 \times 10^6}{d^3 \times 0.6 \times 0.85 \times 0.85}$$

$$\tau_{eq} = \frac{3.2 \times 10^6}{d^3}$$

(iii) Maximum equivalent normal stress

$$\sigma_{eq\,max} = \frac{1}{2}\left[\sigma_{eq} + \sqrt{\sigma_{eq}^2 + 4\tau_{eq}^2}\right] = \frac{\sigma_y}{FOS}$$

$$= \frac{1}{2}\left[\frac{10.16 \times 10^6}{d^3} + \sqrt{\left(\frac{10.16 \times 10^6}{d^3}\right)^2 + 4\left(\frac{3.2 \times 10^6}{d^3}\right)^2}\right] = \frac{360}{2}$$

$\therefore \quad d = 39.49 \simeq 40$ mm

(iv) Maximum equivalent shear stress

$$\tau_{eq\,max} = \frac{1}{2}\sqrt{\sigma_{eq}^2 + 4\tau_{eq}^2} = \frac{\tau_y}{FOS}$$

$$= \frac{1}{2}\sqrt{\left(\frac{10.16 \times 10^6}{d^3}\right)^2 + 4\left(\frac{3.2 \times 10^6}{d^3}\right)^2} = \frac{180}{2}$$

$\therefore \quad d = 40.56$ mm $= 41$

$\therefore$ Recommended diameter, $d = 41$ mm (bigger value).

Problem 12: A simply supported shaft carries a pulley at the centre. The torque on pulley varies between 120 N-m and 200 N-m and the bending moment varies between 300 N-m and –150 N-m. The material of shaft has an ultimate stress of 600 MPa and yield stress of 450 MPa. Endurance stress may be taken as half of the ultimate stress. The stress concentration factor for the shaft is 1.8 in bending and 1.2 in torsion. Take factor of safety as 1.8. The size and surface factors are 0.83 and 0.9 respectively.

Given data: $M_{bmin} = 120$ N-m $= 120 \times 10^3$ N-mm

$M_{bmax} = 200$ N-m $= 200 \times 10^3$ N-mm

$M_{tmax} = 300$ N-m $= 300 \times 10^3$ N-mm

$M_{tmin} = -150$ N-m $= -150 \times 10^3$ N-mm

$\sigma_u = 600$ MPa

$\sigma_y = 450$ MPa

$$\sigma_{-1} = \frac{1}{2}\sigma_u = 300 \text{ MPa}$$

FOS $= 1.8$

$k_{\sigma ben} = k_\sigma = 1.8$

$k_\tau = 1.2$

$k_z = 0.83$

$k_s = 0.9$

Load factor, $k_l = 1.0$ for bending

$k_l = 0.6$ for torsion.

(i) Consider bending

$$\sigma_{max} = \frac{M_{bmax} \cdot C}{I} = \frac{200 \times 10^3 \times \frac{d}{2}}{\left(\frac{\pi d^4}{64}\right)} = \frac{2.037 \times 10^6}{d^3}$$

$$\sigma_{min} = \frac{M_{bmin} \cdot C}{I} = \frac{120 \times 10^3 \times \frac{d}{2}}{\left(\frac{\pi d^4}{64}\right)} = \frac{1.222 \times 10^6}{d^3}$$

$\therefore$ Mean bending stress $= \sigma_{bm} = \dfrac{\sigma_{max} + \sigma_{min}}{2} = \dfrac{1.63 \times 10^6}{d^3}$

Amplitude bending stress, $\sigma_{ba} = \dfrac{\sigma_{max} - \sigma_{min}}{2} = \dfrac{0.407 \times 10^6}{d^3}$

$\therefore$ Equivalent normal stress, $\sigma_{eq} = \sigma_m + \left(\dfrac{\sigma_y}{\sigma_{-1}}\right) \dfrac{k_{\sigma b} \cdot \sigma_a}{k_l k_s k_z}$

($k_l = 1.0$ for bending)

$$\sigma_{eq} = \frac{1.63 \times 10^6}{d^3} + \left(\frac{450}{300}\right) \cdot \frac{1.8 \times 0.407 \times 10^6}{d^3 \times 1 \times 0.83 \times 0.9} = \frac{3.1 \times 10^6}{d^3}$$

(ii) Consider torque

$$\tau_{max} = \frac{M_{t\,max} \cdot r}{J} = \frac{300 \times 10^3 \times \frac{d}{2}}{\left(\frac{\pi d^4}{32}\right)} = \frac{1.528 \times 10^6}{d^3}$$

$$\tau_{min} = \frac{M_{t\,min} \cdot r}{J} = \frac{-150 \times 10^3 \times \frac{d}{2}}{\left(\frac{\pi d^4}{32}\right)} = \frac{-0.764 \times 10^6}{d^3}$$

$\therefore$ Mean shear stress, $\tau_m = \dfrac{\tau_{max} + \tau_{min}}{2} = \dfrac{0.382 \times 10^6}{d^3}$

Amplitude shear stress, $\tau_a = \dfrac{\tau_{max} - \tau_{min}}{2} = \dfrac{1.146 \times 10^6}{d^3}$

$\therefore$ Equivalent shear stress, $\tau_{eq} = \tau_m + \left(\dfrac{\tau_y}{\sigma_{-1}}\right) \dfrac{k_\tau \cdot \tau_a}{k_l k_s k_z}$

$k_l = 0.6$ for torsion, k_s and k_z are same as above

$$\tau_y = 0.5\,\sigma_y = 0.5 \times 450 = 225 \text{ MPa}$$

$\therefore$ $$\tau_{eq} = \frac{0.382 \times 10^6}{d^3} + \left(\frac{225}{300}\right) \frac{1.2 \times 1.146 \times 10^6}{d^3 \times 0.6 \times 0.83 \times 0.9} = \frac{2.683 \times 10^6}{d^3}$$

(iii) Equivalent maximum normal stress

$$\sigma_{eq\,max} = \frac{1}{2}\left[\sigma_{eq} + \sqrt{\sigma_{eq}^2 + 4\tau_{eq}^2}\right] = \frac{\sigma_y}{FOS}$$

$$= \frac{1}{2}\left[\frac{3.1 \times 10^6}{d^3} + \sqrt{\left(\frac{3.1 \times 10^6}{d^3}\right)^2 + 4\left(\frac{2.683 \times 10^6}{d^3}\right)^2}\right] = \frac{450}{1.8}$$

$\therefore$ $d = 26.49 = 27$ mm

(iv) Equivalent maximum shear stress

$$\tau_{eq\,max} = \frac{1}{2}\sqrt{\sigma_{eq}^2 + 4\tau_{eq}^2} = \frac{\tau_y}{FOS}$$

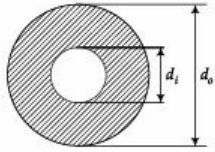
$$= \frac{1}{2}\sqrt{\left(\frac{3.1 \times 10^6}{d^3}\right)^2 + 4\left(\frac{2.683 \times 10^6}{d^3}\right)^2} = \frac{225}{1.8}$$

$\therefore \quad d = 29.16 \simeq 30$ mm

Recommended diamater, $d = 30$ mm

Problem 13: Determine the diameters of a hollow shaft to sustain a twisting moment that fluctuates between 2.5 kN-m and 1.5 kN-m together with a bending moment that fluctuates between 2 kN-m to –2 kN-m. Assume the inner diameter to be 0.6 times the outer diameter. Take normal yield stress as 400 MPa, normal endurance stress as 270 MPa and factor of safety as 2.

Given data:



Moment of inertia, $I = \frac{\pi}{64}(d_o^4 - d_i^4)$

$$I = \frac{\pi}{64}[d_o^4 - (0.6d_o)^4]$$

$$I = \frac{\pi}{64} d_o^4 (0.8704)$$

Polar moment of inertia, $J = \frac{\pi}{32}(d_o^4 - d_i^4)$

$$= \frac{\pi}{32}[d_o^4 - (0.6d_o)^4] = \frac{\pi}{32} d_o^4 (0.8704)$$

$$r = C = \frac{d_o}{2}$$

Maximum bending moment, $M_{b\max} = 2.5$ kN-m $= 2.5 \times 10^6$ N-mm

Minimum bending moment, $M_{b\min} = 1.5$ kN-m $= 1.5 \times 10^6$ N-mm

Maximum torque, $M_{t\max} = 2$ kN-m $= 2 \times 10^6$ N-mm

Minimum torque, $M_{t\min} = -2$ kN-m $= -2 \times 10^6$ N-mm

$\sigma_y = 400$ MPa, $\sigma_{-1} = 270$ MPa, FOS = 2

Assume, $\tau_y = 0.5\,\sigma_y = 200$ MPa

Size factor, $k_z = 0.85$

Surface factor, $k_s = 0.85$

Load factor for bending, $k_l = 1$ and for torsion $k_l = 0.6$

Take stress concentration factor, $k_\sigma = k_\tau = 1.0$

(No change in cross section given)

(i) Consider bending: Bending stress, $\sigma = \frac{M_b C}{I}$

$$\sigma_{\max} = \frac{M_{b\max} \cdot C}{I} = \frac{2.5 \times 10^6 \cdot \frac{d_o}{2}}{\frac{\pi}{64} d_o^4 (0.8704)} = \frac{29.26 \times 10^6}{d_o^3}$$

$$\sigma_{min} = \frac{M_{b\,min} \cdot C}{I} = \frac{1.5 \times 10^6 \times \frac{d_o}{2}}{\frac{\pi}{64} d_0^4 (0.8704)} = \frac{17.55 \times 10^6}{d_o^3}$$

$\therefore$ Mean bending stress, $\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{23.41 \times 10^6}{d_o^3}$

Amplitude bending stress, $\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{5.86 \times 10^6}{d_o^3}$

Equivalent normal stress, $\sigma_{eq} = \sigma_m + \left(\frac{\sigma_y}{\sigma_{-1}}\right) \frac{k_\sigma \sigma_a}{k_l k_s k_z}$

($k_l = 1.0$ for bending)

$$\sigma_{eq} = \frac{23.41 \times 10^6}{d_o^3} + \left(\frac{400}{270}\right) \frac{1 \times 5.86 \times 10^6}{d_o^3 \times 1 \times 0.85 \times 0.85} = \frac{35.42 \times 10^6}{d_o^3}$$

(ii) Consider torque: shear stress, $\tau = \frac{M_t \cdot r}{J}$

$$\tau_{max} = \frac{M_{t\,max} \cdot r}{J} = \frac{2 \times 10^6 \times \frac{d_o}{2}}{\frac{\pi}{32} d_o^4 (0.8704)} = \frac{11.70 \times 10^6}{d_o^3}$$

$$\tau_{min} = \frac{M_{t\,min} \cdot r}{J} = \frac{-2 \times 10^6 \times \frac{d_o}{2}}{\frac{\pi}{32} d_o^4 (0.8704)} = \frac{-11.70 \times 10^6}{d_o^3}$$

$\therefore$ Mean shear stress, $\tau_m = \frac{\tau_{max} + \tau_{min}}{2} = 0$

Amplitude shear stress $= \tau_a = \frac{\tau_{max} - \tau_{min}}{2} = \frac{11.7 \times 10^6}{d_o^3}$

Equivalent shear stress, $\tau_{eq} = \tau_m + \left(\frac{\tau_y}{\sigma_{-1}}\right) \frac{k_\tau \tau_a}{k_l k_s k_z}$

($k_l = 0.6$ for torsion)

$$\tau_{eq} = 0 + \left(\frac{200}{270}\right) \frac{1 \times 11.7 \times 10^6}{d_o^3 \times 0.6 \times 0.85 \times 0.85} = \frac{19.997 \times 10^6}{d_o^3}$$

(iii) Equivalent maximum normal stress

$$\sigma_{eq\,max} = \frac{1}{2}\left[\sigma_{eq} + \sqrt{\sigma_{eq}^2 + 4\tau_{eq}^2}\right] = \frac{\sigma_y}{FOS}$$

$$= \frac{1}{2}\left[\frac{35.42\times10^6}{d_o^3} + \sqrt{\left(\frac{35.42\times10^6}{d_o^3}\right)^2 + 4\left(\frac{19.997\times10^6}{d_o^3}\right)^2}\right] = \frac{400}{2}$$

$\therefore \quad d_o = 60.56$ mm

(iv) Equivalent maximum shear stress

$$\tau_{eq\,max} = \frac{1}{2}\sqrt{\sigma_{eq}^2 + 4\tau_{eq}^2} = \frac{\tau_y}{FOS}$$

$$= \frac{1}{2}\sqrt{\left(\frac{35.42\times10^6}{d_o^3}\right)^2 + 4\left(\frac{19.997\times10^6}{d_o^3}\right)^2} = \frac{200}{2}$$

$d_o = 64.40$

Recommended outside dia $d_o = 64.4 \simeq 65$ mm (bigger value)

$\therefore$ Inside dia $d_i = 0.6, \quad d_o = 39$ mm.

Problem 14: A round rod of diameter $1.2d$ has semicircular groove of diameter $0.2d$. This rod is to sustain a twisting moment that fluctuates between 2.5 kN-m and 1.5 kN-m together with a bending moment that fluctuates between +2 kN-m and –1 kN-m. Take $\sigma_y = 300$ MPa, $\sigma_u = 450$ MPa and factor of safety = 2.5. Find suitable diameter of rod.

Given data: $D = 1.2d$

$d = d$

$r = 0.1d$

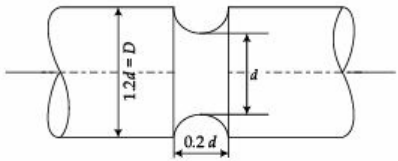


Fig. 4.11

Maximum torque

$m_{t\max} = 2.5$ kN-m $= 2.5\times10^6$ N-mm

Minimum torque

$m_{t\min} = 1.5$ kN-m $= 1.5\times10^6$ N-mm

Maximum bending moment, $M_{b\max} = +2$ kN-m $= 2\times10^6$ N-mm

Minimum bending moment, $M_{b\min} = -1$ kN-m $= -1\times10^6$ N-mm

$\sigma_u = 450$ MPa, $\sigma_y = 300$ MPa FOS = 2.5

Assume, $\sigma_{-1} = \frac{1}{2}\sigma_u = 225$ MPa.

Load factor, $k_l = 1$ for bending

$= 0.6$ for torsion

Assume size factor, $k_z = 0.85$ and

Surface factor, $k_s = 0.85$

(i) Consider bending ...(Fig. 4-16 A/P 4.17)

Ref Fig. 3.10, $\frac{r}{d} = 0.1$ and $\frac{D}{d} = 1.2$

Stress concentration factor $k_\sigma = 1.5 = k_{\sigma a}$

($\because$ q is not given)

Bending stress, $\sigma = \frac{M_b}{\left(\frac{I}{C}\right)}$ where $\frac{I}{C} = \frac{\pi d^3}{32}$

$$\therefore \quad \sigma_{max} = \frac{M_{b\,max}}{(I/C)} = \frac{2 \times 10^6}{\left(\frac{\pi d^3}{32}\right)} = \frac{20.37 \times 10^6}{d^3}$$

$$\sigma_{min} = \frac{M_{b\,min}}{(I/C)} = \frac{-1 \times 10^6}{\left(\frac{\pi d^3}{32}\right)} = \frac{-10.186 \times 10^6}{d^3}$$

Mean bending stress, $\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{5.093 \times 10^6}{d^3}$

Amplitude bending stress, $\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{15.278 \times 10^6}{d^3}$

Equivalent normal, $\sigma_{eq} = \sigma_m + \left(\frac{\sigma_y}{\sigma_{-1}}\right)\frac{k_{\sigma a}\sigma_a}{k_l k_s k_z}$

$$= \frac{5.093 \times 10^6}{d^3} + \left(\frac{300}{225}\right)\frac{1.5 \times 15.278 \times 10^6}{d^3 \times 1 \times 0.85 \times 0.85}$$

$$\sigma_{eq} = \frac{47.39 \times 10^6}{d^3}$$

(ii) Consider torque

Ref Fig. 3.11, for $\frac{r}{d} = 0.1$ and $\frac{D}{d} = 1.2$ ...(Fig. 4-18 A/P 4.18)

Stress concentration factor, $k_\tau = 1.33 = k_{\tau a}$

Shear stress, $\tau = \frac{M_t}{(J/C)} = \frac{M_t}{(\pi d^3/16)}$

$$\therefore \quad \tau_{max} = \frac{M_{t\,max}}{\left(\frac{\pi d^3}{16}\right)} = \frac{2.5 \times 10^6}{\left(\frac{\pi d^3}{16}\right)} = \frac{12.37 \times 10^6}{d^3}$$

$$\tau_{min} = \frac{M_{t\,min}}{\left(\frac{\pi d^3}{16}\right)} = \frac{1.5 \times 10^6}{\left(\frac{\pi d^3}{16}\right)} = \frac{7.64 \times 10^6}{d^3}$$

$\therefore$ Mean shear stress, $\tau_m = \dfrac{\tau_{max} + \tau_{min}}{2} = \dfrac{10.185 \times 10^6}{d^3}$

Amplitude shear stress, $\tau_a = \dfrac{\tau_{max} - \tau_{min}}{2} = \dfrac{2.55 \times 10^6}{d^3}$

Equivalent shear stress, $\tau_{eq} = \tau_m + \left(\dfrac{\tau_y}{\sigma_{-1}}\right)\dfrac{\tau_a k_{ta}}{k_l k_s k_z}$

$$\tau_y = \frac{1}{2} \quad \sigma_y = 150 \text{ MPa}; \quad k_l = 0.6$$

$$= \frac{10.185 \times 10^6}{d^3} + \left(\frac{150}{225}\right)\frac{1.33 \times 2.55 \times 10^6}{d^3 \times 0.6 \times 0.85 \times 0.85} = \frac{15.40 \times 10^6}{d^3}$$

(iii) Equivalent maximum normal stress

$$\sigma_{eq\,max} = \frac{1}{2}\left[\sigma_{eq} + \sqrt{\sigma_{eq}^2 + 4\tau_{eq}^2}\right] = \frac{\sigma_y}{FOS}$$

$$= \frac{1}{2}\left[\frac{47.39 \times 10^6}{d^3} + \sqrt{\left(\frac{47.39 \times 10^6}{d^3}\right)^2 + 4\left(\frac{15.4 \times 10^6}{d^3}\right)^2}\right] = \frac{300}{2.5}$$

$\therefore \quad d = 75.65$ mm

(iv) Equivalent maximum shear stress

$$\tau_{eq\,max} = \frac{1}{2}\sqrt{\sigma_{eq}^2 + 4\tau_{eq}^2} = \frac{\tau_y}{FOS}$$

$$= \frac{1}{2}\sqrt{\left(\frac{47.39 \times 10^6}{d^3}\right)^2 + 4\left(\frac{15.4 \times 10^6}{d^3}\right)^2} = \frac{150}{2.5}$$

$\therefore \quad d = 77.8$

Recommended diameter $d = 77.8 \simeq 78$ mm

(bigger of the above two)

EXERCISE

1. Determine the diameter of a cantilever beam shown in Fig. 4.12. Assume the following:

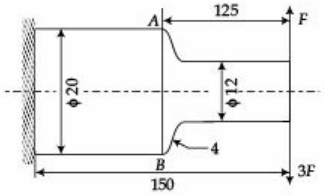


Fig. 4.12

σ_u = 550 MPa, σ_y = 470 MPa, Size factor = 0.85, Surface factor = 0.89, Notch sensitivity = 0.9, Factor of safety = 2.

2. A steel rod (σ_u = 1090 MPa, σ_y = 690 MPa and σ_{et} = 428 MPa) is subjected to a tensile load that fluctuates between 40 MPa and 120 MPa. Find the safe diameter of the rod using Soderberg's criterion. Adopt a factor of safety = 2. Stress concentration factor = unity. The load, size and surface factors may be taken as 0.75, 0.85 and 0.91 respectively.
3. Determine the diameter '*D*' based on Goodman's theory for a machine member load as shown in Fig. 4.13. The ultimate stress and endurance stresses for the material used are 600 MPa and 360 MPa respectively. The notch sensitivity index is 0.92. Factor of safety is 3.

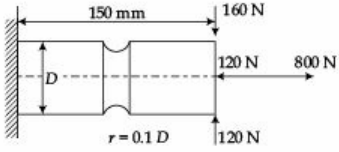


Fig. 4.13

4. A hot rolled steel shaft is subjected to a torsional moment that varies between 300 N-m CW and 120 N-m CCW and at the same time the bending moment varies from 360 N-m to –180 N-m. Determine the suitable diameter of the shaft under factor of safety of 2 based on ultimate stress of 540 MPa and yield stress of 420 MPa.
5. A steel member of circular cross section is subjected to a torsional stress that fluctuates between 18 to 45 MPa and the axial stress varies between 30 MPa tensile and 9 MPa compressive.

   Determine: (a) the maximum equivalent normal stress,
   (b) maximum equivalent shear stress,
   (c) the diameter of the member,

taking yield stress as 400 MPa and endurance stress in reverse bending as 210 MPa. The surface has mirror polish.

CHAPTER

5 Impact Loading

INTRODUCTION

Loads applied suddenly with some velocity are known as *impact loads*. This may be due to a falling weight (potential energy) or a moving weight (kinetic energy).

$$\text{Potential energy} = PE = mgh = W \cdot h$$

W = Weight in N, h = height of fall,

m = mass kg

$$\text{Kinetic energy, } KE = \frac{1}{2} mv^2 = \frac{1}{2} \frac{W}{g} v^2$$

V = velocity of the moving mass, m/s

The energy absorbed by the body is known as *resilience* or *internal energy* stored by it.

The resilience for axial loading is given by

$$U = \frac{\sigma^2}{2E} \text{ (Vol.)}$$

σ = stress

E = modulus of elasticity

Vol = volume = area × length = Al

For bending, the resistance is $U = \dfrac{\sigma^2}{18E}$ (Vol.) (for rectangular c/s)

For shear $U = \dfrac{\tau^2}{2G}$ (Vol.) ...(T3-1/P 3.11)

5.1 IMPACT STRESS FROM AXIAL LOAD

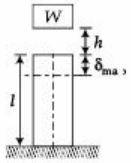


Fig. 5.1

Ref Fig 5.1

W = weight of the falling object, N

h = height of fall, mm

l = length, mm

A = area of c/s, mm^2

δ_{max} = maximum instantaneous deflection under impact, mm

δ_{ST} = static deflection (deflection under static load W), mm

E = modulus of elasticity, MPa

Vol = volume = $A.l$, mm^3

σ_{max} = maximum stress under impact or maximum instantaneous stress or impact stress

σ_{ST} = static stress = $\dfrac{W}{A}$

U = resilience or energy stored.

Figure 5.1 shows a weight W falls form a height of 'h' on to an element of cross sectional area A and length l

δ_{max} = max instantaneous deflection under impact.

Potential energy, $PE = mg(h + \delta_{max}) = W(h + \delta_{max})$

Equate this resilience, U = unit resilience × Vol

For axial loading, $U = \dfrac{\sigma^2}{2E} \cdot \text{vol}$

For impact, $U = \dfrac{\sigma_{max}^2}{2E} \cdot Al$

$\therefore \quad \dfrac{\sigma_{max}^2}{2E} \cdot Al = W(h + \delta_{max})$

$\therefore \quad \sigma_{max}^2 = W\dfrac{2E}{Al} \cdot (h + \delta_{max})$

put $\dfrac{W}{A} = \sigma_{ST}$

For axial loads, deflection = Change in length

$$\delta = \frac{Wl}{AE} = \frac{\sigma l}{E}$$

$\therefore \quad \delta_{max} = \dfrac{\sigma_{max} \cdot l}{E}$

$$\text{Substituting, } \sigma_{max}^2 = \frac{2\sigma_{ST} E}{l}\left(h + \frac{\sigma_{max} l}{E}\right)$$

$$\sigma_{max}^2 = \frac{2\sigma_{ST} Eh}{l} + 2\sigma\sigma_{max}$$

$$\sigma_{max}^2 - 2\sigma_{ST}\sigma_{max} = \frac{2\sigma_{ST} Eh}{l}$$

Add σ_{ST}^2 on both sides

$$\sigma_{max}^2 - 2\sigma_{ST}\sigma_{max} + \sigma_{ST}^2 = \sigma_{ST}^2 + \frac{2\sigma_{ST} Eh}{l}$$

$$(\sigma_{max} - \sigma_{ST})^2 = \sigma_{ST}^2\left(1 + \frac{2Eh}{\sigma_{ST} l}\right)$$

Taking square root on both sides

$$\sigma_{max} - \sigma_{ST} = \sigma_{ST}\sqrt{1 + \frac{2Eh}{\sigma_{ST} l}}$$

$$\therefore \quad \sigma_{max} = \sigma_{ST} + \sigma_{ST}\sqrt{1 + \frac{2Eh}{\sigma_{ST} l}}$$

$$\sigma_{max} = \sigma_{ST}\left(1 + \sqrt{1 + \frac{2Eh}{\sigma_{ST} l}}\right)$$

$$\text{Put } \sigma_{ST} = \frac{W}{A}$$

$$\sigma_{max} = \frac{W}{A} + \left(1 + \sqrt{1 + \frac{2hEA}{Wl}}\right) \qquad \text{...(3-11 a/P 3.3)}$$

Again substituting $\frac{W}{A} = \sigma_{ST}$

$$\text{and } \delta_{ST} = \frac{\sigma_{ST} l}{E} \Rightarrow \frac{E}{\sigma_{ST} l} = \frac{1}{\delta_{ST}}$$

$$\sigma_{max} = \sigma_{ST}\left(1 + \sqrt{1 + \frac{2h}{\delta_{st}}}\right) \qquad \text{...(3-11 c/P 3.3)}$$

$$\text{Impact factor or shock factor IF} = \frac{\sigma_{max}}{\sigma_{ST}} = \left(1 + \sqrt{1 + \frac{2h}{\delta_{ST}}}\right)$$

$$\text{Substituting, } \sigma_{max} = \frac{\delta_{max} \cdot E}{l} \quad \text{and} \quad \sigma_{ST} = \frac{\delta_{ST} \cdot E}{l}$$

In the impact stress equation

$$\frac{\delta_{max} \cdot E}{l} = \frac{\delta_{ST} E}{l}\left(1 + \sqrt{1 + \frac{2h}{\delta_{ST}}}\right)$$

$$\text{or} \quad \delta_{max} = \delta_{ST}\left(1 + \sqrt{1 + \frac{2h}{\delta_{ST}}}\right) \qquad \text{...(3-12 b/P 3.3)}$$

5.2 IMPACT IN BENDING

W = weight of the falling body, N
l = length of the beam, mm
h = height of fall, mm
I = moment of inertia, mm^4
c = distance from neutral axis to the extreme fibre, mm
M_b = bending moment = Wl, N-mm
δ_{ST} = static deflection, mm
δ_{max} = maximum deflection under impact, mm

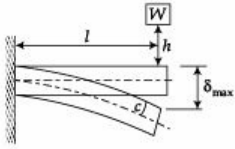


Fig. 5.2

σ_{bST} = static bending stress = $\frac{M_b \cdot C}{I}$

$\sigma_{b\,max}$ = impact stress under bending

Potential energy, $PE = W(h + \delta_{max})$ ...(i)

Equate this to work done by equivalent that produce the same energy $U = \frac{1}{2} Fy$

$$U = \frac{1}{2} W_{max} \delta_{max} \qquad \text{...(ii)}$$

Equating (i) and (ii)

$$\therefore \quad \frac{1}{2} W_{max} \delta_{max} = W(h + \delta_{max})$$

$$\therefore \quad W_{max} = 2W\left(\frac{h + \delta_{max}}{\delta_{max}}\right)$$

For cantilever beam with end load

$$\text{Deflection, } \delta = \frac{Wl^3}{3EI}$$

$$\therefore \quad \delta_{ST} = \frac{Wl^3}{3EI} \quad \text{and} \quad \delta_{max} = \frac{W_{max}\, l^3}{3EI}$$

Substitute the value of W_{max} in δ_{max} Eqn.

$$\delta_{max} = \frac{W_{max}\, l^3}{3EI} = 2W\left(\frac{h+\delta_{max}}{\delta_{max}}\right)\cdot\frac{l^3}{3EI}$$

$$\delta_{max} = 2\left(\frac{h+\delta_{max}}{\delta_{max}}\right)\cdot\frac{Wl^3}{3EI}$$

$$\text{Sub } \frac{Wl^3}{3EI} = \delta_{ST}$$

$$\delta_{max} = \frac{2(h+\delta_{max})}{\delta_{max}}\cdot\delta_{ST}$$

$$\delta_{max}^2 = 2h\cdot\delta_{ST} + 2\delta_{max}\delta_{ST}$$

$$\delta_{max}^2 - 2\delta_{max}\delta_{ST} = 2h\cdot\delta_{ST}$$

Add δ_{ST}^2 on both sides

$$\delta_{max}^2 - 2\delta_{max}\delta_{ST} + \delta_{ST}^2 = \delta_{ST}^2 + 2h\delta_{ST}$$

$$(\delta_{max} - \delta_{ST})^2 = \delta_{ST}^2\left(1+\frac{2h}{\delta_{ST}}\right)$$

Take square root on both sides,

$$\delta_{max} - \delta_{ST} = \delta_{ST}\cdot\sqrt{1+\frac{2h}{\delta_{ST}}}$$

$$\therefore \quad \delta_{max} = \delta_{ST} + \delta_{ST}\sqrt{1+\frac{2h}{\delta_{ST}}}$$

$$\delta_{max} = \delta_{ST}\left(1+\sqrt{1+\frac{2h}{\delta_{ST}}}\right) \qquad \text{...(3-27/P 3.5)}$$

$$\text{WKT } \delta = \frac{Wl^3}{3EI} \quad \text{and} \quad \delta_{max} = \frac{W_{max}\, l^3}{3EI}$$

$$\text{But} \quad \frac{Wl^3}{3EI} = \frac{(Wl)l^2}{I \cdot 3E} = \frac{(Wl)C}{I} \cdot \frac{l^2}{3CE} = \frac{M_b C}{I} \cdot \frac{l^2}{3CE}$$

$$\therefore \quad \delta = \frac{M_b C}{I} \cdot \frac{l^2}{3CE} = \frac{\sigma_b \cdot l^2}{3CE}$$

$$\therefore \quad \delta_{ST} = \sigma_{b_{ST}} \frac{l^2}{3CE} \quad \text{and} \quad \delta_{max} = \sigma_{b_{max}} \frac{l^2}{3CE}$$

Substitutes

$$\sigma_{b_{max}} \frac{l^2}{3CE} = \sigma_{b_{ST}} \frac{l^2}{3CE} \left(1 + \sqrt{1 + \frac{2h}{\sigma_{ST}}}\right)$$

$$\therefore \quad \sigma_{b_{max}} = \sigma_{b_{ST}} \left(1 + \sqrt{1 + \frac{2h}{\sigma_{ST}}}\right) \qquad \text{...(3-26 c/P 3.5)}$$

5.3 IMPACT STRESS IN TORSION

Max. shear stress under impact

$$\tau_{max} = \tau_{ST} \left(1 + \sqrt{1 + \frac{2h}{R\theta_{ST}}}\right) \qquad \text{...(3-31/P 3.6)}$$

Max. angular deflection under impact

$$\theta_{max} = \theta_{ST} \left(1 + \sqrt{1 + \frac{2h}{R\theta_{ST}}}\right) \qquad \text{...(3-32/P 3.6)}$$

τ_{ST} = static shear stress = $\frac{M_t \cdot C}{J}$

M_t = torque = $W \cdot R$

R = radius at which the load W is falling

θ_{ST} = static angular deflection in radians

$$\frac{M_t}{J} = \frac{G\theta}{l} = \frac{\tau}{r} \Rightarrow \theta_{ST} = \frac{M_t \cdot l}{GJ} \quad \text{or} \quad \theta_{ST} = \frac{\tau \cdot l}{Gr}$$

C = distance from neutral axis to outer fibre

J = polar moment of inertia

PROBLEMS

Problem 1: A weight of 3 kN falls through a height of 3 mm before it strikes the round '*k*' of a round rod 30 mm diameter and 500 mm long as shown in Fig. 5.3. Modulus of elasticity of the material of the rod is 210 GPa. Find:

(i) the maximum stress induced,

(ii) compare with the static stress.

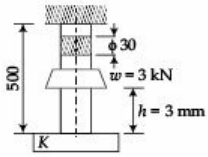


Fig. 5.3

Given data: Weight, $W = 3 \text{ kN} = 3 \times 10^3 \text{ N}$

Height of fall, $h = 3$ mm

Diameter, $d = 30$ mm

$$\therefore \quad A = \frac{\pi d^2}{4} = \frac{\pi \times 30^2}{4} = 706.86 \text{ mm}^2$$

Length, $l = 500$ mm

$E = 210 \text{ GPa} = 210 \times 10^3 \text{ MPa}$

(i) Impact stress, $\sigma_{max} = \frac{W}{A}\left(1 + \sqrt{1 + \frac{2hAE}{Wl}}\right)$ ...(3-11 a/P 3.3)

$$= \frac{3000}{706.86}\left(1 + \sqrt{1 + \frac{2 \times 3 \times 706.86 \times 210 \times 10^3}{300 \times 500}}\right)$$

$\sigma_{max} = 107.75$ MPa

(ii) Static stress, $\sigma_{ST} = \frac{W}{A} = \frac{3000}{706.86} = 4.244$ MPa

Comparing, $\frac{\sigma_{max}}{\sigma_{ST}} = \frac{107.75}{4.244} = 25.39.$

The impact stress is 25.39 times the static stress.

Problem 2: A bar of rectangular cross section with sides ratio as 2, is 300 mm long. It is subjected to an axial impact by a load of 1.5 kN that falls on to it from a height of 10 mm. Determine the sectional dimensions of the bar if the allowable stress is 120 MPa.

Given data: $W = 1.5 \text{ kN} = 1500 \text{ N}$

$h = 10$ mm

$l = 300$ mm

$\frac{b}{d} = 2 \quad \therefore \quad A = bd = 2d^2$

$\sigma_{max} = 120$ MPa

Assume, $E = 210 \text{ GPa} = 210 \times 10^3$ MPa.

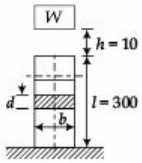


Fig. 5.4

$$\text{Impact axial stress, } \sigma_{max} = \frac{W}{A}\left(1+\sqrt{1+\frac{2hAE}{Wl}}\right) \quad \text{...(3-11 a/P 3.3)}$$

$$120 = \frac{1500}{A} = \left(1+\sqrt{1+\frac{2\times 10\times A\times 210\times 10^3}{1500\times 300}}\right)$$

$$\frac{120A}{1500} = 1+\sqrt{1+9.33A}$$

$$0.08A - 1 = \sqrt{1+9.33A}$$

Squaring on both sides

$$(0.08A-1)^2 = 1+9.33A$$

$$0.0064A^2 + 1 - 0.16A = 1 + 9.33A$$

$\therefore$ $$0.0064A^2 = 9.33A + 0.16A$$

$$A = \frac{9.49}{0.0064} = 1482.81 = 2d^2$$

$\therefore$ $$d = 27.23 \text{ mm} \simeq 28 \text{ mm}$$

$$b = 2d = 56 \text{ mm}.$$

Problem 3: A steel rod 1.5 m long has to resist longitudinally an impact of 2.5 kN falling under gravity at a velocity of 0.9925 m/s. The maximum computed stress is to be limited to 150 MPa. Determine the diameter of the round rod.

Given data: $l = 1.5 \text{ m} = 1500 \text{ mm}$, $W = 2.5 \text{ kN} = 2.5\times W^3 \text{ N}$

$v = 0.9925 \text{ m/s}$, $\sigma_{max} = 150 \text{ MPa}$

Find d = ?

To find height of fall 'h'.

We know that $v^2 - u^2 = 2as$

For falling under gravity, $u = 0$, $a = g$, $s = h$

$\therefore$ $$V^2 = 2gh \Rightarrow h = \frac{V^2}{2g} = \frac{0.9925^2}{2\times 9.81} = 0.0502 \text{ m}$$

$\therefore$ height of fall, $h = 50.2$ mm

Assume, $E = 210\times 10^3$ MPa

$$\text{Impact axial stress, } \sigma_{max} = \frac{W}{A}\left(1+\sqrt{1+\frac{2hAE}{Wl}}\right) \quad \text{...(3-11 a/P 3.3)}$$

$$150 = \frac{2500}{A}\left(1+\sqrt{1+\frac{2\times 50.2\times A\times 210\times 10^3}{2500\times 1500}}\right)$$

$$\frac{150A}{2500} - 1 = \sqrt{1+5\cdot 6224A}$$

Squaring on both sides

$$(0.06A - 1)^2 = 1 + 5.6224A$$

$$0.0036A^2 + 1 - 0.12A = 1 + 5.6224A$$

$$\therefore \quad A = \frac{5.6224 + 0.12}{0.0036} = 1595.11 \text{ mm}^2$$

But for round rod, $A = \frac{\pi d^2}{4} = 1595{\cdot}11$

$$\therefore \quad \text{dia of rod, } d = 45.066 \text{ mm} \simeq 45 \text{ mm.}$$

Problem 4: An unknown weight falls through 15 mm on a collar rigidly attached to the lower end of a vertical bar 3 m long and 500 mm^2 in section. If the maximum instantaneous extension is known to be 2 mm, what is the corresponding stress and the value of unknown weight. Take E = 200 kN/mm^2.

Given data:

$h = 15$ mm

$l = 3$ m $= 3000$ mm

$A = 500$ mm^2

$\delta_{max} = 2$ mm

$E = 200$ kN/mm^2 $= 200 \times 10^3$ N-mm^2

We know that elongation $\quad \delta = \frac{Wl}{AE} = \frac{\sigma_l}{E}$

$$\therefore \quad \delta_{max} = \frac{\sigma_{max}\, l}{E} \Rightarrow 2 = \frac{\sigma_{max}\times 3000}{200\times 10^3}$$

$$\therefore \quad \text{max stress, } \sigma_{max} = \frac{2\times 200\times 10^3}{3000} = 133.33 \text{ MPa}$$

For axial impact, $\quad \sigma_{max} = \frac{W}{A}\left(1+\sqrt{1+\frac{2hAE}{Wl}}\right)$ ...(3-11 a/P 3.3)

$$133.33 = \frac{W}{500}\left(1+\sqrt{1+\frac{2\times 15\times 500\times 200\times 10^3}{W\times 3000}}\right)$$

$$\frac{66,666.67}{W} - 1 = \sqrt{1+\frac{1\times 10^6}{W}}$$

Squaring on both sides

$$\frac{4.44\times 10^9}{W^2} + 1 - \frac{133.33\times 10^3}{W} = 1 + \frac{1\times 10^6}{W}$$

$$\frac{4.44\times 10^9}{W^2} = \frac{10^6}{W} + \frac{133.33\times 10^3}{W}$$

$$\frac{4.44\times 10^9}{W^2} = \frac{1.133\times 10^6}{W}$$

$\therefore$ $W = 3918.8$ N.

Problem 5: A load of 100 N falls by gravity from a vertical distance of 20 mm when it is suddenly stopped by a collar at the end of a vertical rod of length 600 mm and a diameter of 20 mm. Then the load is lifted and made to fall from a height of 10 mm. Then the cycle repeats with negligible gap. The top of the bar is rigidly attached. Calculate the factor of safety by Soderberg's criterion, if the yield stress of the member is 360 MPa and corrected endurance strength is 125 MPa. Assume $E = 200\times 10^3$ N/mm^2.

Given data: $W = 100$ N

$l = 600$ mm

$d = 20$ mm

$\therefore$ $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}20^2 = 314.16$ mm^2

Height of first fall, $h_1 = 20$ mm

$E = 200 \times 10^3$ MPa

Height of second fall, $h_2 = 10$ mm

$\sigma_y = 360$ MPa

σ_{-1} actual $= 125$ MPa

(Since this is corrected endurance stress, no need to consider load, surface and size factors)

$$\sigma_{yd} = \frac{\sigma_y}{\text{FOS}} = \frac{360}{N} \quad \text{when} \quad N = \text{FOS}$$

$$\sigma_{-1d} = \frac{\sigma_{-1}k_l k_s k_z}{\text{FOS}} = \frac{\sigma_{-1\text{actual}}}{\text{FOS}} = \frac{125}{N}$$

(i) Consider max. height h = 20 mm

$$\text{Impact stress, } \sigma_{max} = \frac{W}{A}\left(1+\sqrt{1+\frac{2hAE}{Wl}}\right) \quad \text{...(3-11 a/P 3.3)}$$

$$\sigma_{max} = \frac{100}{314.16}\left(1+\sqrt{1+\frac{2\times 20\times 314.16\times 200\times 10^3}{100\times 600}}\right)$$

$$\sigma_{max} = 65.47 \text{ MPa}$$

(ii) Consider min. height, h = 10 mm

$$\text{Impact stress, } \sigma_{min} = \frac{W}{A}\left(1+\sqrt{1+\frac{2hAE}{Wl}}\right)$$

$$\sigma_{min} = \frac{100}{314.16}\left(1+\sqrt{1+\frac{2\times 10\times 314.16\times 200\times 10^3}{100\times 600}}\right)$$

$$\sigma_{min} = 46.39 \text{ MPa}$$

$$\text{Mean stress, } \sigma_m = \frac{\sigma_{max}+\sigma_{min}}{2} = 55.93 \text{ MPa}$$

$$\text{Amplitude stress} = \sigma_a = \frac{\sigma_{max}-\sigma_{min}}{2} = 9.54 \text{ MPa}$$

Stress concentration factor $k_\sigma = 1$

From Soderberg's relation

$$\frac{K_\sigma \sigma_a}{\sigma_{-1d}} + \frac{\sigma_m}{\sigma_{yd}} = 1 \quad \text{...(5-40/P 5.10)}$$

$$\frac{1\times 9.54}{\left(\frac{125}{N}\right)} + \frac{55.93}{\left(\frac{360}{N}\right)} = 1$$

$\therefore$ Factor of safety, N = 4.32.

Problem 6: An elevator car carrying a load of 10 kN and is descending by means of a steel rope at a speed of 1.0 m/s. The cross-sectional area of the rope is 400 mm^2. The rope is suddenly brought to rest by braking after 30 seconds of descent. Calculate the stress induced in the rope due to sudden stoppage taking Young's moduly for the rope as 80 GPa.

Given data:

$W = 10 \text{ kN} = 10 \times 10^3 \text{ N}$

$V = 1 \text{ m/s} = 1000 \text{ mm/s}$

$A = 400 \text{ mm}^2$

time, $t = 30$ sec

$E = 80 \text{ GPa} = 80 \times 10^3 \text{ MPa}$

$\therefore$ Distance moved by load in 30 sec $S = Vt = 1 \times 30 = 30$ m

When the load moves by a distance 'S' the rope will unwind by equal length.

$\therefore$ Length of rope after 30 sec, $l = S = 30 \times 10^3$ mm

$$\text{Kinetic energy, } KE = \frac{1}{2}\frac{W}{g}V^2 = \frac{1}{2} \times \frac{10 \times 10^3}{9810} \times 1000^2 = 509.68 \times 10^3 \text{ N-mm}$$

$$\text{Equating this to resilience, } U = \frac{\sigma_{max}^2}{2E} \cdot Al$$

$$\frac{\sigma_{max}^2}{2 \times 80 \times 10^3} \times 400 \times 30 \times 10^3 = 509.68 \times 10^3$$

$\therefore$ $\sigma_{max} = 82.44$ MPa

Stress induced in the rope, $\sigma_{max} = 82.44$ MPa.

Problem 7: A mass of 500 kg is being lowered by means of steel wire rope having cross sectional area of 250 mm^2. The velocity of weight is 0.5 m/sec. When the length of the extended rope is 20 m, the sheave gets stuck up. Determine the stresses induced in the rope due to sudden stoppage of the sheave. Neglect friction.

Given data:

Mass, $m = 500$ kg

Area, $A = 250 \text{ mm}^2$

velocity, $V = 0.5$ m/sec

length, $l = 20 \text{ m} = 20 \times 10^3$ mm

Assume, $E = 200 \text{ GPa} = 200 \times 10^3$ MPa

$$\text{Kinetic energy, } KE = \frac{1}{2}mv^2 = \frac{1}{2}500 \times 0.5^2 \text{ Nm}$$

$$KE = 62.5 \text{ Nm} = 62.5 \times 10^3 \text{ N-mm}$$

Equate this to resilience (energy stored)

$$U = \frac{\sigma_{max}^2}{2E} \cdot (\text{vol}) = 62.5 \times 10^3$$

Vol = Volume = Area $\times$ Length

$$\frac{\sigma_{max}^2}{2 \times 200 \times 10^3} \times (250 \times 20 \times 10^3) = 62.5 \times 10^3$$

$\therefore$ Max. stress induced, $\sigma_{max} = 70.71$ MPa.

Problem 8: A cantilever beam of section width 50 mm and depth 150 mm, 1.5 m long is struck by a weight of 1 kN that falls on to it from a height of 10 mm at its free end. Take $E = 206$ GPa, find:

(i) The maximum instantaneous stress,
(ii) The maximum instantaneous load,
(iii) The maximum instantaneous deflection, and
(iv) The impact factor.

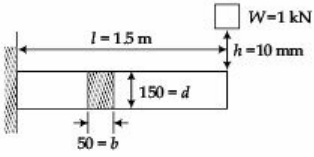


Fig. 5.5

Given data:

Width, $b = 50$ mm

Depth, $d = 150$ mm

Length, $l = 1.5$ m $= 1500$ mm

Load, $W = 1$ kN $= 1000$ N

$E = 206$ GPa $= 206 \times 10^3$ MPa

(i) Impact bending stress = maximum instantaneous stress

$$\sigma_{b_{max}} = \sigma_{b_{ST}} \left(1 + \sqrt{1 + \frac{2h}{\sigma_{ST}}}\right) \quad \text{...(3-26 c/P 3.5)}$$

where $\sigma_{b_{ST}}$ = static bending stress $= \dfrac{M_b \cdot C}{I}$

$$C = \frac{d}{2} = \frac{150}{2} = 75 \text{ mm (distance from } NA \text{ to outer fibre)}$$

$$I = \text{moment of inertia} = \frac{bd^3}{12} = \frac{50 \times 150^3}{12} = 14.0625 \times 10^6 \text{ mm}^4$$

$$M_b = \text{bending moment} = W \times l = 1000 \times 1500 = 1.5 \times 10^6 \text{ N-mm}$$

$$\therefore \quad \sigma_{b_{ST}} = \frac{1.5 \times 10^6 \times 75}{14.0625 \times 10^6} = 8 \text{ MPa}$$

δ_{ST} = static deflection ...(T 2-8/P 2.34)

For cantilever with and load, $\delta_{ST} = \dfrac{Wl^3}{3EI} = \dfrac{1000 \times 1500^3}{3 \times 206 \times 10^3 \times 14.0625 \times 10^6} = 0.388$ mm

$$\therefore \quad \sigma_{b_{max}} = 8\left(1 + \sqrt{1 + \frac{2 \times 10}{0.388}}\right) = 65.96 \text{ MPa}$$

(ii) Maximum instantaneous load

$$W_{max} = W\left(1+\sqrt{1+\frac{2h}{\sigma_{ST}}}\right)$$

$$W_{max} = 1000\left(1+\sqrt{1+\frac{2\times10}{0.388}}\right) = 8245.69 \text{ N}$$

(iii) Maximum instantaneous deflection

$$\delta_{max} = \delta_{ST}\left(1+\sqrt{1+\frac{2h}{\sigma_{ST}}}\right) \quad \text{...(3-27/P 3.5)}$$

$$= 0.388\left(1+\sqrt{1+\frac{2\times10}{0.388}}\right) = 3.2 \text{ mm}$$

(iv) Impact factor, $I_F = \dfrac{\sigma_{b_{max}}}{\sigma_{b_{ST}}} = \dfrac{65.97}{8} = 8.246$

Problem 9: A machine element in the form of a cantilever beam of span 800 mm having rectangular cross section of depth 200 mm is subjected to an impact by a weight 120 kg that falls on to it from a height of 40 mm. The material has the yield stress of 294.2 MPa. Taking factor of safety as 2, determine the required width of cross section.

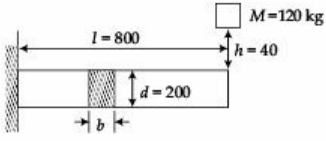


Fig. 5.6

Given data:

Length, $l = 800$ mm

Depth, $d = 200$ mm

Height of fall, $h = 40$ mm

Mass, $m = 120$ kg $\quad \therefore \quad W = mg = 120 \times 9.81 = 1177.2$ N

$\sigma_y = 294.2$ MPa $\quad$ FOS = 2

$\therefore$ Allowable stress, $\sigma_{max} = \dfrac{\sigma_y}{\text{FOS}} = \dfrac{294.2}{2} = 147.1$ MPa

Assume, $E = 210$ GPa $= 210 \times 10^3$ MPa

Impact bending stress

$$\sigma_{b_{max}} = \sigma_{b_{ST}}\left(1+\sqrt{1+\frac{2h}{\sigma_{ST}}}\right) \quad \text{...(3-26 c)}$$

where $\sigma_{b_{ST}} = \dfrac{M_b \cdot C}{I}$

M_b = Bending moment = $W \cdot l = 1177.2 \times 800$

$M_b = 941.76 \times 10^3$ N-mm

$$C = \frac{d}{2} = \frac{200}{2} = 100 \text{ mm}$$

$$I = \frac{bd^3}{12} = \frac{b \times 200^3}{12} = 666666.67(b)$$

$\therefore$ $$\sigma_{b_{ST}} = \frac{941.76 \times 10^3 \times 100}{666666.67(b)} = \frac{141.264}{b}$$

For cantilever, $$\delta_{ST} = \frac{Wl^3}{3EI} = \frac{1177.2 \times 800^3}{3 \times 210 \times 10^3 \times 666666.67(b)}$$

$$\delta_{ST} = \frac{1.435}{b} \text{ mm.}$$

$\therefore$ $$\sigma_{b_{max}} = \sigma_{b_{ST}} \left(1 + \sqrt{1 + \frac{2h}{\delta_{ST}}}\right)$$

$$147.1 = \frac{141.264}{b} \left(1 + \sqrt{1 + \frac{2 \times 40}{\left(\frac{1.435}{b}\right)}}\right)$$

$$\frac{147.1(b)}{141.264} - 1 = \sqrt{1 + 55.75b}$$

$$[1{\cdot}041(b) - 1] = \sqrt{1 + 55.75(b)}$$

Squaring on both sides

$$1{\cdot}0843b^2 + 1 - 2{\cdot}0826(b) = 1 + 55{\cdot}75(b)$$

$$1{\cdot}0843b^2 = 55{\cdot}75(b) + 2{\cdot}082(b)$$

$\therefore$ $$b = 55.34 \text{ mm} \simeq 54 \text{ mm}$$

Width, $b = 54$ mm.

Problem 10: A machine member in the form of a cantilever beam is made of a rod of circular cross section diameter 40 mm with a length of 300 mm. It is made of C40 steel (σ_y = 328.6 MPa). Determine the safe value of the transverse load that falls at the free end from a height of 25 mm. Take factor of safety = 2.5.

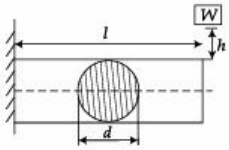
Given data: Cantilever, $d = 40$ mm

$l = 300$ mm

$\sigma_y = 328.6$ MPa FOS = 2.5

Assume, $E = 210 \times 10^3$ MPa

Height of fall, $h = 25$ mm



$$\text{Allowable stress, } \sigma_{b\max} = \frac{\sigma_y}{\text{FOS}} = \frac{328.6}{2.5} = 131.44 \text{ MPa}$$

$$\text{For bending, impact stress, } \sigma_{b\max} = \sigma_{bST}\left(1 + \sqrt{1 + \frac{2h}{\delta_{ST}}}\right) \quad \text{...(3-26 c)}$$

$$\text{Static bending stress, } \sigma_{bST} = \frac{M_b \cdot C}{I}$$

M_b = bending moment = $W \cdot l = W \times 300$ N-mm

$$C = \frac{d}{2} = \frac{40}{2} = 20 \text{ mm}, \quad I = \frac{\pi d^4}{64} = \frac{\pi \times 40^4}{64} = 125.664 \times 10^3 \text{ mm}^4$$

$$\therefore \quad \sigma_{b_{ST}} = \frac{W \times 300 \times 20}{125.664 \times 10^3} = 0.04775 \text{ W}$$

For cantilever, $$\delta_{ST} = \frac{Wl^3}{3EI} = \frac{W \times 300^3}{3 \times 210 \times 10^3 \times 125.664 \times 10^3} = 3.41 \times 10 - 410 \text{ W}$$

Substituting in $\sigma_{b\max}$ equation

$$131.44 = 0.04775 \text{ W}\left(1 + \sqrt{1 + \frac{2 \times 25}{3.41 \times 10^{-4} W}}\right)$$

$$\frac{131.44}{0.04775 W} - 1 = \sqrt{1 + \frac{146.61 \times 10^3}{W}}$$

$$\frac{2752.7}{W} - 1 = \sqrt{1 + \frac{146.61 \times 10^3}{W}}$$

Squaring on both sides

$$\frac{7.77 \times 10^6}{W^2} + 1 - \frac{5505.34}{W} = 1 + \frac{146.61 \times 10^3}{W}$$

$$\frac{7.577 \times 10^6}{W^2} = \frac{146.61 \times 10^3 + 5505.34}{W}$$

$$\therefore \quad W = \frac{7.577 \times 10^6}{152.115 \times 10^3} = 49.81 \text{ N}$$

Safe load $W = 49.81$ N.

Problem 11: An element in the form of a cantilever beam has a rectangular cross section 40 mm wide and 120 mm deep. Its length is 600 mm. A transverse load of 5 kN falls on to it from an unknown height at its free end. Determine the safe value of height 'h' limiting maximum stress to 120 MPa. Take E = 200 GPa.

Given data: Width, $b = 40$ mm, Depth, $d = 120$ mm

Span, $l = 600$ mm, $W = 5$ kN $= 5000$ N

$\sigma_{b\max} = 120$ MPa, $E = 200 \times 10^3$ MPa

$$\text{Impact bending stress, } \sigma_{b\max} = \sigma_{bST}\left(1 + \sqrt{1 + \frac{2h}{\delta_{ST}}}\right) \qquad \text{...(3-26 c)}$$

$$\sigma_{bST} = \text{static bending stress} = \frac{M_b \cdot C}{I}$$

$$M_b = Wl = 5000 \times 600 = 3 \times 10^6 \text{ N-mm}$$

$$C = \frac{d}{2} = \frac{120}{2} = 60 \text{ mm} \quad I = \frac{bd^3}{12} = \frac{40 \times 120^3}{12} = 5.76 \times 10^6 \text{ mm}^4$$

$$\therefore \quad \sigma_{bST} = \frac{3 \times 10^6 \times 60}{5.76 \times 10^6} = 31.25 \text{ MPa}$$

$$\delta_{ST} = \frac{Wl^3}{3EI} = \frac{5000 \times 600^3}{3 \times 200 \times 10^3 \times 5.76 \times 10^6} = 0.3125 \text{ mm}$$

$$\therefore \quad 120 = 31.25\left(1 + \sqrt{1 + \frac{2 \times h}{0.3125}}\right)$$

$$\frac{120}{31.25} - 1 = \sqrt{1 + 6.4h}$$

$$3.84 - 1 = \sqrt{1 + 6.4h}$$

$$2.84 = \sqrt{1 + 6.4h}$$

Squaring on both sides $8.0656 = 1 + 6.4h$

$$h = 1.104 \text{ mm.}$$

Problem 12: A beam of I-section 250 mm depth has a moment of inertia of 60×10^6 mm^4. It is simply supported at the ends at a distance of 3 m apart. A weight of 3 kN falls at its middle from an unknown height. Take $E = 210$ GPa, determine the safe height 'h' taking the allowable stress as 90 MPa.

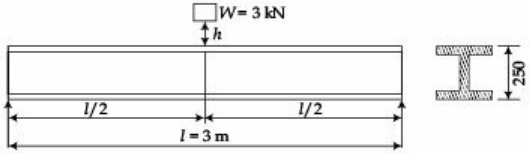


Fig. 5.7

Given data:

$$d = 250 \text{ mm}$$
$$I = 60 \times 10^6 \text{ mm}^4$$
$$l = 3000 \text{ mm}$$
$$W = 3 \text{ kN} = 3 \times 10^3 \text{ N}$$
$$E = 210 \times 10^3 \text{ MPa}$$
$$\sigma_{b\max} = 90 \text{ MPa}$$

For bending, impact stress, $\sigma_{b\max} = \sigma_{bST}\left(1 + \sqrt{1 + \frac{2h}{\delta_{ST}}}\right)$ ...(3-26 c)

$$\sigma_{bST} = \frac{M_b C}{I}$$

For simply supported beam with central load ...(T 2-8/P 2.34)

$$M_b = \frac{Wl}{4} = \frac{3000 \times 3000}{4} = 2.25 \times 10^6 \text{ N-mm}$$

$$C = \frac{d}{2} = \frac{250}{2} = 125 \text{ mm}$$

$\therefore$ $$\sigma_{bST} = \frac{2.25 \times 10^6 \times 125}{60 \times 10^6} = 4.6875 \text{ MPa}$$

Static deflection, $\delta_{ST} = \frac{Wl^3}{48EI}$ for simply supported beam with central load ...(T 2-8/P 2.34)

$$\delta_{ST} = \frac{3000 \times 3000^3}{48 \times 210 \times 10^3 \times 60 \times 10^6} = 0{\cdot}1339 \text{ mm}$$

$$\therefore \qquad 90 = 4.6875\left(1+\sqrt{1+\frac{2h}{0{\cdot}1339}}\right)$$

$$\frac{90}{4.6875} - 1 = \sqrt{1+14.93(h)}$$

$$h = 22.12 \text{ mm.}$$

Problem 13: A weight of 1.2 kN falls from a height of 30 mm at the middle of a simply supported beam of circular cross section 1.8 long. The allowable stress for steel used is 120 MPa and modulus of elasticity = 206 GPa. Find the required diameter of the rod.

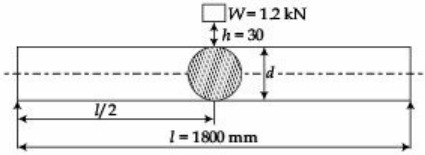


Fig. 5.8

Given data:

$$W = 1200 \text{ N}$$
$$h = 30 \text{ mm}$$
$$l = 1800 \text{ mm}$$
$$\sigma_{b\max} = 120 \text{ MPa}$$
$$E = 206 \times 10^3 \text{ MPa}$$

$$C = \frac{d}{2}, \quad I = \frac{\pi d^4}{64} \text{ (For circular c/s)}$$

For simply supported beam with central load

$$\sigma_{bST} = \frac{M_b \cdot C}{I} \text{ and } \delta_{ST} = \frac{Wl^3}{48EI}, \quad M_b = \frac{Wl}{4} \quad \text{...(T 2-8/P 2.34)}$$

$$M_b = \frac{Wl}{4} = \frac{1200 \times 1800}{4} = 540 \times 10^3 \text{ N-mm}$$

$$\therefore \qquad \sigma_{b_{ST}} = \frac{540 \times 10^3}{(\pi d^4/64)} \times \frac{d}{2} = \frac{5.5 \times 10^6}{d^3}$$

$$\delta_{ST} = \frac{1200 \times 1800^3}{48 \times 206 \times 10^3 \times (\pi d^4/64)} = \frac{14.4185 \times 10^6}{d^4}$$

Impact bending stress, $\sigma_{b\max} = \sigma_{bST}\left[1+\sqrt{1+\frac{2h}{\delta_{ST}}}\right]$ ...(3-26 c)

$$120 = \frac{5.5\times10^6}{d^3}\left[1+\sqrt{1+\frac{2\times30}{(14.4185\times10^6/d^4)}}\right]$$

$$\frac{120d^3}{5.5\times10^6} - 1 = \sqrt{1+4.1613\times10^{-6}d^4}$$

$$2.182\times10^{-5}d^3 - 1 = \sqrt{1+4.1613\times10^{-6}d^4}$$

Squaring on both sides.

$$4.76\times10^{-10}d^6 + 1 - 4.3636\times10^{-5}d^3 = 1 + 4.1613\times10^{-6}d^4$$

$$4.70\times10^{-10}d^6 - 4.1613\times10^{-6}d^4 - 4.3636\times10^{-5}d^3 = 0$$

$$d^3(4.76\times10^{-10}d^3 - 4.1613\times10^{-6}\times d - 4.3636\times10^{-5}) = 0$$

$\therefore$ $4.76\times10^{-10}d^3 - 4.1613\times10^{-6}d - 4.3636\times10^{-5} = 0$ (dividing by d^3)

Solving $d = 98.36$ mm $\simeq$ 99 mm.

Problem 14: A cam shaft 20 mm diameter is mounted at ends carries a cam midway between the bearings. The distance between ends is 150 mm. The follower which weight 2.5 kN is out of adjustment so that impact is produce when it is contacted by the cam. The height of fall is 0.25 mm. Determine the bending stress due to impact. Take E = 206 GPa.

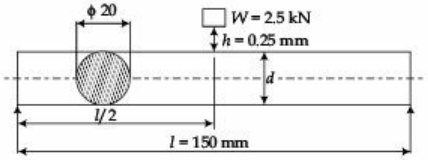


Fig. 5.9

Given data:

$d = 20$ mm

$l = 150$ mm

$W = 2500$ N

$h = 0.25$ mm

$E = 206$ GPa $= 206\times10^3$ N-mm

Impact bending stress, $\sigma_{b\max} = \sigma_{bST}\left[1+\sqrt{1+\frac{2h}{\delta_{ST}}}\right]$

$$\sigma_{bST} = \frac{M_b \cdot C}{I} \quad \text{and} \quad M_b = \frac{Wl}{4} = \frac{2500 \times 150}{4} = 93{,}750 \text{ N-mm}$$

$$C = \frac{d}{2} = \frac{20}{2} = 10 \text{ mm}, \quad I = \frac{\pi d^4}{64} = \frac{\pi \times 20^4}{64} = 7854 \times 10^3 \text{ mm}^4$$

$$\therefore \quad \sigma_{bST} = \frac{93{,}750 \times 10}{7.854 \times 10^4} = 119.37 \text{ MPa}$$

For simply supported beam with central load

$$\delta_{ST} = \frac{Wl^3}{48EI} = \frac{2500 \times 150^3}{48 \times 206 \times 10^3 \times 7.854 \times 10^3} = 0.1086 \text{ mm}$$

$$\therefore \quad \sigma_{max} = 119.37\left(1 + \sqrt{1 + \frac{2 \times 0.25}{0.1086}}\right) = 401.9 \text{ MPa}.$$

Problem 15: A 5 kg block is dropped from a height of 200 mm on to a beam as shown in Figure 5.10. The material has an allowable stress of 50 MPa. Determine the dimensions of the rectangular cross section whose depth is 1.5 times the width. Take E = 70 GPa.

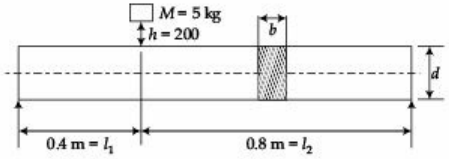


Fig. 5.10

Given data:

$$m = 5 \text{ kg}$$

$$\therefore \quad W = mg = 5 \times 9.81 = 49.05 \text{ N}$$

$$l = 0.4 + 0.8 = 1.2 \text{ m} = 1200 \text{ mm}$$

$$\sigma_{b\,max} = 50 \text{ MPa}$$

$$h = 200 \text{ MPa}, \quad E = 70 \times 10^3 \text{ MPa}$$

$$\frac{d}{b} = 1.5 \Rightarrow d = 1.5b$$

$$\text{Impact bending stress, } \sigma_{b\,max} = \sigma_{bST}\left(1 + \sqrt{1 + \frac{2h}{\delta_{ST}}}\right)$$

$$\text{where} \quad \sigma_{bST} = \frac{M_b \cdot C}{I}, \quad M_b = \frac{Wl_1 l_2}{l} \qquad \text{...(T 2-8/P 2.35)}$$

$$\text{where} \quad l_1 = 400 \text{ mm} \quad l_2 = 800 \text{ mm} \quad l = l_1 + l_2 = 1200 \text{ mm}$$

$$M_b = \frac{49.05 \times 400 \times 800}{1200} = 13{,}080 \text{ N-mm}$$

$$C = \frac{d}{2} = \frac{1.5b}{2} = 0.75b, \quad I = \frac{bd^3}{12} = \frac{b(1.5b)^3}{12} = 0.28125b^4$$

$\therefore$ $$\sigma_{bST} = \frac{13{,}080 \times 0.75b}{0.28125b^4} = \frac{34880}{b^3}$$

$$\delta_{ST} = \frac{WI_1^2 I_2^2}{3EIl}$$

for simply supported beam with load other than centre.

$$\delta_{ST} = \frac{49.05 \times 400^2 \times 800^2}{3 \times 70 \times 10^3 \times 0.28125b^4 \times 1200} = \frac{70.876 \times 10^3}{b^4}$$

substituting

$$50 = \frac{34{,}880}{b^3}\left(1 + \sqrt{1 + \frac{2 \times 200}{\left(\frac{70.876 \times 10^3}{b^4}\right)}}\right)$$

$$\frac{50b^3}{34{,}880} - 1 = \sqrt{1 + 5.644 \times 10^{-3} b^4}$$

$$1.4335 \times 10^{-3} b^3 - 1 = \sqrt{1 + 5.644 \times 10^{-3} b^4}$$

Squaring on both sides

$$(1.4335 \times 10^{-3} b^3 - 1)^2 = 1 + 5.664 \times 10^{-3} b^4$$

$$2.055 \times 10^{-6} b^6 + 1 - 2.867 \times 10^{-3} b^3 = 1 + 5.664 \times 10^{-3} b^4$$

$$2.055 \times 10^{-6} b^6 - 5.664 \times 10^{-3} b^4 - 2.867 \times 10^{-3} b^3 = 0$$

Divide by b^3

$$2.055 \times 10^{-6} b^3 - 5.664 \times 10^{-3}(b) - 2.867 \times 10^{-3} = 0$$

solving, $b = 52.75 \simeq 53$ mm

$\therefore$ $d = 1{\cdot}5b = 79{\cdot}5$ mm $= 80$ mm

$b = 53$ and $d = 80$ mm.

Problem 16: Figure 5.11 shows a box spanner used to tighten nuts. The diameter of rod is 30 mm and its length is 300 mm. The radius of crank is 150 mm. At the end of crank an impact is induced by a hammer that strikes the crank from a height of 70 mm. The weight of hammer is 50 N. Determine the impact stress induced in the rod. Take $G = 80$ GPa. Also find the maximum angle of twist in the rod.

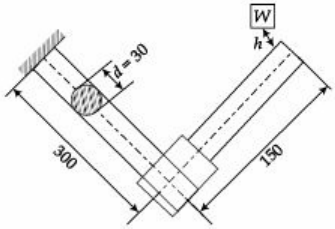


Fig. 5.11

Given data:

Diameter, $d = 30$ mm

Length, $l = 300$ mm

Radius of crank, $R = 150$ mm

$W_t = W = 50$ N

ht of fall, $h = 70$ mm

$G = 80 \times 10^3$ MPa

Impact shear stress, $\tau_{max} = \tau_{ST}\left(1 + \sqrt{1 + \dfrac{2h}{R\theta_{ST}}}\right)$

τ_{ST} = static shear stress = $\dfrac{M_t r}{J}$

M_t = static torque = $W \cdot R$

$= 50 \times 150$ N-mm $= 7500$ N-mm

$r = \dfrac{d}{2} = \dfrac{30}{2} = 15$ mm $\quad J = \dfrac{\pi d^4}{32} = \dfrac{\pi \times 30^4}{32}$ mm^4

$\therefore \quad \tau_{ST} = \dfrac{7500 \times 15}{(\pi \times 30^4/32)} = 1.415$ MPa

WKT $\dfrac{\tau}{r} = \dfrac{G\theta}{l} \quad \therefore \quad \theta_{ST} = \dfrac{\tau_{ST} \cdot l}{G \cdot r}$

$$\theta_{ST} = \frac{1.415 \times 300}{80 \times 10^3 \times 15} = 3.54 \times 10^{-4} \text{ radians.}$$

$$\therefore \quad \tau_{max} = 1.415\left[1 + \sqrt{1 + \frac{2 \times 70}{150 \times 3.54 \times 10^{-4}}}\right]$$

$$\tau_{max} = 74.12 \text{ MPa}$$

Maximum angular deflection under impact

$$\theta_{max} = \theta_{ST}\left(1 + \sqrt{1 + \frac{2h}{R\theta_{ST}}}\right)$$

$$= 3.54 \times 10^{-4}\left[1 + \sqrt{1 + \frac{2 \times 70}{150 \times 3.54 \times 10^{-4}}}\right]$$

$$\theta_{max} = 0.0185 \text{ radians} = 0.0185 \times \frac{180}{\pi} = 1.0619 \text{ degs.}$$

EXERCISE

1. A simply supported beam of span 5 m has to resist an impact of 6 kN that falls under gravity at a velocity of 30 m/min at its centre. The beam is of box section 40 mm deep and has a moment of inertia of 10^8 mm^4, modulus of elasticity is 2×10^5 N/mm^2. Determine the maximum stress induced in the beam and compare with static stress.
2. A circular rod 0.3 m long is subjected to an impact due to a mass of 500 kg that falls on to it longitudinally from a height of 30 mm. Determine the diameter of the rod taking the maximum stress as 100 MPa. Modulus of elasticity of the material is 207 GPa.
3. A power hammer of mass 500 kg strikes an anvil supported at the mid-point a beam supported at its 5 m apart. The height through which the hammer strikes is 100 mm. Determine the width of the rectangular cross section taking the depth as 210 mm.
4. A weight of 1 kN is dropped from a height of 50 mm at the free end of a cantilever beam of length 300 mm. Determine the size of square cross section taking the allowable stress as 80 MPa.
5. A steel rod is 900 mm long. It has to resist longitudinally an impact of 200 N falling under gravity at a velocity of 0.5 m/s. The maximum computed stress is limited to 240 MPa. Determine the diameter of the rod. Also find the impact strength if the length is decreased to one-third. Take E = 200 GPa.
6. A 500 N weight falls from a height of 25 mm and impact the centre of 300 mm long simply supported round beam. The maximum allowable bending stress is 90 MPa. Find the diameter of the beam taking E = 200 GPa.

CHAPTER

6 Shafts

INTRODUCTION

Shaft is usually a round member, solid or hollow cross section that rotates and transmits power. It carries machine elements like gears, pulleys, cams, sprockets, couplings etc.

Shaft design primarily consists of finding the size (diameter) in order to satisfy the strength and rigidity, while transmitting power. Shafts are usually subjected to torsion, bending and axial loads.

SYMBOLS

P = power in kW
n = speed in rpm
M_b = maximum bending moment on shaft, N-mm
M_t = maximum torque on shaft, N-mm
(torque is also known as twisting moment or turning moment)
D = diameter for solid shaft, mm
D_o = outside diameter of hollow shaft, mm
D_i = inside diameter of hollow shaft, mm
K = ratio of diameters for hollow shaft = $\frac{D_i}{D_o} < 1$
(K = 0 for solid shaft)
F = axial or thrust load, N
$K_b = C_m$ = combined shock and endurance factor in bending
$K_t = C_t$ = combined shock and endurance factor in torsion

For rotating shafts with gradually applied to loads

$K_b = 1.5$ and

$K_t = 1.0$

Suddenly applied loads

with minor shocks, $K_b = 1.5$ to 2.0

$K_t = 1$ to 1.5

Suddenly applied loads

with heavy shocks, $K_b = 2.0$ to 3.0

$K_t = 1.5$ to 3.0 ...(T 14-2/P 14.13)

σ = allowable tensile stress for shaft material, MPa

τ = allowable shear stress for shaft material, MPa

ASME code for commercial shafts

$\tau = 55$ MPa

$\sigma = 110$ MPa without key way

$\tau = 40$ MPa and $\sigma = 80$ MPa with key way

$\tau = 30\%$ of σ_y or 18% σ_u without key way

The values are to be reduced by 25% if key way is present in the shaft (i.e., $\tau = 75\%$ of $(0.3\sigma_y$ or $0.18\sigma_u)$.

Standard Sizes of Shafts

Shaft of standard size is to be preferred wherever possible. Typical standard size of shafts are:

- in steps of 0.5 mm upto 25 mm
- increment of 1 mm from 26 to 50 mm
- increment of 2 mm from 51 to 100 mm ...(T 14-6/P 14-13)
- increment of 5 mm more than 100 mm

6.1 SHAFTS SUBJECTED TO PURE TORSION

(a) *Solid shaft:* Dia of shaft, $D = \left(\frac{16 K_t M_t}{\pi\tau}\right)^{1/3}$ ...(14-9/P 14.2)

(b) *Hollow shaft:* Outside diameter of shaft, $D_o = \left(\frac{16 K_t M_t}{\pi\tau(1 - K^4)}\right)^{1/3}$ ...(14-34/P 14.4)

6.2 SHAFTS SUBJECTED TO PURE BENDING

(a) *Solid shaft:* Dia of shaft, $D = \left(\frac{32K_bM_b}{\pi\sigma}\right)^{1/3}$ ...(14-10/P 14.2)

(b) *Hollow shaft:* Outside diameter, $D_o = \left(\frac{32K_bM_b}{\pi\sigma(1-K^4)}\right)^{1/3}$. ...(14-35/P 14.4)

Inside diameter, $D_i = K D_o$

6.3 SHAFTS SUBJECTED TO COMBINED BENDING AND TORSION

(a) *Solid shaft*

(i) According to maximum normal stress theory

$$\text{Diameter of shaft, } D = \left[\frac{16}{\pi\sigma}\left\{K_bM_b + \sqrt{(K_bM_b)^2 + (K_tM_t)^2}\right\}\right]^{1/3} \quad ...(14\text{-}11/P\ 14.2)$$

(ii) According to maximum shear stress theory

$$\text{Diameter of shaft, } D = \left[\frac{16}{\pi\tau}\sqrt{(K_bM_b)^2 + (K_tM_t)^2}\right]^{1/3} \quad ...(14\text{-}12/P\ 14.2)$$

(b) *Hollow shaft*

(i) According to maximum normal stress theory outside diameter of shaft

$$D_o = \left[\frac{16}{\pi\sigma(1-K^4)}\left\{K_bM_b + \sqrt{(K_bM_b)^2 + (K_tM_t)^2}\right\}\right]^{1/3} \quad ...(14\text{-}36/P\ 14.4)$$

(ii) According to maximum shear stress theory outside diameter of shaft

$$D_o = \left[\frac{16}{\pi\tau(1-K^4)}\sqrt{(K_bM_b)^2 + (K_tM_t)^2}\right]^{1/3} \quad ...(14\text{-}37/P\ 14.4)$$

6.4 SHAFTS SUBJECTED TO COMBINED AXIAL LOAD, BENDING MOMENT AND TORSION

(a) *Solid shaft*

(i) According to maximum normal stress theory

$$\text{Diameter of shaft, } D = \left[\frac{16}{\pi\sigma}\left\{\left(K_bM_b + \alpha\frac{FD}{8}\right) + \sqrt{\left(K_bM_b + \alpha\frac{FD}{8}\right)^2 + (K_tM_t)^2}\right\}\right]^{1/3}$$

...(14-14/P 14.2)

(ii) According to maximum shear stress theory

$$\text{Diameter of shaft, } D = \left[\frac{16}{\pi\tau}\sqrt{\left(K_b M_b + \alpha\frac{FD}{8}\right)^2 + (K_t M_t)^2}\right]^{1/3} \quad \text{...(14-15/P 14.2)}$$

(b) *Hollow shaft*

(i) According to maximum normal stress theory outside diameter of the shaft

$$D_o = \left[\frac{16}{\pi\sigma(1-K^4)}\left\{\left(K_b M_b + \frac{\alpha FD_o}{8}(1+K^2)\right) + \sqrt{\left(K_b M_b + \frac{\alpha FD_o}{8}(1+K^2)\right)^2 + (K_t M_t)^2}\right\}\right]^{1/3}$$

...(14-39/P 14.4)

(ii) According to maximum shear stress theory outside diameter of the shaft

$$D_o = \left[\frac{16}{\pi\tau(1-K^4)}\sqrt{\left(K_b M_b + \frac{\alpha FDo(1+K^2)}{8}\right)^2 + (K_t M_t)^2}\right]^{1/3} \quad \text{...(14-40/P 14.5)}$$

where α = column action factor or buckling factor
= 1 for tensile loads

$$\alpha = \frac{1}{1 - 0.0044\left(\frac{l}{k}\right)} \quad \text{for } \frac{l}{k} < 115 \quad \text{...(14-64/P 14.7)}$$

$$\alpha = \frac{\sigma_y}{\pi^2 hE}\left(\frac{l}{k}\right) \quad \text{for } \frac{l}{k} > 115 \quad \text{...(14-65/P 14.7)}$$

σ_y = yield stress in compression
n = end condition coefficient
= 1 for hinged ends
= 2.25 for fixed ends
= 1.6 for ends in bearings
l = length of shaft

k = radius of gyration = $\sqrt{\frac{I}{A}}$

I = moment of inertia
A = area of cross section

PROBLEMS

Problem 1: Determine diameter of a solid shaft required to sustain a bending moment of 1.2 kN-m and a twisting moment of 0.9 kN-m. The allowable tensile and shear stresses for the shaft material are 120 MPa and 75 MPa respectively.

Given data: Bending moment, $M_b = 1.2$ kN-m $= 1.2 \times 10^6$ N-mm

Twisting moment, $M_t = 0.9$ kN-m $= 0.9 \times 10^6$ N-mm

Allowable tensile stress, $\sigma = 120$ MPa

Allowable shear stress, $\tau = 75$ MPa

Assume combined shock and endurance factors in bending and torsion $K_b = 1.75$ and $K_t = 1.25$

(i) According to maximum normal stress theory, dia of shaft

$$D = \left[\frac{16}{\pi\sigma}\{K_bM_b + [(K_bM_b)^2 + (K_tM_t)^2]^{1/2}\}\right]^{1/3} \quad ...(14\text{-}11)$$

$$= \left[\frac{16}{\pi \times 120}\{1.75 \times 1.2 \times 10^6 + [(1.75 \times 1.2 \times 10^6)^2 + (1.25 \times 0.9 \times 10^6)^2]^{1/2}\}\right]^{1/3}$$

$D = 57.513$ mm

(ii) According to maximum shear stress theory, dia of shaft

$$D = \left[\frac{16}{\pi\tau}\{(K_bM_b)^2 + (K_tM_t)^2\}^{1/2}\right]^{1/3} \quad ...(14\text{-}12)$$

$$= \left[\frac{16}{\pi \times 75}\{(1.75 \times 1.2 \times 10^6)^2 + (1.25 \times 0.9 \times 10^6)^2\}^{1/2}\right]^{1/3}$$

$D = 54.489$ mm

Recommended diameter, $D = 57.513$ mm (bigger value) $\simeq 58$ mm.

Problem 2: Determine the diameters of a hollow shaft to sustain a twisting moment of 600 N-m and a bending moment of 750 N-m limiting the angle of twist to 0.5 deg/m length. The diameters ratio is 0.6.

Given data: Hollow shaft, diameter's ratio, $K = \frac{D_i}{D_o} = 0.6$

Twisting moment, $M_t = 600$ N-m $= 600 \times 10^3$ N-mm

Bending moment, $M_b = 750$ N-m $= 750 \times 10^3$ N-mm

Angle of twist, $\theta = 0.5^\circ = 0.5 \times \frac{\pi}{180}$ radians

Length, $l = 1$ m $= 1000$ mm

(θ is given per metre length, hence take, $l = 1$ m)

Assume allowable tensile stress, $\sigma = 110$ MPa

Allowable shear stress, $\tau = 55$ MPa

Select $K_b = 1.75$ and $K_t = 1.25$

(i) According to maximum normal stress theory, outside dia of shaft

$$D_o = \left[\frac{16}{\pi\sigma(1-K^4)}\{K_bM_b + ((K_bM_b)^2 + (K_tM_t)^2)^{1/2}\}\right]^{1/3} \quad ...(14\text{-}36)$$

$$= \left[\frac{16}{\pi \times 110(1-0.6^4)}\{1.75 \times 750 \times 10^3 + ((1.75 \times 750 \times 10^3)^2 + (1.25 \times 600 \times 10^3)^2\}^{1/2}\right]^{1/3}$$

$= 53.16$ mm

(ii) According to maximum shear stress theory, outside diameter of shaft

$$D_o = \left[\frac{16}{\pi\tau(1-K^4)}\{(K_bM_b)^2 + (K_tM_t)^2)^{1/2}\}\right]^{1/3} \quad ...(14\text{-}37)$$

$$D_o = \left[\frac{16}{\pi \times 55(1-0.6^4)}\{(1.75 \times 750 \times 10^3)^2 + (1.25 \times 600 \times 10^3)^2)^{1/2}\}\right]^{1/3}$$

$D_o = 54.38$ mm

(iii) According to angle of twist,

$$\frac{M_t}{J} = \frac{G\theta}{l}$$

Assume rigidity modulus, $G = 80$ GPa $= 80 \times 10^3$ MPa

$$J = \frac{\pi}{32}(D_o^4 - D_i^4) = \frac{\pi}{32}(D_o^4 - (0.6D_o)^4)$$

$$= \frac{\pi}{32}D_o^4(0.8704)$$

$$\therefore \quad \frac{600 \times 10^3}{\frac{\pi}{32}D_o^4(0.8704)} = \frac{80{,}000}{1000} \times \frac{0.5\pi}{180}$$

$$\therefore \quad D_o = 56.32 \text{ mm}$$

Recommended diameter, $D_o = 56.32$ mm (biggest of above 3 values) $= 57$ mm

$$D_i = 0.6 \times D_o = 0.6 \times 57 = 34.2 = 34$$

$$D_o = 57 \text{ mm}$$

$D_i = 34$ mm (inner dia is to be rounded off to lower value)

Problem 3: Determine the diameter of a solid shaft required to transmit 50 kW at 1000 rpm. The allowable shear stress may be taken as 90 MPa. Replace this solid shaft with a hollow one assuming the diameter's ratio as 0.6, made of same material and factor of safety. As a result of replacement, determine:

(i) percentage of reduction in weight assuming same length for both shafts,

(ii) the ratio of torsional stiffness of the hollow shaft to that of the solid shaft.

Given data: Power, $P = 50$ kW

Speed, $n = 1000$ rpm

Allowable shear stress, $\tau = 90$ MPa

For hollow shaft, $\frac{D_i}{D_o} = 0.6 = K$

Same material, length and FOS.

$$\text{Torque, } M_t = \frac{9.55 \times 10^6 P}{n} = \frac{9.55 \times 10^6 \times 50}{1000}$$

$$M_t = 477.5 \times 10^3 \text{ N-mm.}$$

(i) Solid shaft, dia, $D = \left[\frac{16 M_t \cdot K_t}{\pi \tau}\right]^{1/3}$ ...(14-9)

Assume $K_t = 1.0$ (no shocks)

$$D = \left[\frac{16 \times 477.5 \times 10^3 \times 1}{\pi \times 90}\right]^{1/3} = 30 \text{ mm}$$

(ii) For hollow shaft with $K = 0.6$, outside dia

$$D_o = \left[\frac{16 K_t M_t}{\pi \tau (1 - K^4)}\right] \quad \text{...(14-26)}$$

$$\therefore \quad = \left[\frac{16 \times 1 \times 477.5 \times 10^3}{\pi \times 90 (1 - 0.6^4)}\right]^{1/3} = 31.43 \simeq 32 \text{ mm}$$

$$D_i = KD_o = 0.6 \times 32 = 19.2 \simeq 19 \text{ mm}$$

(iii) % saving in weight $= \left(\frac{W_S - W_H}{W_S}\right) \times 100$

W_s = weight of solid shaft = $A_s l w$

$$W_s = \left(\frac{\pi}{4} D^2\right) l w$$

ω = Specific weight of shaft material.

$$W_s = \frac{\pi}{4} 30^2 l w.$$

W_H = weight of hollow shaft = $A_H l w$

$$W_H = \frac{\pi}{4}(D_o^2 - D_i^2)lw = \frac{\pi}{4}(32^2 - 19^2)lw$$

$$\therefore \quad \% \text{ Saving} = \frac{\frac{\pi}{4}30^2 lw - \frac{\pi}{4}(32^2 - 19^2)lw}{\frac{\pi}{4}30^2 lw} \times 100$$

$$= 26.33\%$$

(iv) Ratio of torsional stiffnesses

$$\text{Torsional stiffness} \left(\frac{M_t}{\theta}\right) = \frac{GJ}{l}$$

$$\text{For solid shaft} \left(\frac{M_t}{\theta}\right)_s = \frac{GJ_s}{l} = \frac{G\frac{\pi}{32}D^4}{l}$$

$$\text{For hollow shaft} \left(\frac{M_t}{\theta}\right)_H = \frac{GJ_H}{l} = \frac{G\cdot\frac{\pi}{32}(D_o^4 - D_i^4)}{l}$$

$$\text{Comparing} \quad \frac{(M_t/\theta)_S}{(M_t/\theta)_H} = \frac{(G\pi D^4/32l)}{(G\pi(D_o^4 - D_i^4/32l))} = \frac{30^4}{32^4 - 19^4} = 0.882$$

Stiffness of solid shaft is 0.882 times that of hollow one.

Problem 4: A hollow shaft of 50 mm outside diameter and 30 mm inside diameter, 300 mm long is subjected to a torque of 4 N-m. What is the angle of twist if modules of rigidity is 90 GPa.

Given data:

$D_o = 50$ mm, $D_i = 30$ mm

$l = 300$ mm, $M_t = 4$ N-m $= 4 \times 10^3$ N-mm

$G = 90$ GPa $= 90 \times 10^3$ MPa

$$\text{WKT} \frac{M_t}{J} = \frac{G\theta}{l} \Rightarrow \frac{4000}{(\pi/32)(D_o^4 - D_i^4)} = \frac{90 \times 10^3 \theta}{300}$$

$$\therefore \quad \theta = 2.4965 \times 10^{-5} \text{ radians} = 2.4965 \times 10^{-5} \times \frac{180}{\pi}$$

$$\theta = 1.43 \times 10^{-3} \text{ degrees.}$$

Problem 5: A simply supported shaft has the distance between supports as 600 mm. The load at the centre is 15 kN. If the deflection at the centre is to be limited to 0.02 mm, what should be diameter of the shaft. If the shaft diameter is doubled, what will be the deflection at the centre. The modulus of elasticity is 210 GPa.

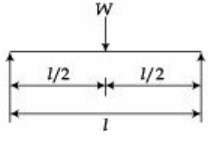


Fig. 6.1

Given data: $l = 600$ mm.

$W = 15$ kW $= 15 \times 10^3$ N, deflection at the centre $y = 0.02$ mm

$E = 210$ GPa $= 210 \times 10^3$ MPa

For simply supported beam with central load,

(i) Deflection $y = \dfrac{Wl^3}{48EI} \Rightarrow 0.02 = \dfrac{15000 \times 600^3}{48 \times 210 \times 10^3 \times (\pi D^4/64)}$ ...(T 2-8/P 2.34)

$\therefore$ Diameter, $D = 134.515$ mm

$D \simeq 135$ mm

(ii) If the diameter is doubled, $D = 2 \times 135 = 270$ mm

$$\text{Deflection, } y = \frac{15 \times 10^3 \times 600^3}{48 \times 210 \times 10^3 \times (\pi \times 270^4/64)} = 1.232 \times 10^{-3} \text{ mm.}$$

Problem 6: A shaft rotating at 300 rpm transmits 18 kN. It is supported on bearings at a distance of 3 m apart and carries a central load of 2 kN. Selecting C40 steel for shaft ($\sigma_y = 328.6$ MPa) and factor of safety = 2.5, find suitable diameter of the shaft.

Given data: Speed, $n = 300$ rpm

Power, $P = 18$ kW

Span $= l = 3$ m $= 3000$ mm

Central load, $W = 2$ kN $= 2000$ N

$\sigma_y = 328.6$ MPa, FOS $= 2.5$

$$\text{Allowable tensile stress, } \sigma = \frac{\sigma_y}{\text{FOS}} = \frac{328.6}{2.5} = 131.44 \text{ MPa}$$

Allowable shear stress, $\tau = 0.5$, $\sigma = 0.5 \times 131.44 = 65.72$ MPa

Assume $K_b = 1.75$ and $K_t = 1.25$ considering minor shock

$$\text{Torque, } M_t = \frac{9.55 \times 10^6 P}{n} = \frac{9.55 \times 10^6 \times 18}{300} = 573 \times 10^3 \text{ N-mm}$$

For simply supported beam with central load,

$$M_b = \frac{Wl}{4} = \frac{2000 \times 3000}{4}$$ ...(T 2-8/P 2.34)

Bending moment

$$M_b = 1.5 \times 10^6 \text{ N-mm}$$

(i) According to maximum normal stress theory, diameter of shaft

$$D = \left[\frac{16}{\pi\sigma}\{K_b M_b + [(K_b M_b)^2 + (K_t M_t^2)]^{1/2}\}\right]^{1/3}$$ ...(14-11)

$$= \left[\frac{16}{\pi \times 131.44}\{1.75 \times 1.5 \times 10^6 + [(1.75 \times 1.5 \times 10^6)^2 + (1.25 \times 573 \times 10^3)^2]^{1/2}\}\right]^{1/3}$$

$D = 59.168$ mm

(ii) According to maximum shear stress theory, diameter of the shaft

$$D = \left[\frac{16}{\pi\tau}\{(K_b M_b)^2 + (K_t M_t)^2\}^{1/2}\right]^{1/3} \quad ...(14\text{-}12)$$

$$= \left[\frac{16}{\pi \times 65.72}\{(1.75 \times 1.5 \times 10^6)^2 + (1.25 \times 573 \times 10^3)^2\}^{1/2}\right]^{1/3}$$

$$D = 59.52$$

Recommended diameter $D = 59.52 \simeq 60$ mm (bigger value).

Problem 7: The shaft of a motor is supported at two bearings 800 mm apart. The armature of the motor can be considered a *UDL* of 15 N/mm, centrally spread over a length of 0.5 m. Select suitable material and appropriate value of factor of safety to determine the diameter of the shaft. The motor develops 15 kW at 1500 rpm.

Given data:

$P = 15$ kW

$n = 1500$ rpm

UDL $w = 15$ N-mm

Length of *UDL* = 500 mm

Centrally placed for 500 mm. (Ref. Fig. 6.2)

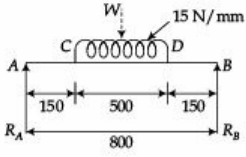


Fig. 6.2

Total load $W = wl_1 = 15 \times 500 = 7500$ N

Since the load is in the middle, both reactions are equal

$$R_A = R_B = \frac{W}{2} = \frac{7500}{2} = 3750 \text{ N}$$

Maximum bending moment

at A and B, $M_{b_A} = M_{b_B} = 0$

at C = at D $M_{b_C} = M_{b_D} = R_A \times 150 = 3750 \times 150$

$= 562.5 \times 10^3$ N-mm

at the middle, $M_b = R_A \times 400 - (15 \times 250) \times \frac{250}{2}$

$$= 3750 \times 400 - \frac{15 \times 250^2}{2} = 1.031 \times 10^6 \text{ N-mm}$$

$\therefore$ Maximum bending moment on shaft, $M_b = 1.031 \times 10^6$

$$\text{Torque, } M_t = \frac{9.55 \times 10^6 P}{n} = \frac{9.55 \times 10^6 \times 15}{1500}$$

$= M_t = 95500$ N-mm

Assuming minor shocks, $K_b = 1.75$ and $K_t = 1.25$

(i) According to maximum normal stress theory, diameter of the shaft

$$D = \left[\frac{16}{\pi\sigma}\{K_b M_b + [(K_b M_b)^2 + (K_t M_t)^2]^{1/2}\}\right]^{1/3} \quad ...(14\text{-}11)$$

Select C40 steel for shaft, yield stress $\sigma_y = 328.6$ MPa assume FOS = 2.5

$\therefore$ Allowable tensile stress, $\sigma = \dfrac{\sigma_y}{FOS} = \dfrac{328.6}{2.5} = 131.44$ MPa

Assume, $\tau = 0.5 \times \sigma = 65.72$ MPa.

$$D = \left[\frac{16}{\pi \times 131.44}\{1.75 \times 1.031 \times 10^6 + [(1.75 \times 1.031 \times 10^6)^2 + (1.25 \times 95500)^2]^{1/2}\}\right]^{1/3}$$

$$D = 51.92 \text{ mm}$$

(ii) According to maximum shear stress theory, diameter of the shaft

$$D = \left[\frac{16}{\pi\tau}\{(K_b M_b)^2 + (K_t M_t)^2\}^{1/2}\right]^{1/3} \quad ...(14\text{-}12)$$

$$= \left[\frac{16}{\pi \times 65.72}\{(1.75 \times 1.031 \times 10^6)^2 + (1.25 \times 95500)^2\}^{1/2}\right]^{1/3}$$

$$D = 51.94 \text{ mm}$$

Recommended dia of shaft, $D = 51.94 \simeq 52$ mm

Problem 8: A hollow shaft of diameters 300 mm and 400 mm is mounted on two bearing 9 m apart and transmits 90 kW at 1800 rpm. The shaft weighs 60 kN. Determine the stresses induced in the shaft. Take $C_M = C_t = 1.5$.

Given data:

$D_i = 300$ mm

$D_o = 400$ mm

$l = 9000$ mm

Power, $P = 90$ kW

Speed, $n = 1800$ rpm

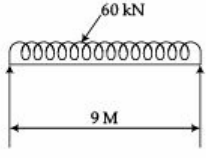


Fig. 6.3

Wt of shaft = 60 kN is to considered *UDL* (self weight)

$C_m = K_b = 1.5$

$C_t = K_t = 1.5$

$$\text{Torque, } M_t = \frac{9.55 \times 10^6 P}{n} = \frac{9.55 \times 10^6 \times 90}{1800} = 477.5 \times 10^3 \text{ N-mm}$$

For simply supported beam with *UDL* ...(T 2-8)

$$\text{Maximum bending moment, } M_b = \frac{Wl}{8} = \frac{60 \times 1000 \times 9000}{8} = 67.5 \times 10^6 \text{ N-mm}$$

$$\text{Diameters ratio, } K = \frac{D_i}{D_o} = \frac{300}{400} = 0.75$$

(i) According to maximum normal stress theory outside diameter

$$D_o = \left[\frac{16}{\pi\sigma(1-K^4)}\{K_b M_b + [(K_b M_b)^2 + (K_t M_t)^2]^{1/2}\}\right]^{1/3} \quad ...(14\text{-}36)$$

$$\therefore \quad \sigma = \frac{16}{\pi \times 400^3 (1-0.75^4)}\{1.5 \times 67.5 \times 10^6 + [1.5 \times 67.5 \times 10^6)^2 + (1.5 \times 477.5 \times 10^3)^2]^{1/2}\}$$

$$\text{Normal stress, } \sigma = 23.57 \text{ MPa}$$

(ii) According to maximum shear stress theory outside diameter

$$D_o = \left[\frac{16}{\pi\tau(1-K^4)}\{(K_b M_b)^2 + (K_t M_t)^2\}^{1/2}\right]^{1/3} \quad ...(14\text{-}37)$$

$$\text{Shear stress, } \tau = \frac{16}{\pi \times 400^3 (1-0.75^4)}[(1.5 \times 67.5 \times 10^6)^2 + (1.5 \times 477.5 \times 10^3)^2]^{1/2}$$

$$\tau = 11.79 \text{ MPa.}$$

Problem 9: With usual notations, prove that a hollow shaft is always stronger than solid shaft made of the same material, length and weight when subjected to simple torsion and hence deduce

$$\frac{\text{Torque of hollow shaft}}{\text{Torque of solid shaft}} = \frac{1+K^2}{\sqrt{1-K^2}} > 1 \quad \text{where} \quad K = \frac{D_i}{D_o}$$

Given data: Material is same:

Hence, specific *wt* '*w*' is same.

Allowable shear stress τ is same and rigidity modulus '*G*' is same for both solid and hollow shafts.

Length is same '*l*'

Weight is same $W_s = W_H$

$$\left(\frac{\pi}{4}D^2\right) lw = \frac{\pi}{4}(D_o^2 - D_i^2) lw$$

$$D^2 = D_o^2 - D_i^2$$

$$\therefore \quad D^2 = D_o^2 - (KD_o)^2 = D_o^2(1-K^2)$$

$$D = D_o\sqrt{1-K^2}$$

$$\text{WKT } \frac{M_t}{J} = \frac{\tau}{r} \quad \therefore \quad \text{torque } M_t = \frac{\tau \cdot J}{r}$$

$$\text{For solid shaft, } M_{t_s} = \frac{\tau \cdot J_s}{r_s} = \frac{\tau \cdot \pi D^4/32}{D/2} = \frac{\tau \pi D^3}{16}$$

$$\text{Substitute, } D = D_o\sqrt{1-K^2}$$

$$\therefore \quad M_{t_s} = \frac{\tau \cdot \pi (D_o\sqrt{1-K^2})^3}{16} = \frac{\tau \cdot \pi D_o^3}{16}(\sqrt{1-K^2})^3$$

$$\text{For hollow shaft,} \quad M_{t_H} = \frac{\tau \cdot J_H}{r_H} = \frac{\tau \cdot \frac{\pi}{32}(D_o^4 - D_i^4)}{\frac{D_o}{2}}$$

$$M_{t_H} = \frac{\tau \cdot \pi}{16} = \frac{D_o^4(1-K^4)}{Do} = \frac{\tau\pi}{16} D_o^3(1-K^4)$$

$$\therefore \quad \frac{\text{Torque of hollow shaft}}{\text{Torque of solid shaft}} = \frac{\tau \cdot \frac{\pi}{16} D_o^3(1-K^4)}{\tau \cdot \frac{\pi}{16} D_o^3(1-K^2)^3} = \frac{1-K^4}{(\sqrt{1-K^2})^3}$$

$$= \frac{(1+K^2)(1-K^2)}{(1-K^2)\sqrt{1-k^2}} = \frac{1+K^2}{\sqrt{1-K^2}}$$

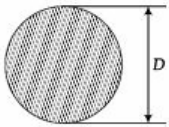
This is always > 1 since $K < 1$ and $1 + K > 1 - K$.

Hence, hollow shaft is stronger than solid shaft of the same material, weight and length.

Problem 10: Compare the weight, strength and stiffness of a hollow shaft with that of solid shaft. Both shafts have same external diameter length and made of the same material. The internal diameter of the hollow shaft is 0.6 times the external diameter.

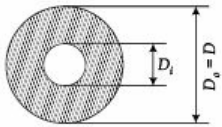
Given data:

Solid Shaft



Area $A = \frac{\pi}{4} D^2$

Hollow Shaft



$$K = \frac{D_i}{D_o} = 0.6$$

$$A = \frac{\pi}{4}(D_o^2 - D_i^2) = \frac{\pi}{4} D_o^2(1-K^2)$$

$$r = \frac{D}{2}$$

$$J = \frac{\pi}{32} D^4$$

$$A = \frac{\pi}{4} D_o^2 (1 - 0.6^2) = \frac{\pi}{4} D_o^2 (0.64)$$

$$A = \frac{\pi}{4} D^2 (0.64)$$

$$r = \frac{D_o}{2} = \frac{D}{2}$$

$$J = \frac{\pi}{32} (D_o^4 - D_i^4) = \frac{\pi}{32} D_o^4 (1 - K^4)$$

$$J = \frac{\pi}{32} D^4 (1 - 0.6^4) = \frac{\pi}{32} D^4 (0.8704)$$

(i) **Weight** $W = Alw$

Since material and length are same for both G, τ, sp. weight w and l are same.

$$\text{For solid shaft} = W_s = \frac{\pi}{4} D^2 lw$$

$$\text{Hollow shaft, } W_H = \frac{\pi}{4} D^2 (0.64) lw$$

$$\text{Comparing } \frac{W_H}{W_S} = \frac{\frac{\pi}{4} D^2 (0.64) lw}{\frac{\pi}{4} D^2 lw} = 0.64$$

Weight of the hollow shaft is 0.64 times that of solid shaft.

(ii) **Strength:** Strength for a shaft is torque capacity

$$M_t = \tau \cdot \frac{J}{C}$$

$$\text{For solid shaft, } M_{t_S} = \tau \cdot \left(\frac{J}{r}\right)_S = \frac{\tau \cdot \frac{\pi D^4}{32}}{d/2} = \tau \cdot \frac{\pi D^3}{16}$$

$$\text{For hollow shaft, } M_{t_H} = \tau \cdot \left(\frac{J}{r}\right)_H = \frac{\tau \cdot \frac{\pi D^4}{32} (0.8704)}{D/2}$$

$$M_{t_H} = \tau \cdot \frac{\pi D^3}{16} (0.8704)$$

Comparing, $$\frac{M_{t_H}}{M_{t_S}} = \frac{0.8704\dfrac{\pi D^3}{16}\tau}{\left(\dfrac{\pi D^3}{16}\tau\right)} = 0.8704$$

The strength of the hollow shaft is 0.8704 times that of solid shaft

(iii) **Stiffness:** Stiffness is $\left(\dfrac{M_t}{\theta}\right) = \dfrac{GJ}{l}$

For solid shaft $$\left(\frac{M_t}{\theta}\right)_S = \frac{G \cdot J_S}{l} = \frac{G}{l}\frac{\pi D^4}{32}$$

For hollow shaft $$\left(\frac{M_t}{\theta}\right)_H = \frac{GJ_H}{l} = \frac{G}{l}\frac{\pi D^4 (0.8704)}{32}$$

Comparing $$\frac{\left(\dfrac{M_t}{\theta}\right)_H}{\left(\dfrac{M_t}{\theta}\right)_S} = \frac{\dfrac{G}{l}\dfrac{\pi D^4}{32}(0.8704)}{\left(\dfrac{G}{l}\dfrac{\pi D^4}{32}\right)} = 0.8704$$

The stiffness of the hollow shaft is 0.8704 times the stiffness of the solid shaft.

Problem 11: One solid and one hollow shaft are to be of equal strength in torsion. The outside diameter of the hollow shaft is to be 10% larger than the solid shaft. What will be the ratio weight of the hollow shaft to that of solid shaft. Both shafts are made of the same material.

Given data: Dia of solid shaft = D

Outside dia of hollow shaft, $D_o = 1.1D$ (10% more than dia of solid shaft)

Let D_i = inside dia of hollow shaft

τ = allowable shear stress (same for both since material is same)

(i) Strength in torsion = $M_t = \dfrac{\tau \cdot J}{r}$

For solid shaft, $M_{t_S} = \dfrac{\tau \cdot J_S}{r_s}$

$$J_s = \frac{\pi}{32}D^4 \quad \text{and} \quad r_s = \frac{D}{2}$$

$$\therefore \quad M_{t_S} = \frac{\tau \cdot \dfrac{\pi D^4}{32}}{(D/2)} = \frac{\pi D^3}{16}\tau$$

For hollow shaft, $M_{t_H} = \dfrac{\tau \cdot J_H}{r_H}$

$$J_H = \frac{\pi}{32}(D_o^4 - D_i^4) = \frac{\pi}{32} D_o^4 (1 - K^4) = \frac{\pi}{32}(1.1D)^4 (1 - K^4)$$

where $K = \dfrac{D_i}{D_o}$

$$r = \frac{D_o}{2} = \frac{1.1D}{2}$$

$$\therefore \quad M_{t_H} = \tau \cdot \frac{\frac{\pi}{32}(1 \cdot iD)^4 (1 - K^4)}{\left(\frac{1 \cdot 1D}{2}\right)} = \frac{\pi}{16} D^4 (1 - K^4)(1.331)$$

For equal strength, equate these two, $M_{t_S} = M_{t_H}$

$$\frac{\pi D^3}{16} \tau = \frac{\pi}{16} D^3 (1 - K^4)(1.331)$$

$$\therefore \quad K = 0.706 = \frac{D_i}{D_o}$$

$$\therefore \quad D_i = KD_o = 0.706 (1 \cdot 1D) = 0.777D$$

(ii) Weight W = vol × sp. weight = $(Al)w$

For solid shaft, $W_S = \dfrac{\pi D^2}{4} lw$

For hollow shaft, $W_H = \dfrac{\pi}{4}(D_o^2 - D_i^2) lw$

$$= \frac{\pi}{4}[(1.1D)^2 - (0.777D)^2]\, lw$$

$$= \frac{\pi}{4} D^2 (0.606D)$$

Comparing $\dfrac{W_H}{W_S} = 0.606.$

6.5 EFFECT OF BELT AND GEAR DRIVE ON SHAFT

As seen from the above problems, bending and torsional moments are main factors influencing the shaft design. Hence it is essential to find the various forces acting on the shaft. Then find bending moments at critical points on the shaft to arrive at maximum bending moment on the shaft.

6.5.1 Forces Acting Shaft Due to Pulley and Belt Drive

T_1 = tight side tension, N

T_2 = slack side tension, N

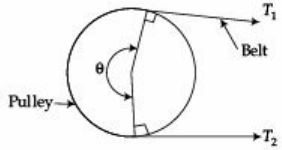
θ = angle of contact, radians

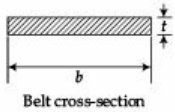
R_P = radius of pulley

b = width of belt

t = thickness of belt

σ_{belt} = allowable stress in belt material





$$\text{Tension's ratio} = \frac{T_1}{T_2} = e^{\mu\theta} \text{ for flat belt}$$

and $$\frac{T_1}{T_2} = e^{(\mu\theta/\sin\alpha)} \quad \text{for } v\text{–belts}$$

μ = coefficient of friction.

α = semi groove angle for V-pulley

Torque transmitted by belt $M_t = (T_1 - T_2)R_P$

From these two relations find T_2 and T_1

$\therefore$ Force action on shaft due to belt pull = $T_1 + T_2$

This acts along the line joining the centres of pulleys.

T_1 = tight side tension or allowable tension

T_1 = area of belt $\times$ σ_{belt} = $(bt)\sigma_{belt}$.

6.5.2 Forces Acting on Shaft Due to Gear Drive

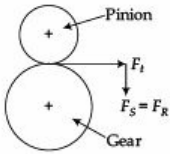
Smaller gear is pinion

Bigger gear is gear

F_t = Tangential force acts along common tangent of pitch circles

$F_s = F_R$ = Separating or radial force, acts perpendicular to F_t (opp on each gear)

$$F_t = \frac{\text{Torque}}{\text{Radius of gear or pinion}} = \frac{M_t}{R_g}$$



$F_S = F_t \tan\alpha$ where α = pressure angle standard pressure angles are 14.5° and 20°.

Dia of gear or pinion = D_g = module $\times$ No. of both

R_g = Radius of pinion or gear = $\frac{D_g}{2}$

6.5.3 Effect of Weights

Any weight due to gravity always acts vertical downward only.

6.5.4 Torque $M_t = \frac{9.55 \times 10^6 P}{n}$ N-mm

P = power in kW.

n = speed in rpm.

Problem 12: A section of commercial shaft 2 m long between bearings carries a 900 N pulley at its midpoint. The shaft transmits 21 kW at 300 rpm. The belt drive is horizontal. The sum of tensions is 6 kN. Find suitable diameter of shaft and angle of twist between the bearings. Taking G = 80 GPa.

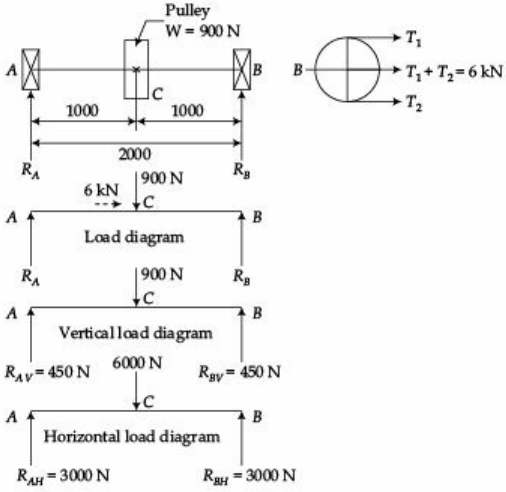


Fig. 6.4

Given data: Power, P = 21 kW, Speed, n = 300 rpm.

For commercial shaft with keyway,

Allowable tensile stress, σ = 80 MPa and

Allowable shear stress, τ = 40 MPa

Weight of pulley, W = 900 N

Sum of tensions, $T_1 + T_2$ = 6000 N.

Weight always acts in vertical direction

Consider vertical load diagram.

Since the load is in the middle, both reactions will be equal.

$$R_A = R_B = \frac{900}{2} = 450 \text{ N}$$

For simply supported beam with central load, ...(T 2-8/P 2.34)

Vertical bending moment, $M_{b_V} = \frac{Wl}{4} = \frac{900 \times 2000}{4} = 450 \times 10^3$ N-mm

Consider, horizontal load diagram reactions, $R_{A_H} = R_{B_H} = \frac{6000}{2} = 3000$ N

Horizontal bending moment, $M_{b_H} = \frac{W_H l}{4} = \frac{6000 \times 2000}{4}$ 3×10^6 N-mm

Resultant bending moment, $M_b = \sqrt{M_{bv}^2 + M_{bH}^2}$

$$M_b = \sqrt{(450 \times 10^3)^2 + (3 \times 10^6)^2} = 3.0336 \times 10^6 \text{ N-mm}$$

Torque on shaft $$M_t = \frac{9.55 \times 10^6 P}{n}$$

$$M_t = \frac{9.55 \times 10^6 \times 21}{300} = 668.5 \times 10^3 \text{ N-mm}$$

Assuming minor shocks, $K_b = 1.75$ and $K_t = 1.25$

(i) According to maximum normal stress theory, diameter of the shaft

$$D = \left[\frac{16}{\pi\sigma}\left\{K_b M_b + \left[(K_b M_b)^2 + (K_t M_t)^2\right]^{1/2}\right\}\right]^{1/3} \quad ...(14\text{-}11)$$

$$= \left[\frac{16}{\pi \times 80}\{1.75 \times 3.0336 \times 10^6 + [(1.75 \times 3.0336 \times 10^6)^2 + (1.25 \times 668.5 \times 10^3)^2]^{1/2}\}\right]^{1/3}$$

$D = 87.94 \simeq 88$ mm.

(ii) According to the maximum shear stress theory, diameter of the shaft

$$D = \left[\frac{16}{\pi\tau}\left((K_b M_b)^2 + (K_t M_t)^2\right)^{\frac{1}{2}}\right]^{\frac{1}{3}} \quad ...(14\text{-}12)$$

$$D = \left[\frac{16}{\pi \times 40}\left\{(1.75 \times 3.0336 \times 10^6)^2 + (1.25 \times 668.5 \times 10^3)^2\right\}^{\frac{1}{2}}\right]^{\frac{1}{3}}$$

$D = 88.12$ m = 89 mm

Recommended diameter $D = 89 = 90$ std dia

Angle of twist: $\frac{M_t}{J} = \frac{G\theta}{l}$

$$\theta = \frac{M_{t\text{-}l}}{JG}$$

l = 1000 mm from pulley to any bearing.

$G = 80\ GPa = 80 \times 10^3$ MPa, $J = \frac{\pi D^4}{32} = \frac{\pi \times 90^4}{32}$

$$\theta = \frac{668.5 \times 10^3 \times 1000}{\left(\frac{\pi}{32} 90^4\right) \times 80 \times 10^3} = 1.2973 \times 10^{-3} \text{ radians}$$

$$\theta = 1.2973 \times 10^3 \times \frac{180}{\pi} = 0.0743 \text{ degrees.}$$

Problem 13: A M.S shaft transmits 15 kW at 300 rpm. It is supported on two bearings 1.2 m apart. The shaft receives power through a 450 mm diameter pulley mounted at 300 mm to the right of right bearing. The power is given out through a 300 mm diameter gear mounted at 250 mm to the right of left bearing. The belt drive is horizontal and the gear drives with a downward tangential force. Find suitable diameter of the shaft if yield stress for shaft material is 234 MPa and factor of safety = 2. Take $K_b = K_t = 1.5$. Tension's ratio of belt is 3.

Given data:

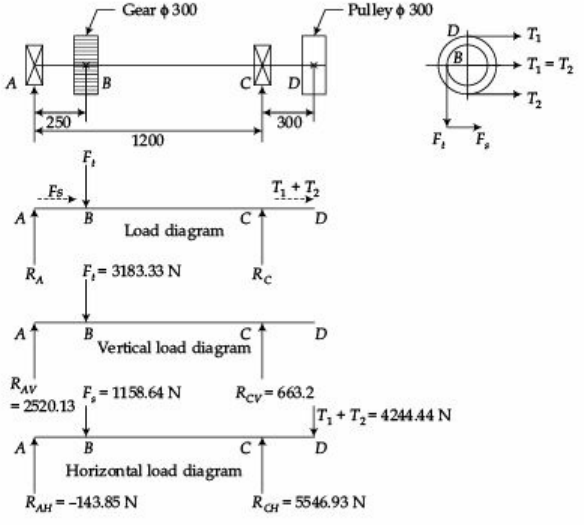


Fig. 6.5

Power, $P = 15$ kW, Speed $n = 300$ rpm

$\sigma_y = 234$ MPa

$\therefore \quad \sigma = \dfrac{\sigma_y}{FOS} = \dfrac{234}{2} = 117$ MPa.

Assume $\tau = 0.5\,\sigma = 0.5 \times 117 = 58.5$ MPa.

$K_b = K_t = 1.5$

$$\text{Torque, } M_t = \frac{9.55 \times 10^6 P}{n} = \frac{9.55 \times 10^6 \times 15}{300} = 477.5 \times 10^3 \text{ N-mm}$$

Consider gear at B

$$\text{Tangential load, } F_t = \frac{M_t}{Rg} = \frac{477.5 \times 10^3}{(300/2)} = 3183.33 \text{ (Vertical)}$$

Separating force, $F_s = F_t \times \tan \alpha$

Assume pressure angle for gear, $\alpha = 20°$

$F_s = 3183.33 \tan 20 = 1158.64$ N (Horizontal)

Consider pulley at D

$$\text{Tension's ratio, } \frac{T_1}{T_2} = 3 \Rightarrow T_1 = 3T_2$$

Torque transmitted by belt, $M_t = (T_1 - T_2)R_p$

$$477.5 \times 10^6 = (3T_2 - T_2)\frac{450}{2}$$

$\therefore \quad T_2 = 1061.11$ N

$T_1 = 3T_2 = 3183.33$ N

$\therefore$ Sum of tensions acting on shaft $= T_1 + T_2 = 4244.44$ N (Horizontal)

Now consider vertical load diagram

Reactions $R_{AV} + R_{CV} = 3183.33$ N

Taking moments about A,

$3183.33 \times 250 - R_{CV} \times 1200 = 0 \quad \therefore \quad R_{CV} = 663.2$ N

$\therefore \quad R_{AV} = 3183.33 - R_{CV} = 2520.13$ N

Vertical bending moments

at A, $M_{bVA} = 0$

at B, $M_{bVB} = 2520.13 \times 250 = 630.034 \times 10^3$ N-mm

at C, $M_{bVC} = 0$

at D, $M_{bVD} = 0$

Consider horizontal load diagram

Reactions $R_{A_H} + R_{C_H} = 1158.64 + 4244.44 = 5403.08$

Taking moments about A,

$1158.64 \times 250 - R_{C_H} \times 1200 + 4244.44 \times 1500 = 0$

$\therefore \quad R_{C_H} = 5546.93$ N

$\therefore \quad R_{A_H} = 4341.97 - R_{C_H} = -143.85$ N

(Since $-ve$, change direction of R_{AH} and sign)

Horizontal bending moments

at A, $M_{b_{H_A}} = 0$

at B, $M_{b_{H_B}} = 143.85 \times 250 = 35.96 \times 10^3$ N-mm

at C, $M_{b_{H_C}} = 4244.44 \times 300 = 1.273 \times 10^6$ N-mm

at D, $M_{b_{H_D}} = 0$

Resultant bending moments

at A, $M_{b_A} = \sqrt{M_{b_{VA}}^2 + M_{b_{H_A}}^2} = 0$

at B, $M_{b_B} = \sqrt{(630.04 \times 10^3)^2 + (35.96 \times 10^3)^2} = 631.07 \times 10^3$ N-mm

at C, $M_{b_C} = \sqrt{0^2 + (1.273 \times 10^6)^2} = 1.273 \times 10^6$ N-mm

at D, $M_{b_D} = 0$

$\therefore$ Maximum bending moment on shaft, $M_b = 1.273 \times 10^6$ N-mm

Torque on shaft, $M_t = 477.5 \times 10^3$ N-mm

$K_b = K_t = 1.5$

$\sigma = 117$ MPa, $\tau = 58.5$ MPa.

(i) According to maximum normal stress theory, diameter of shaft

$$D = \left[\frac{16}{\pi\sigma}\{K_bM_b + ((K_bM_b)^2 + (K_tM_t)^2)^{1/2}\}\right]^{1/3} \quad \text{...(14-11)}$$

$$D = \left[\frac{16}{\pi \times 117}\left\{\left(1.5 \times 1.273 \times 10^6 + \left[(1.5 \times 1.273 \times 10^6)^2 + (1.5 \times 477.5 \times 10^3)^2\right]^{\frac{1}{2}}\right)\right\}\right]^{\frac{1}{3}}$$

$D = 55.60$ mm

(ii) According to maximum shear stress theory, diameter of the shaft

$$D = \left[\frac{16}{\pi\tau}\{(K_b M_b)^2 + (K_t M_t)^2)^{1/2}\}\right]^{1/3} \quad ...(14\text{-}12)$$

$$= \left[\frac{16}{\pi \times 58.5}\{(1.5 \times 1.273 \times 10^6)^2 + (1.5 \times 477.5 \times 10^3)^2\}^{1/2}\right]^{1/3}$$

$$D = 56.20 \text{ mm}$$

recommended dia $D = 56.20 \simeq 57$ mm.

Problem 14: A machine shaft running at 600 rpm is supported on bearings 750 mm apart. 15 kW of power is supplied to the shaft through a 450 mm pulley located at 250 mm to the right of right bearing. The power is transmitted from the shaft through a 200 mm gear located 250 mm to the left of left bearing. The belt drive is at an angle 60° downward with horizontal. The pulley weighs 800 N to provide some flywheel effect. The angle of contact of the belt is 157° and coefficient of friction between belt and pulley is 0.4. The gear is in mesh with another gear located directly above the shaft. If the shaft material selected has an ultimate strength of 500 MPa and yield stress of 310 MPa, determine the necessary diameter of the shaft.

Given data: Power, $P = 15$ kW, $n = 600$ rpm.

Angle of contact, $\theta = 157°$, $\mu = 0.4$

Weight of pulley, $W_p = 800$ N

$\sigma_u = 500$ MPa, $\sigma_y = 310$ MPa.

Allowable shear stress $\tau = 0.18$, σ_u or $0.3\sigma_y$ whichever is small

$\tau = 0.18 \times 500 = 90$ MPa or

$\tau = 0.3 \times 310 = 93$ MPa

$\tau = 90$ MPa (smaller)

with key way, $\tau = 75\%$ of $\tau = 0.75 \times 90 = 67.5$

$$\text{Torque, } M_t = \frac{9.55 \times 10^6 P}{h} = \frac{9.55 \times 10^6 \times 15}{600} = 238.75 \times 10^3 \text{ N-mm}$$

Consider gear at A

$$\text{Tangential force, } F_t = \frac{M_t}{R_g} = \frac{238.75 \times 10^3}{\left(\frac{200}{2}\right)} = 2387.5 \text{ N}$$

Separating force, $F_s = F_t \tan \alpha$

Assume, α = Pressure angle = 20° (std)

$F_s = 2387.5 \tan 20° = 868.98$ N

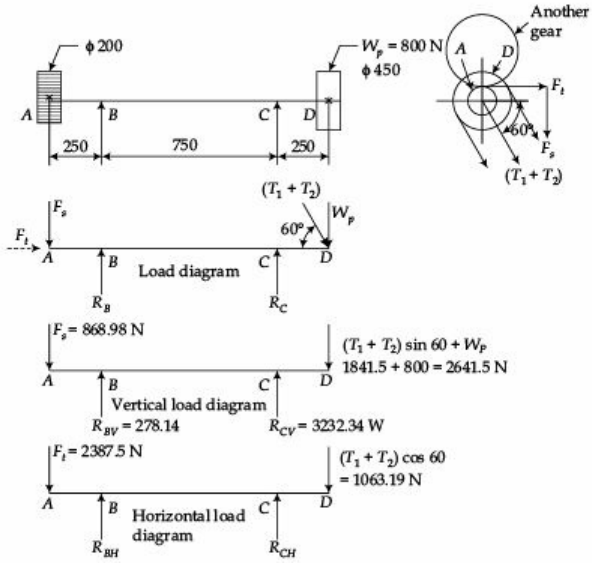


Fig. 6.6

Consider pulley at D

$$\text{Tension's ratio } \frac{T_1}{T_2} = e^{\mu\theta} = e^{0.4\left(157\frac{\pi}{180}\right)} = 2.992$$

$$\therefore \quad T_1 = 2.992\ T_2$$

Torque transmitted by the belt, $M_t = (T_1 - T_2)R_P$

$$238.75 \times 10^3 = (2.992T_2 - T_2)\frac{450}{2}$$

$$\therefore \quad T_2 = 532.69 \text{ N}$$

$$T_1 = 2.992 \times T_2 = 1593.8 \text{ N}$$

$\therefore$ Load acting on the shaft due to belt drive

$$= T_1 + T_2 = 2126.49 \text{ N}$$

$$\therefore \quad (T_1 + T_2) \sin 60 = 1841.5 \text{ N (vertical)}$$

$$(T_1 + T_2) \cos 60 = 1063.19 \text{ N (horizontal)}$$

Consider vertical load diagram

$$R_{B_V} + R_{C_V} = 868.98 + 2641.5 = 3510.48 \text{ N}$$

Taking moments about B,

$$-868.98 \times 250 - R_{C_V} \times 750 + 2641.5 \times 1000 = 0$$

$$\therefore \quad R_{C_V} = 3232.34 \text{ N}$$

$$R_{B_V} = 278.14 \text{ N}$$

Vertical bending moments

at A, $M_{b_{V_A}} = 0$

at B, $M_{b_{V_B}} = 868.98 \times 250 = 217.245 \times 10^3$ N-mm

at C, $M_{b_{V_C}} = 2641.5 \times 250 = 660.375 \times 10^3$ N-mm

at D, $M_{b_{V_D}} = 0$

Consider horizontal load diagram

$$R_{B_H} + R_{C_H} = 2387.5 + 1063.19 = 3450.69 \text{ N}$$

Taking moment about B,

$$-2387.5 \times 250 - R_{C_H} \times 750 + 1063.19 \times 1000 = 0$$

$$R_{C_H} = 621.75 \text{ N}$$

$$R_{B_H} = 3450.69 - 621.75 = 2828.94 \text{ N}$$

Horizontal bending moments

at A, $M_{b_{H_A}} = 0$

at B, $M_{b_{H_B}} = 2387.5 \times 250 = 596.88 \times 10^3$ N-mm

at C, $M_{b_{H_C}} = 1063.19 \times 250 = 265.8 \times 10^3$ N-mm

at D, $M_{b_{H_D}} = 0$

Resultant bending moments

at A, $M_{b_A} = 0$

at B,

$$M_{b_B} = \sqrt{M_{b_{V_B}}^2 + M_{b_{H_B}}^2}$$

$$= \sqrt{(217.254 \times 10^3)^2 + (596.88 \times 10^3)^2}$$

$$= 635.19 \times 10^3 \text{ N-mm}$$

at C, $M_{b_C} = \sqrt{M_{b_{VC}}^2 + M_{b_{HC}}^2}$

$= \sqrt{(660.375 \times 10^3)^2 + (265.8 \times 10^3)^2} = 711.86 \times 10^3$ N-mm

at D, $M_{b_D} = 0$

$\therefore$ Maximum bending moment, $M_b = 711.86 \times 10^3$ N-mm

Torque, $M_t = 238.75 \times 10^3$ N-mm

$\tau = 67.5$ MPa

Assuming minor shocks, $K_b = 1.75$ and $K_t = 1.25$

According to the maximum shear stress theory, diameter of the shaft

$$D = \left[\frac{16}{\pi\tau}\{(k_b M_b)^2 + (k_t M_t)^2\}^{1/2}\right]^{1/3} \quad ...(14\text{-}12)$$

$$D = \left[\frac{16}{\pi 67.5}\{(1.75 \times 711.86 \times 10^3)^2 + (1.25 \times 238.75 \times 10^3)^2\}^{1/2}\right]^{1/3}$$

Diameter of shaft $D = 45.89 \simeq 46$ mm.

Problem 15: A hollow shaft is supported by two bearings placed 1.2 m apart. A 600 mm diameter pulley is mounted at a distance of 200 mm to the right of left hand bearing and this drives a pulley directly below it with the help of a belt having maximum tension of 2 kN. Another pulley of 400 mm diameter is placed 200 mm to the left of right bearing and is driven with the help of a motor and belt which is placed at 45° to vertical and downward towards the observer. The angle of contact for both pulleys is 180° and coefficient of friction is 0.25. Determine the suitable diameter for hollow shaft having diameters ratio of 0.5. The allowable tensile and shear stresses are 63 MPa and 42 MPa respectively and assume heavy shock condition.

Given data: Angle of contact, $\theta = 180° = \pi$ radians

Coefficient of friction, $\mu = 0.25$

Diameter ratio for hollow shaft, $K = \frac{D_i}{D_o} = 0.5$

Allowable tensile stress, $\sigma = 63$ MPa

Allowable shear stress, $\tau = 42$ MPa

Assume, $K_b = 2.5$ and $K_t = 2.0$ for heavy shocks. ...$\left(\frac{\text{T14-2}}{\text{P14-13}}\right)$

Consider pulley B

Maximum tension, $T_{1B} = 2000$ N

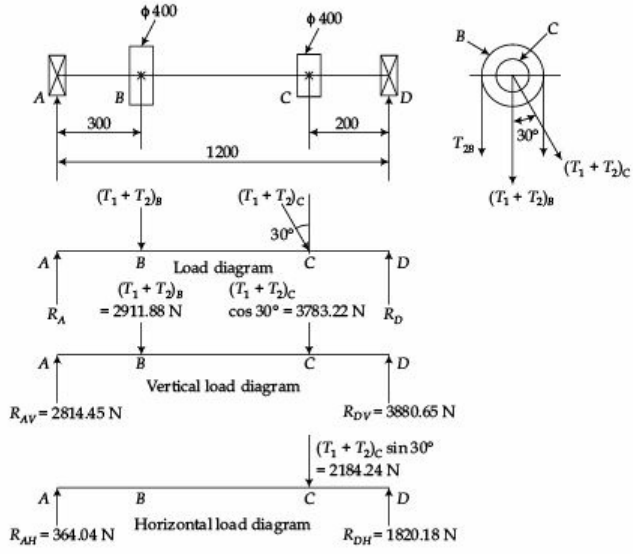


Fig. 6.7

$$\text{Tension's ratio } \left(\frac{T_1}{T_2}\right)_B = e^{\mu\theta} = e^{0.25\pi} = 2.1933$$

$$\therefore \quad T_{2B} = \frac{T_{1B}}{2.1933} = \frac{2000}{2.1933} = 911.88 \text{ N}$$

∴ For θ acting on the shaft at B due to belt pull

$$= (T_1 + T_2)_B = 2000 + 911.88 = 2911.88 \text{ N}$$

Torque capacity of the belt and pulley at B

$$M_t = (T_1 - T_2)_B R_{PB}$$

where R_{PB} = Radius of pulley at $B = \frac{600}{2}$ mm

$$\therefore \quad M_t = (2000 - 911.88)\frac{600}{2} = 326.44 \times 10^3 \text{ N-mm}$$

Consider the same torque capacity for shaft and pulley C also.

New for pulley C: Tension ratio $\left(\frac{T_1}{T_2}\right)_C = e^{\mu\theta}$

$$\left(\frac{T_1}{T_2}\right)_C = e^{0.25 \times \pi} = 2.193$$

$\therefore$ $$T_{1C} = 2.193\ T_{2C}$$

Torque capacity, $M_{tC} = (T_1 - T_2)_C R_{PC}$

$$R_{PC} = \text{Radius of pulley at } C = \frac{400}{2} \text{ mm}$$

$$326.44 \times 10^3 = (2.193T_2 - T_2)_C \times \frac{400}{2}$$

$\therefore$ $$T_{2C} = 1368.14 \text{ N}$$

$$T_{1C} = 3000.32 \text{ N}$$

$\therefore$ $$(T_1 + T_2)_C = 4368.46 \text{ N}$$

$\therefore$ Vertical component $(T_1 + T_2)_C \cos 30° = 3783.22$ N

Horizontal component $(T_1 + T_2)_C \sin 30° = 2184.24$ N

Consider vertical load diagram

$$R_{AV} + R_{DV} = 2911.88 + 3783.22 = 6695.10 \text{ N}$$

Taking moments about A,

$$2911.88 \times 300 + 3783.22 \times 1000 - R_{DV} \times 1200 = 0$$

$\therefore$ $$R_{DV} = 3880.65 \text{ N}$$

$$R_{AV} = 6695.10 - 3880.65 = 2814.45 \text{ N}$$

Vertical bending moments

at A, $M_{bVA} = 0$

at B, $M_{bVB} = 2814.45 \times 300 = 844.335 \times 10^3$ N-mm

at C, $M_{bVC} = 3880.65 \times 200 = 776.13 \times 10^3$ N-mm

at D, $M_{bVD} = 0$

Consider horizontal load diagram

$$R_{AH} + R_{DH} = 2184.22 \text{ N}$$

Taking moments about A,

$$2184.22 \times 1000 - R_{DH} \times 1200 = 0$$

$\therefore$ $$R_{DH} = 1820.18 \text{ N}$$

$$R_{AH} = 2184.22 - 1820.18 = 364.04 \text{ N}$$

Horizontal bending moments

at A, $M_{bHA} = 0$

at B, $M_{bHB} = 364.04 \times 300 = 109.211 \times 10^3$ N-mm

at C, $M_{bHC} = 1820.18 \times 200 = 364.036 \times 10^3$ N-mm

at D, $M_{bHD} = 0$

Resultant bending moments

at A, $M_{b_A} = 0$

at B, $M_{b_B} = \sqrt{(844.335 \times 10^3)^2 + (109.211 \times 10^3)^2}$

$= 851.37 \times 10^3$ N-mm

at C, $M_{b_C} = \sqrt{(776.13 \times 10^3)^2 + (364.036 \times 10^3)^2}$

$= 857.26 \times 10^3$ N-mm

at D, $M_{b_D} = 0$

$\therefore$ Maximum bending moment on the shaft $M_b = 857.26 \times 10^3$ N-mm

Torque $M_t = 326.44 \times 10^3$ N-mm

(i) According to maximum normal stress theory, outside diameter of the shaft

$$D_0 = \left[\frac{16}{\pi\sigma(1-K^4)}\{k_b M_b + (k_b M_b)^2 + (k_t M_t)^2\}^{1/2}\right]^{1/3} \quad \text{...(14-11)}$$

$$D_0 = \left[\frac{16}{\pi \times 63(1-0.5^4)}\{2.5 \times 857.26 \times 10^3 + ((2.5 \times 857.26 \times 10^3)^2 + (2 \times 326.44 \times 10^3)^2)^{1/2}\}\right]^{1/3}$$

$D_0 = 72.303$ mm

(ii) According to the maximum shear stress theory, outside diameter of the shaft

$$D_0 = \left[\frac{16}{\pi\tau(1-K^4)}((k_b M_b)^2 + (k_t M_t)^2)^{1/2}\right]^{1/3} \quad \text{...(14-12)}$$

$$= \left[\frac{16}{\pi \times 42(1-0.5^4)}((2.5 \times 857.26 \times 10^3)^2 + (2 \times 326.44 \times 10^3)^2)^{1/2}\right]^{1/3}$$

$D_0 = 66.175$ mm

Recommended outside diameter, $D_0 = 72.303$ mm $\simeq 75$ mm

$\therefore$ $D_1 = 0.5$, $D_0 = 0.5 \times 75 = 37.5$ mm ≈ 37 mm.

Problem 16: A steel shaft 0.9 m long between bearings receives 18 kW at 900 rpm through a 20° involute gear of 2 mm module and 100 teeth, located at 250 mm to the left of left bearing and is driven by a gear placed directly behind it. The power is transmitted by a 400 mm diameter pulley to another pulley placed towards front and above it at an angle of 45° to horizontal. The pulley is located at a distance of 300 mm to the left of right bearing. The tension ratio is 2.7. Design a hollow shaft taking allowable shear stress as 72 MPa. Take diameters ratio as 2.

Given data: Power, $P = 18$ kW

Speed, $n = 900$ rpm

Pressure angle, $\alpha = 20°$

Module, $m = 2$ mm, No. of teeth $z = 100$

$\therefore$ Pitch diameter of the gear, $d_g = mz = 200$ mm

Tensions ratio for the belt, $\frac{T_1}{T_2} = 2.7$

Allowable shear stress, $\tau = 72$ MPa

Diameters ratio $= 2 = \frac{D_o}{D_i}$

$\therefore$ $K = \frac{D_i}{D_o} = \frac{1}{2} = 0.5$

Assume, $K_b = 1.75$ and $K_t = 1.25$.

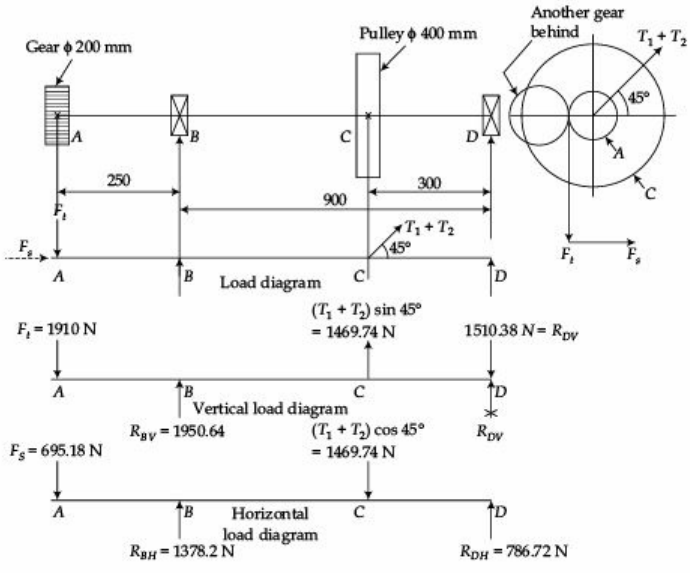


Fig. 6.8

$$\text{Torque, } M_t = \frac{9.55 \times 10^6 P}{n} = \frac{9.55 \times 10^6 \times 18}{900} = 191 \times 10^3 \text{ N-mm}$$

Consider gear at A

$$\text{Tangential force, } F_t = \frac{M_t}{R_g}$$

$$F_t = \frac{191 \times 10^3}{(200/2)} = 1910 \text{ N} \quad \text{(downward)}$$

$\therefore$ Separating force, $F_s = F_t \tan \alpha = 1910 \times \tan 20° = 695.18$ N (towards front)

Consider pulley at C

$$\text{Tension's ratio, } \frac{T_1}{T_2} = 2.7$$

$$\therefore \quad T_1 = 2.7T_2$$

Torque transmitted by belt, $M_t = (T_1 - T_2)R_P$

$$191 \times 10^3 = (2.7T_2 - T_2)\frac{400}{2}$$

$$\therefore \quad T_2 = 561.76 \text{ N}$$

$$T_1 = 2.7 \quad T_2 = 1516.76 \text{ N}$$

$\therefore$ Force acting on shaft due to belt drive

$$T_1 + T_2 = 2078.52 \text{ N}$$

$$\text{Vertical component} = (T_1 + T_2) \sin 45 = 1469.74 \text{ N} \quad \text{(upwards)}$$

$$\text{Horizontal component} = (T_1 + T_2) \cos 45 = 1469.74 \text{ N} \quad \text{(towards front)}$$

Consider vertical load diagram

$$R_{BV} + R_{DV} = 1910 - 1469.74 = 440.26 \text{ N}$$

Taking moments about B,

$$-1910 \times 250 - R_{DV} \times 900 - 1469.74 \times 600 = 0$$

$$R_{DV} = -1510.38 \text{ N}$$

Since –ve, the assumed direction is wrong. Hence change the direction and sign of R_{DV}.

$$\therefore \quad R_{DV} = 1510.38\text{N (downward)}$$

$$R_{BV} = 440.26 + 1510.38 = 1950.64 \text{ N}$$

Vertical bending moments

at A, $\quad M_{b_{VA}} = 0$

at B, $\quad M_{b_{VB}} = 1910 \times 250 = 477.5 \times 10^3$ N-mm

at C, $M_{b_{VC}} = 1510.38 \times 300 = 453.11 \times 10^3$ N-mm

at D, $M_{b_{VD}} = 0$

Consider horizontal load diagram

$$R_{BH} + R_{DH} = 695.18 + 1469.74 = 2164.92 \text{ N}$$

Taking moments about B,

$$-695.18 \times 250 + 1469.74 \times 600 - R_{DH} \times 900 = 0$$

$$R_{DH} = 786.72 \text{ N}$$

$$R_{BH} = 1378.2 \text{ N}$$

Horizontal bending moments

at A, $M_{b_{HA}} = 0$

at B, $M_{b_{HB}} = 695.18 \times 250 = 173.795 \times 10^3$ N-mm

at C, $M_{b_{HC}} = 786.72 \times 300 = 236.016 \times 10^3$ N-mm

at D, $M_{b_{HD}} = 0$

Resultant bending moments

at A, $M_{b_A} = 0$

at B, $M_{b_B} = \sqrt{(477.5 \times 10^3)^2 + (173.795 \times 10^3)^2}$

$= 508.15 \times 10^3$ N-mm

at C, $M_{b_C} = \sqrt{(453.11 \times 10^3)^2 + (236.016 \times 10^3)^2}$

$= 510.89 \times 10^3$ N-mm

at D, $M_{b_D} = 0$

$\therefore$ Maximum bending moment, $M_b = 510.89 \times 10^3$ N-mm

According to the maximum shear stress theory, outside diameter of hollow the shaft

$$D_0 = \left[\frac{16}{\pi\tau(1-k^4)}\{(k_b M_b)^2 + (k_t M_t)^2\}^{1/2}\right]^{1/3} \quad \text{...(14-37)}$$

$$D_0 = \left[\frac{16}{\pi \times 72(1-0.5^4)}\{(1.75 \times 510.89 \times 10^3)^2 + (1.25 \times 191 \times 10^3)^2\}^{1/2}\right]^{1/3}$$

$D_0 = 41.18 \simeq 42$ mm

$\therefore$ Inside diameter, $D_i = 0.5\, D_o = 21$ mm

Problem 17: A power transmission shaft 1500 mm long is supported at two points A and B. A is at extreme left end of the shaft and B is at a distance of 400 mm from the right extreme end of the shaft. The shaft receives a power of 50 kW at a rated speed of 500 rpm through a belt drive located at the right extreme end of the shaft. The pulley has a diameter of 400 mm and weighs 1 kN. The belt moving on the pulley is directed away from the observer below the horizontal and inclined at 45° to it. The ratio of belt tensions is 3. This power is given out through a gear drive located at a distance of 500 mm from the right supports. The gear mounted on the shaft has a pitch diameter of 200 mm and weighs 400 N. The other gear, which mashes with this gear is placed exactly in front. The gear teeth have a pressure angle of 20°. Taking yield stress as 353 MPa and factor of safety as 2.5, determine the suitable diameter of the shaft.

Given data:

Power, $P = 50$ kW

Speed, $n = 500$ rpm

Tensions ratio, $\frac{T_1}{T_2} = 3$

Pressure angle, $\alpha = 20°$

Yield stress, $\sigma_y = 353$ MPa

Factor of stress = 2.5

$\therefore$ Allowable tensile stress, $\sigma = \frac{\sigma_y}{FOS} = \frac{353}{2.5} = 141.2$ MPa

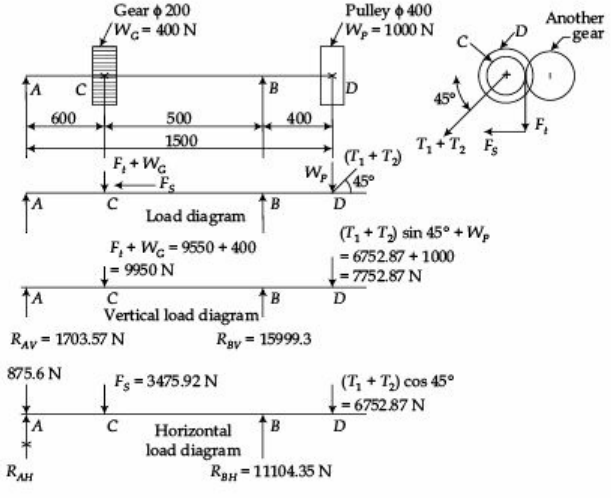


Fig. 6.9

$$\sigma = 141.2 \text{ MPa}$$

$$\text{Assume } \tau = 0.5\sigma = 70.6 \text{ MPa}$$

Considering minor shocks, select $K_b = 1.75$

$$K_t = 1.25$$

$$\text{Torque, } M_t = \frac{9.55 \times 10^6 P}{n} = \frac{9.55 \times 10^6 \times 50}{500} = 955 \times 10^3 \text{ N-mm}$$

Consider gear at C

$$\text{Tangential force, } F_t = \frac{M_t}{R_g} = \frac{955 \times 10^3}{(200/2)} = 9550 \text{ N}$$

$$\text{Separating force, } F_s = F_t \tan \alpha = 9550 \tan 20$$

$$F_s = 3475.92 \text{ N}$$

Consider pulley at D

$$\text{Tension's ratio} = \frac{T_1}{T_2} = 3$$

$$\therefore \quad T_1 = 3T_2$$

$$\text{Torque capacity of belt, } M_t = (T_1 - T_2)R_p$$

$$955 \times 10^3 = (3T_2 - T_2) \times \frac{400}{2}$$

$$\therefore \quad T_2 = 2387.5 \text{ N}$$

$$T_1 = 7162.5 \text{ N}$$

Force acting on the shaft due to belt pull $(T_1 + T_2) = 9550$ N

$\therefore$ Vertical component $(T_1 + T_2) \sin 45° = 6752.87$ N

Horizontal component $(T_1 + T_2) \cos 45° = 6752.87$ N

Consider vertical load diagram

$$R_{AV} + R_{BV} = 9950 + 7752.87 = 17702.87 \text{ N}$$

Taking moments about A,

$$9950 \times 600 - R_{BV} \times 1100 + 7752.82 \times 1500 = 0$$

$$\therefore \quad R_{BV} = 15999.3 \text{ N}$$

$$R_{AV} = 1703.57 \text{ N}$$

Vertical bending moments

at A, $M_{b_{VA}} = 0$

at C, $M_{b_{VC}} = 1703.57 \times 600 = 1.022 \times 10^6$ N-mm

at B, $M_{b_{VB}} = 7752.82 \times 400 = 3.10 \times 10^6$ N-mm

at D, $M_{b_{VD}} = 0$

Consider horizontal load diagram

$$R_{AH} + R_{BH} = 3475.92 + 6752.82 = 10228.74 \text{ N}$$

Taking moments about A,

$$3475.92 \times 600 - R_{BH} \times 1100 + 6752.82 \times 1500 = 0$$

$\therefore$ $$R_{BH} = 11104.35 \text{ N}$$

$$R_{AH} = -875.6 \text{ N (since } -ve \text{ change the direction and sign)}$$

Horizontal bending moments

at A, $M_{b_{HA}} = 0$

at C, $M_{b_{HC}} = 875.6 \times 600 = 525.36 \times 10^3$ N-mm

at B, $M_{b_{HC}} = 6752.82 \times 400 = 2.70 \times 10^6$ N-mm

at D, $M_{b_{HD}} = 0$

Resultant bending moments,

at A, $M_{b_A} = 0$

at C, $M_{b_C} = \sqrt{(1.022 \times 10^6)^2 + (525.36 \times 10^3)^2} = 1.149 \times 10^6$ N-mm

at B, $M_{b_B} = \sqrt{(3.10 \times 10^6)^2 + (2.70 \times 10^6)^2} = 4.11 \times 10^6$ N-mm

at D, $M_{b_D} = 0$

Maximum bending moment $M_b = 4.11 \times 10^6$ N-mm

(i) According to maximum normal stress theory, diameter of the shaft

$$D = \left[\frac{16}{\pi\sigma}\{k_b M_b + ((k_b M_b)^2 + (k_t M_t)^2)^{1/2}\}\right]^{1/3} \quad \text{...(14-11)}$$

$$= \left[\frac{16}{\pi \times 141.2}\{1.75 \times 4.11 \times 10^6 + ((1.75 \times 4.11 \times 10^6)^2 + (1.25 \times 955 \times 10^3)^2)^{1/2}\}\right]^{1/3}$$

$D = 80.54$ mm

(ii) According to the maximum shear stress theory, diameter of the shaft

$$D = \left[\frac{16}{\pi\tau}\{(k_b M_b)^2 + (k_t M_t)^2\}^{1/2}\right]^{1/3} \quad \text{...(14-12)}$$

$$= \left[\frac{16}{\pi \times 70.6}\{(1.75 \times 4.11 \times 10^6)^2 + (1.25 \times 955 \times 10^3)^2\}^{1/2}\right]^{1/3}$$

$D = 80.72$ mm

$\therefore$ Recommended diameter $D = 80.72 \simeq 81$ mm (bigger value).

Problem 18: A hoisting drum 0.5 m in diameter is keyed to a shaft which is supported on 2 bearings and driver through a 12:1 reduction ratio by an electric motor. Determine the power of the motor if a maximum load of 8 kN is hoisted at a speed of 50 m/min and the efficiency of drive is 80%. Determine the torque on the drum and speed of the motor in rpm. Also design a suitable shaft having allowable stress as 115 MPa in tension and 50 MPa in shear. The drive gear is 450 mm in diameter and mounted at 150 mm over-hanging the nearest bearing. The combined shack and endurance factors in bending and torsion may be taken as 2 and 1.5 respectively.

Given data:

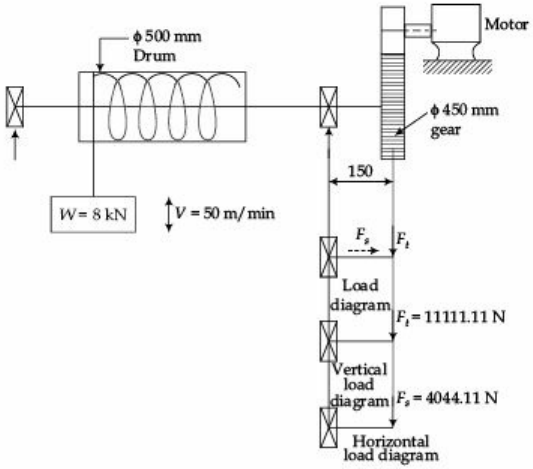


Fig. 6.10

Weight, $W = 8000$ N

$$\text{Velocity, } v = 50 \text{ m/min} = \frac{50}{60} \text{ m/s}$$

$$\therefore \quad \text{Power at drum} = \frac{WV}{1000} \text{ kW}$$

$$P_{\text{Drum}} = \frac{8000 \times 50}{60 \times 1000} = 6.67 \text{ kW}$$

With 80% efficiency of drive

$$\text{Power of motor, } P_{\text{Motor}} = P = \frac{P_{\text{Drum}}}{\eta} = \frac{6.67}{0.8}$$

$$P = 8.33 \text{ kW}$$

To find speed of the drum and motor

$$V_{\text{Drum}} = \frac{\pi D n}{60000}$$

$$\frac{50}{60} = \frac{\pi \times 500\, n_{\text{Drum}}}{60000} \Rightarrow \text{speed of Drum } n_{\text{Drum}} = 31.83 \text{ rpm}$$

$\therefore$ Speed of motor = $n_{\text{Drum}} \times$ Gear ratio

$$n_{\text{motor}} = 31.83 \times 12 = 381.97 \text{ rpm}$$

$$\sigma = 115 \text{ MPa}$$

$$\tau = 50 \text{ MPa}$$

$$K_b = 2$$

$$K_t = 1.5$$

(Since the distance between bearinsgs is not given, neglect the bending effect due to weight W)

$$\text{Torque on drum shaft} = M_t = \frac{9.55 \times 10^6 \times P_{\text{Drum}}}{n_{\text{Drum}}}$$

$$M_t = \frac{9.55 \times 10^6 \times 6.67}{31.83} = 2.0 \times 10^6 \text{ N-mm}$$

To find F_t and F_s consider the power developed at the motor that is being transmitted by the gears.

$$M_{t\,\text{gear}} = \frac{9.55 \times 10^6 \times 8.33}{31.83}$$

$$M_{t_g} = 2.500 \times 10^6 \text{ N-mm}$$

$\therefore$ $$F_t = \frac{M_{t_g}}{R_g} = \frac{2.5 \times 10^6}{(450/2)} = 11111.11 \text{ N}$$

Assuming pressure angle $\alpha = 20^\circ$

$$F_s = F_t \tan \alpha = 11111.11 \times \tan 20 = 4044.11 \text{ N}$$

The shaft is like a cantilever under bending.

Consider vertical load diagram

$$M_{b_V} = F_t \cdot 150 = 11{,}111.11 \times 150 = 1.67 \times 10^6 \text{ N-mm}$$

Horizontal load diagram

$$M_{b_H} = F_s \cdot 150 = 4044.11 \times 150 = 606.66 \times 10^3 \text{ N-mm}$$

Resultant bending moment

$$M_b = \sqrt{M_{b_{V^2}} + M_{b_{H^2}}} = 1.774 \times 10^6 \text{ N-mm}$$

(i) According to the maximum normal stress theory, diameter of the shaft

$$D = \left[\frac{16}{\pi \times 115}\{2 \times 1.774 \times 10^6 + [(2 \times 1.774 \times 10^6)^2 + (1.5 \times 2 \times 10^6)^2]^{1/2}\}\right]^{1/3} \quad ...(14\text{-}11)$$

$= 71.32$ mm

(ii) According to the maximum shear stress theory, diameter of the shaft

$$D = \left[\frac{16}{\pi \times 50}\{(2 \times 1.774 \times 10^6)^2 + (1.5 \times 2 \times 10^6)^2\}^{1/2}\right]^{1/3} \quad ...(14\text{-}12)$$

$D = 77.93$ mm

Recommending the bigger value, diameter of the shaft

$D = 77.93 = 78$ mm.

Problem 19: A power transmission shaft 1800 mm long is supported at two points A and B. A is at 300 mm from left extreme end and B is at right extreme end. A power of 30 kW is received at 600 rpm through a gear drive located at the left extreme end of shaft. This gear has a pitch diameter of 300 mm and weighs 700 N. The drive gear is placed exactly behind this. 18 kW of this power is given out through the drive located at a distance of 600 mm from the left support. The pulley mounted on the shaft has a diameter of 400 mm and weighs 100 N. The belt is directed towards the observer below horizontal and inclined at 45° to it. The ratio of the belt tension is 3. The remaining power is given out through a gear drive at a distance of 400 mm from the right support. The gear has a pitch diameter of 200 mm and weighs 500 N. Find suitable diameter of the shaft taking allowable stresses as as 120 MPa in tension and 75 MPa in shear.

Given data:

Power at $C = P_C = 30$ kW

Power at $D = P_D = 18$ kW

Remaining power at $E = P_E = 30 - 18 = 12$ kW

Speed of the shaft, $n = 600$ rpm

$$\text{Tension's ratio} = \frac{T_1}{T_2} = 3 \Rightarrow T_1 = 3T_2$$

Assume pressure angle for both gears = 20°

Assume the direction of tangential force at E as horizontal (not given).

Selecting minor shocks, $K_b = 1.75$

$K_t = 1.25$

Consider gear at C:

Power, $P_C = 30$ kW

$$\text{Torque, } M_{t_C} = \frac{9.55 \times 10^6 P_C}{n} = \frac{9.55 \times 10^6 \times 30}{600}$$

$$M_{t_C} = 477.5 \times 10^6 \text{ N-mm}$$

$$\therefore \quad F_{t_C} = \frac{M_{t_C}}{R_{g_C}} = \frac{477.5 \times 10^6}{(300/2)} = 3183.33 \text{ N}$$

$$F_{a_C} = F_{t_C} \times \tan \alpha = 1158.64 \text{ N}$$

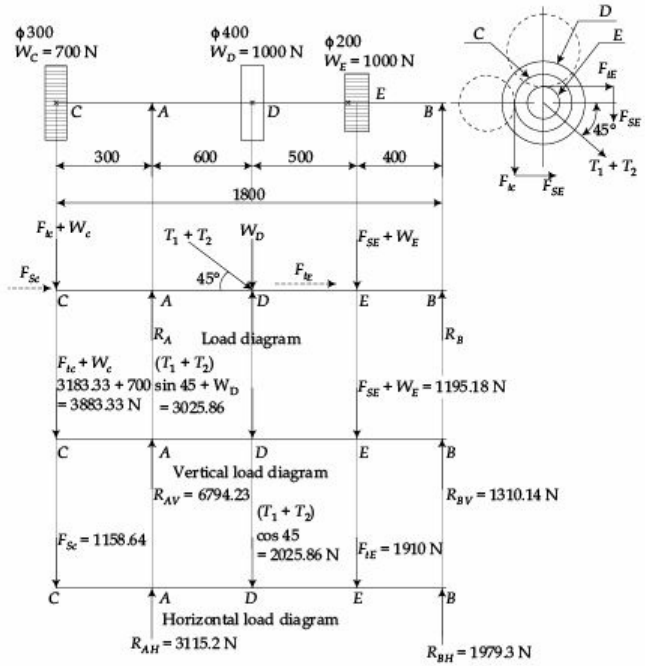


Fig. 6.11

Vertical load at C = $F_{tc} + W_c$ = 3183.33 + 700 = 3883.33 N

Consider pulley at D Power, P_D = 18 kW

$$\therefore \quad \text{Torque, } M_{t_D} = \frac{9.55 \times 10^6 \times 18}{600} = 286.5 \times 10^3 \text{ N-mm}$$

But $$M_{t_D} = (T_1 - T_2)R_p$$

$$286.5 \times 10^3 = (3T_1 - T_2) \times \frac{400}{2}$$

$$\therefore \quad T_2 = 716.25 \text{ N}$$

$$T_1 = 3T_2 = 2148.75 \text{ N}$$

$\therefore$ Load acting on the shaft due to belt pull

$$= T_1 + T_2 = 2865 \text{ N}$$

$\therefore \quad (T_1 + T_2) \sin 45 = (T_1 + T_2) \cos 45 = 2025.86 \text{ N}$

$(T_1 + T_2) \sin 45 + W_D = 2025.86 + 1000 = 3025.86 \text{ N}$

Consider gear at E Power, $P_E = 12$ kW

$$\therefore \quad M_{t_E} = \frac{9.55 \times 10^6 \times 12}{600} = 191 \times 10^3 \text{ N-mm}$$

$$\therefore \quad F_{t_E} = \frac{M_{t_E}}{R_{g_E}} = \frac{191 \times 10^3}{(200/2)} = 1910 \text{ N}$$

$$F_{S_E} = F_{t_E} \times \tan \alpha = 695.18 \text{ N}$$

$$\therefore \quad F_{SE} + W_E = 695.18 + 500 = 1195.18 \text{ N}$$

Consider vertical load diagram

$$R_{A_V} + R_{B_V} = 3883.33 + 3025.86 + 1195.18 = 8104.37 \text{ N}$$

Taking moments about *A*,

$$-3883.33 \times 300 + 3025.86 \times 600 + 1195.18 \times 1100 - R_{B_V} \times 1500 = 0$$

$$R_{B_V} = 1310.14 \text{ N}$$

$$R_{A_V} = 6794.23 \text{ N}$$

Vertical bending moments

at *C*, $M_{b_{V_C}} = 0$

at *A*, $M_{b_{V_A}} = 3883.33 \times 300 = 1.165 \times 10^6$ N-mm

at *D*, $M_{b_{V_D}} = 3883.33 \times 900 - 6794.23 \times 600 = -0.582 \times 10^6$ N-mm

at *E*, $M_{b_{V_E}} = 1310.14 \times 400 = 0.524 \times 10^6$ N-mm

at *B*, $M_{b_{V_B}} = 0$

Consider horizontal load diagram

$$R_{AH} + R_{BH} = 1158.64 + 2025.86 + 1910 = 5094.5 \text{ N}$$

Taking moment about *A*,

$$-1158.64 \times 300 + 2025.86 \times 600 + 1910 \times 1100 - R_{BH} \times 1500 = 0$$

$$\therefore \quad R_{BH} = 1979.3 \text{ N}$$

$$R_{AH} = 3115.2 \text{ N}$$

Horizontal bending moments

at C, $M_{b_{H_C}} = 0$

at A, $M_{b_{H_A}} = 1158.64 \times 300 = 0.348 \times 10^6$ N-mm

at D, $M_{b_{H_D}} = 1158.64 \times 900 - 3115.2 \times 600 = -0.826 \times 10^6$ N-mm

at E, $M_{b_{H_E}} = 1979.3 \times 400 = 0.792 \times 10^6$ N-mm

at B, $M_{b_{H_B}} = 0$

Resultant bending moments

at C, $M_{b_C} = 0$

at A, $M_{b_A} = \sqrt{(1.165 \times 10^6)^2 + (0.348 \times 10^6)^2} = 1.216 \times 10^6$ N-mm

at D, $M_{b_D} = \sqrt{(-0.582 \times 10^6)^2 + (-0.826 \times 10^6)^2} = 0.01 \times 10^6$ N-mm

at E, $M_{b_E} = \sqrt{(0.524 \times 10^6)^2 + (0.792 \times 10^6)^2} = 0.95 \times 10^6$ N-mm

at B, $M_{b_B} = 0$.

$\therefore$ Maximum bending moment, $M_b = 1.216 \times 10^6$ N-mm

Maximum torque $= M_{t_C} = M_t = 477.5 \times 10^3$ N-mm

(i) According to the maximum normal stress theory. Diameter of the shaft

$$D = \left[\frac{16}{\pi \times 120}\{1.75 \times 1.216 \times 10^6 + ((1.75 \times 1.216 \times 10^6)^2 + (1.25 \times 477.5 \times 10^3)^2)^{1/2}\}\right]^{1/3}$$

$D = 51.46$ mm

(ii) According to the maximum shear stress theory, Diameter of the shaft

$$D = \left[\frac{16}{\pi \times 75}((1.75 \times 1.216 \times 10^6)^2 + (1.25 \times 477.5 \times 10^3)^2)^{1/2}\}\right]^{1/3}$$

$D = 53.14$

Recommending the bigger diameter

$$D = 53.14 = 54 \text{ mm}.$$

Problem 20: A hollow shaft of 400 mm outside dia and 250 mm inside diameter is supported on two bearings 3 m apart. The shaft rotates at 180 rpm and receives a thrust load of 300 kN and transmits 120 kW. The weight of the shaft is 90 kW. Determine the maximum normal and shear

stresses induced.

Given data:

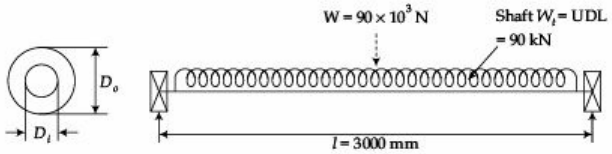


Fig. 6.12

Power $P = 120$ kW, $n = 180$ rpm

$D_i = 250$ mm, $D_o = 400$ mm, $F = 300 \times 10^3$ N

$$K = \frac{D_i}{D_o} = \frac{250}{400} = 0.625. \quad \text{Assume } K_b = 1.75, \quad K_t = 1.25$$

$$I = \frac{\pi}{64}(D_o^4 - D_i^4) = \frac{\pi}{64}(400^4 - 250^4) = 1.065 \times 10^9 \text{ mm}^4$$

$$A = \frac{\pi}{4}(D_o^2 - D_i^2) = \frac{\pi}{4}(400^2 - 250^2) = 76.576 \times 10^3 \text{ mm}^2$$

$\therefore$ Radius of gyration, $k = \sqrt{\frac{I}{A}} = 117.93$ m

$\therefore$ $$\frac{l}{k} = \frac{3000}{117.93} = 25.44 < 115 \quad \ldots\left(\frac{14-64}{P14.7}\right)$$

$\therefore$ $$\alpha = \frac{1}{1 - 0.0044(l/k)} = \frac{1}{1 - 0.0044(25.44)} = 1.126$$

(i) According to the maximum normal stress theory, outside diameter of the shaft

$$D_o = \left[\frac{16}{\pi\sigma(1-K^4)}\left\{K_b M_b + \frac{\alpha F D_o(1+K^2)}{8} + \left(\left(\frac{K_b M_b + \alpha F D_o(1+K^2)}{8}\right)^2 + (K_t M_t)^2\right)^{1/2}\right\}\right]^{1/3} \quad \ldots(14\text{-}39)$$

where M_b = bending moment = $\frac{Wl}{8}$ for *UDL* and simply supported beam (weight of shaft is *UDL*)

$$M_b = \frac{90000 \times 3000}{8} = 33.75 \times 10^6 \text{ N-mm}$$

$$\text{Torque, } M_t = \frac{9.55 \times 10^6 p}{n} = \frac{9.55 \times 10^6 \times 120}{180}$$

$$= 6.367 \times 10^6 \text{ N-mm}$$

$$F = 300 \times 10^3 \text{ N}$$

$$D_o = 400, \quad K = 0.625, \quad K_b = 1.75, \quad K_t = 1.25$$

$$\alpha = 1.126$$

$$400 = \left[\frac{16}{\pi\sigma(1 - 0.625^4)} \left\{ 1.75 \times 33.75 \times 10^6 + \frac{1.126 \times 300 \times 10^3 \times 400(1 + 0.625^2)}{8} + \right. \right.$$

$$\left. \left. \left(\left(1.75 \times 33.75 \times 10^6 + \frac{1.126 \times 300 \times 10^3 + 400\,(1 + 0.625^2)^2}{8} \right)^2 + (1.25 \times 6.367 \times 10^6)^2 \right)^{1/2} \right\} \right]^{1/3}$$

Normal stress $\sigma = 15.54$ MPa

(ii) According to the maximum shear stress theory, outside diameter of the shaft

$$D_o = \left[\frac{16}{\pi\tau(1 - K^4)} \left\{ \left(K_b M_b + \frac{\alpha F D_o (1 + K^2)}{8} \right)^2 + (K_t M_t)^2 \right\}^{1/2} \right]^{1/3} \quad \text{...(14-40)}$$

$$400 = \left[\frac{16}{\pi\tau(1 - 0.625^4)} \left\{ \left(1.75 \times 33.75 \times 10^6 + \frac{1.126 \times 300 \times 10^3 + 400(1 + 0.625^2)}{8} \right)^2 \right. \right.$$

$$\left. \left. + \left(1.25 \times 6.367 \times 10^6 \right)^2 \right\}^{1/2} \right]^{1/3}$$

Shear stress $\tau = 7.787$ MPa.

EXERCISE

1. Power is transmitted by a shaft 0.9 m long and is supported at the ends. A pulley of diameter 420 mm is placed at 150 mm to the left of right bearing and another pulley of diameter 270 mm is mounted midway between bearings. The belt drives are at right angles to each other with tensions ratio as 3 : 1. Determine the dia of the shaft required to transmit 24 kW at 300 rpm. The ultimate and yield stresses for the shaft material are 630 MPa and 360 MPa respectively.
2. A MS hollow shaft is supported on two bearings 1.2 m apart. A 275 mm diameter gear of 20° pressure angle is mounted at 300 mm to the right of left bearing and is driven by another gear placed directly over it. A pulley 450 mm diameter is keyed to the shaft at 400 mm to the

left of the right bearing. The belt drive is downward at an angle of 30° to vertical. Tension's ratio is 2.8 : 1. The diameters ratio is 0.6. The shaft is required to transmit 30 kW at 450 rpm. Take $K_b = 1.5$ and $K_t = 1.0$ and determine suitable diameters of the shaft. The allowable tensile and shear stresses are 120 MPa and 80 MPa respectively.

3. A solid steel shaft 1200 mm long is required to transmit 18 kW at 600 rpm. A gear 180 mm diameter is keyed to the shaft at 300 mm to the left of the right bearing and driven by another gear from behind. Another gear of 300 mm dia weighing 500 N is mounted on the shaft at 200 mm to the right of the left bearing drives a gear placed directly above it. Pressure angle for both the gears is 20°. The allowable shear stress for shaft material is 72 MPa. Determine suitable diameter of the shaft under medium shock condition.
4. A shaft is supported on bearings *A* and *B*, 800 mm apart. *A* 20° spur gear having 100 teeth of 6 mm module is located at 200 mm to the right of left hand bearing and a 700 mm diameter pulley is mounted 250 mm to the left of right bearing. The gear is driven by a pinion with downward tangential force while the belt drive is horizontal having 180° angle of contact. The belt used is 200 mm wide, 10 mm thick having allowable stress of 2.1 MPa. The coefficient of friction is 0.3. Determine the dia of the shaft taking $\sigma_u = 550$ MPa and $\sigma_y = 400$ MPa.

   (Hint: $T_1 = \tau_{belt} \times$ width $\times$ thickness $= 2.1 \times 200 \times 10 = 4200$ N)
5. A horizontal piece of commercial shaft is supported on two bearings 1.5 m apart. A keyed gear 20° involute 175 mm diameter is located 400 mm to the left of right bearing and is driven by a gear with downward tangential force. A 600 mm pulley is mounted on the shaft at 600 mm to the right of left bearing and drives a pulley with horizontal belt. The tension ratio is 3 : 1. The drive transmits 15 kW at 500 rpm. Find suitable diameters of the hollow shaft if the diameters ratio is 2.
6. A solid steel shaft of length 1.8 m between bearing rotates at 250 rpm in clockwise seen flom right. A $14\frac{1}{2}$ deg involute gear *D*, 300 mm diameter is located at 150 mm to the left of right bearing. Two pulleys *B* and *C* are located at distances 450 mm and 1200 mm respectively to the right of left bearing. Pulley *B* is of 600 mm and pulley *C* is of 750 mm diameter. The ratio of belt tensions = 2 for both. A power of 29.5 kW is supplied to the gear *D* by another gear directly below it. At pulley *B*, power of 18.5 kW is taken off by means of belt drive inclined at 60° with horizontal below the shaft. The remaining power 11 kW is taken off at pulley *C* by a belt arranged vertically below it. The allowable stresses are 80 MPa in tension and 40 MPa in shear. The combined shock and fatigue factors in bending and torsion are 2.0 and 1.5 respectively. Find the diameter of the shaft.

CHAPTER

7 Keys and Couplings

7.1 KEYS

Keys are used to prevent relative motion between a shaft and the connected member through which torque is being transmitted.

Common Types of Keys

The common types of keys are: square key, rectangular key, feather key, round key, gib head key, taper key, kennedy key, barth key, saddle key, and wood ruff key as shown in Fig. 7.1.

Design of square and flat keys

The keys are subjected to shear stress and crushing or bearing stress while transmitting torque.

b = width of key, h = thickness, l = length of key.

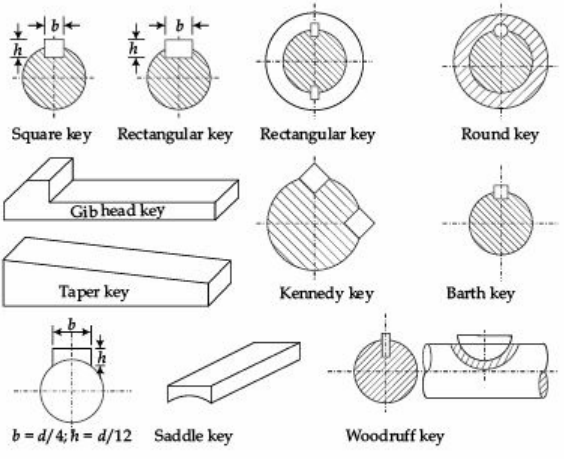
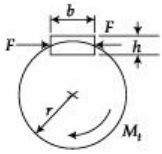


Fig. 7.1 Types of keys.

Considering the shearing of key



$$\text{Shear stress, } \tau = \frac{F}{\text{Area of shear}} = \frac{F}{bl}$$

where $$F = \text{Force} = \frac{\text{Torque}}{\text{Radius of shaft}} = \frac{M_t}{r} = \frac{M_t}{(d/2)} = \frac{2M_t}{d}$$

$$\therefore \quad \tau = \frac{2M_t}{dlb} \Rightarrow \text{width of key } b = \frac{2M_t}{dl\tau} \qquad \text{...(19-50/P 19.6)}$$

Considering crushing of key

$$\text{Crushing stress, } \sigma_c = \frac{F}{\text{Area of crushing}} = \frac{F}{(h/2)l}$$

$$\sigma_c = \frac{2M_t}{d(h/2)l} = \frac{4M_t}{dhl} \Rightarrow \text{thickness of key } h = \frac{4M_t}{dl\sigma_c} \qquad \text{...(19-51)}$$

(Crushing stress σ_c is also called bearing stress σ'_b)

For square key $b = h$

$$b = \frac{2M_t}{dl\tau} \quad \text{and} \quad h = b = \frac{4M_t}{dl\sigma_C}$$

$$M_t = \frac{bdl\tau}{2} \text{ in shear} \quad \text{and} \quad M_t = \frac{bdl\sigma_C}{4} \text{ in crushing}$$

For steels, $$\tau = \frac{\sigma_C}{2}$$

$$\therefore \quad M_t = \frac{bdl\sigma_C}{4} \text{ in shear, and } M_t = \frac{bdl\sigma_C}{4} \text{ in crushing}$$

Hence, square key is equally strong in shear and crushing.

Procedure – Type 1: Length of Key/Hub not Available

Step 1: Find torque, $M_t = \frac{9.55 \times 10^6 \times P}{n}$ N-mm

P = power in kW, n = speed, rpm.

Step 2: Diameter of shaft, $d = \sqrt[3]{\frac{16M_t}{\pi\tau_S}}$

τ_s = allowable shear stress for shaft material.

Step 3: Based on the shaft diameter, from Table 7.1, select standard key cross section, b and h.

Step 4: Find length of key from ...(T 17-1/P 17.10)

(a) Width of key, $b = \frac{2M_t}{dl\tau_K}$

Find l = ?

τ_k = allowable shear stress for key

(b) Thickness of key, $h = \frac{4M_t}{dl\sigma_b'}$

Find l = ?

Recommend the bigger one as length.

σ_b' = allowable crushing stress or bearing stress for key.

Procedure – Type 2: Length of Key / Hub Given

Step 1: Torque, $M_t = \frac{9.55 \times 10^6 \times P}{n}$

Step 2: Diameter of shaft, $d = \sqrt[3]{\frac{16M_t}{\pi\tau_S}}$

Step 3: Design of key

(a) Length of key = length of hub = l

(b) Width of key, $b = \frac{2M_t}{dl\tau_K} \Rightarrow$ find b

(c) Thickness of key, $h = \frac{4M_t}{dl\sigma_b'} \Rightarrow$ find h

Ref. Table 7.1 and recommend standard key cross section based on b and h.

Table 7.1 Dimensions (mm) of parallel keys and preferred lengths (mm).

Shaft dia	*Above*	*6*	*8*	*10*	*12*	*17*	*22*	*30*	*38*	*44*	*50*	*58*	*65*	*75*
	Upto	*8*	*10*	*12*	*17*	*22*	*30*	*38*	*44*	*50*	*58*	*65*	*75*	*85*
Key	***b***	*2*	*3*	*4*	*5*	*6*	*8*	*10*	*12*	*14*	*16*	*18*	*20*	*22*
	h	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*8*	*9*	*10*	*11*	*12*	*14*
Length		6	6	8	10	14	18	22	28	36	45	50	56	63
		8	8	6	12	16	20	25	32	40	40	56	63	70
		10	10	8	14	18	22	28	36	45	45	63	70	80
		12	12	10	16	20	25	32	40	50	50	70	80	90
		14	14	12	18	22	28	36	45	56	56	80	90	100
		16	16	14	20	25	32	40	50	63	63	90	100	110
		18	18	16	22	28	36	45	56	70	70	100	110	125
		20	20	18	25	32	40	50	63	80	80	110	125	140
		-	22	20	28	36	45	56	70	90	90	125	140	160
		-	25	22	32	40	50	63	80	100	100	140	160	180
		-	28	25	36	45	56	70	90	110	110	160	180	200
		-	32	28	40	50	63	80	100	125	125	180	200	220
		-	36	32	45	56	70	90	110	140	140	200	220	250
		-	-	36	50	63	80	100	125	160	160	-	-	-
		-	-	40	56	70	90	110	140	-	180	-	-	-
		-	-	45	-	-	-	-	-	-	-	-	-	-

Shaft dia	*Above*	*85*	*95*	*110*	*130*	*150*	*170*	*200*	*230*	*260*	*290*	*330*	*380*	*440*
	Upto	*95*	*110*	*130*	*150*	*170*	*200*	*230*	*260*	*290*	*330*	*380*	*440*	*500*
Key	***b***	*25*	*28*	*32*	*36*	*40*	*45*	*50*	*56*	*63*	*70*	*80*	*90*	*100*
	h	*14*	*16*	*18*	*20*	*22*	*25*	*28*	*32*	*32*	*36*	*40*	*45*	*50*
Length		70	80	90	100	110	125	140	160	180	200	220	250	280
		80	90	100	110	125	140	160	180	200	220	250	280	320
		90	100	110	125	140	160	180	200	220	250	280	320	360
		100	110	125	140	160	180	200	220	250	280	320	360	400
		110	125	140	160	180	200	220	250	280	320	360	400	-
		125	140	160	180	200	220	250	280	320	360	400	-	-
		140	160	180	200	220	250	280	320	360	400	-	-	-
		160	180	200	220	250	280	320	360	400	-	-	-	-
		180	200	220	250	280	320	360	400	-	-	-	-	-
		200	220	250	280	320	360	400	-	-	-	-	-	-
		220	250	280	320	360	400	-	-	-	-	-	-	-
		250	280	320	360	400	-	-	-	-	-	-	-	-
		280	320	360	400	-	-	-	-	-	-	-	-	-

Designation: A parallel key of width 12 mm, height 8 mm and length 50 mm shall be designated as: parallel key $12 \times 8 \times 50$.

PROBLEMS

Problem 1: Select a rectangular parallel key to transmit 9 kW at 300 rpm. The yield stress for the steel used is 310 MPa. Take factor of safety as 2.5.

Given data: Power, $P = 9$ kW, Speed, $n = 300$ rpm

$\sigma_y = 310$ MPa, FOS = 2.5

$\therefore$ Allowable stress, $\sigma = \dfrac{\sigma_y}{FOS} = \dfrac{310}{2.5} = 124$ MPa $= \sigma'_b$

assume $\tau = 0.5\sigma = 62$ MPa $= \tau_k$

with key way in the shaft assume, $\tau_s = 0.75\tau$

$\tau_s = 0.75 \times 62 = 46.5$ MPa.

Step 1: Torque, $M_t = \dfrac{9.55 \times 10^6 P}{n} = \dfrac{9.55 \times 10^6 \times 9}{300} = 286.5 \times 10^3$ N-mm

Step 2: Dia of shaft, $d = \left[\dfrac{16M_t}{\pi\tau_S}\right]^{1/3}$

$$d = \left[\frac{16 \times 286.5 \times 10^3}{\pi \times 46.5}\right]^{1/3} = 31.54 \simeq 32 \text{ mm}$$

Step 3: Ref to Table 7.1, select standard key ...(Table 17-1/P 17.10)

For shaft dia, $d = 32$

Select, $b = 10$ mm

$h = 8$ mm

Step 4: To find length of the key

(a) Width, $b = \dfrac{2M_t}{dl\tau_K}$

$$10 = \frac{2 \times 286.5 \times 10^3}{32 \times l \times 62} \Rightarrow l = 28.88$$

(b) Thickness, $h = \dfrac{4M_t}{dl\sigma'_b}$

$$8 = \frac{4 \times 286.5 \times 10^3}{32 \times l \times 124} = 36.1 \text{ mm}$$

Recommended length, $l = 36.1 \simeq 37$ mm (bigger value).

Problem 2: Determine the required length of square key for a shaft diameter of 35 mm. The key is 8 × 8 mm. Allowable shear and crushing stresses for key material are 55 MPa and 120 MPa respectively. The shaft transmits 639 Nm of torque.

Given data:

$d = 35$ mm.

$b = h = 8$ mm (square key)

$\tau_k = 55$ MPa

$\sigma_b' = 120$ MPa

$M_t = 639 \times 10^3$ N-mm

(a) Width of key, $b = \dfrac{2M_t}{dl\tau_K}$

$$l = \frac{2 \times 639 \times 10^3}{35 \times 8 \times 55} = 82.99 \text{ mm}$$

(b) Thickness of key, $h = \dfrac{4M_t}{dl\sigma_b'}$

$$l = \frac{4 \times 639 \times 10^3}{35 \times 8 \times 120} = 76.07$$

Recommended length, $l = 82.99 \simeq 83$ mm.

Problem 3: A rectangular cross section key 8 × 7 × 36 is used to transmit 6 kW at 1200 rpm. The shaft diameter is 30 mm. If the allowable shear and crushing stress for key material are 60 MPa and 135 MPa respectively, find whether key is safe or not.

Given data:

$P = 6$ kW, $n = 1200$ rpm, $d = 30$ mm

key c/s $b = 8$ mm $h = 7$ mm, $l = 36$ mm

$\tau_k = 60$ MPa, $\sigma_b' = 135$ MPa

$$\text{Torque, } M_t = \frac{9.55 \times 10^6 P}{n} = \frac{9.55 \times 10^6 \times 6}{1200} = 47.75 \times 10^3 \text{ N-mm}$$

$$b = \frac{2M_t}{dl\tau_K} \Rightarrow \tau_k = \frac{2 \times 47.75 \times 10^3}{30 \times 8 \times 36} = 11.05 \text{ MPa}$$

This is less than allowable shear stress, hence safe

$$h = \frac{4M_t}{dl\sigma_b'} \Rightarrow \sigma_b' = \frac{4 \times 47.75 \times 10^3}{30 \times 7 \times 36} = 25.26 \text{ MPa}$$

This is also less than allowable crushing stres, hence the key is safe.

Problem 4: A shaft is required to transmit 16 kW at 500 rpm. Select a suitable key of rectangular cross section, if the hub length is 60 mm. Take allowable shear and crushing stresses for material used as 72 MPa and 140 MPa respectively.

Given data: $P = 16$ kW, $n = 500$ rpm, $l = 60$ mm

$\tau_s = \tau_k = 72$ MPa, $\sigma'_b = 140$ MPa

Step 1: Torque, $M_t = \dfrac{9.55 \times 10^6 \times P}{n}$

$$= \frac{9.55 \times 10^6 \times 16}{500} = 305.6 \times 10^3 \text{ N-mm}$$

Step 2: Diameter of shaft, $d = \left(\dfrac{16M_t}{\pi\tau_S}\right)^{1/3} = \left(\dfrac{16 \times 305.6 \times 10^3}{\pi \times 75}\right)^{1/3}$

$d = 27.48 \simeq 28$ mm.

Step 3: Key cross section:

(a) Length of key $= l = 60$ mm

(b) Width, $b = \dfrac{2M_t}{dl\tau_K}$

$\therefore$ $$b = \frac{2 \times 305.6 \times 10^3}{28 \times 60 \times 72} = 5.05 \text{ mm}$$

Thickness of key, $h = \dfrac{4M_t}{dl\sigma'_b}$

$$h = \frac{4 \times 305.6 \times 10^3}{28 \times 60 \times 140} = 5.19$$

Ref. Table 7.1, select standard key c/s ...(T 17-1/P 17.10)

$b = 6$ mm

$h = 6$ mm (based on the required b and h).

Problem 5: Select a rectangular key for transmitting a power of 50 kW at 500 rpm, to mount a hub of length 60 mm on a solid circular shaft of diameter 50 mm.

Given data: $P = 50$ kW, $n = 500$ rpm, $d = 50$ mm

Hub length = key of length, $l = 60$ mm

Step 1: Torque, $M_t = \dfrac{9.55 \times 10^6 P}{n}$

$$M_t = \frac{9.55 \times 10^6 \times 50}{500} = 955 \times 10^3 \text{ N-mm}$$

Step 2: Dia of shaft, $d = 50$ mm (given)

Step 3: Selection of key

(a) Length of key $= l = 60$ mm

(b) Width of key, $b = \dfrac{2M_t}{dl\tau_K}$

Assume τ_k 55 MPa and $\sigma_b' = 110$ MPa

$$b = \frac{2 \times 955 \times 10^3}{50 \times 60 \times 55} = 11.58 \text{ mm}$$

(c) Thickness of key, $h = \dfrac{4M_t}{dl\sigma_b'}$

$$h = \frac{4 \times 955 \times 10^3}{50 \times 60 \times 110} = 11.56 \text{ mm}$$

From Table 7.1, select standard key of 20×12 mm.

7.2 COUPLINGS

Couplings are used to connect two shafts and to transmit power from one shaft to another. Unlike clutches, this is a permanent connection between shafts.

Couplings are of two types: Rigid couplings and flexible coupling. Muff coupling, marine type flange coupling, C.I. flange coupling are the examples of rigid couplings. Examples of flexible couplings are universal coupling (Hooke's joint), pin type or bush type flexible coupling, Oldham's coupling etc.

Flexible couplings are used to accommodate a small misalignment in the shafts.

1. Muff or Sleeve Coupling

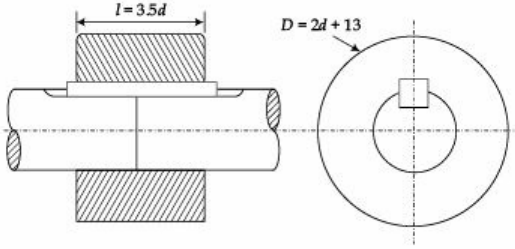


Fig. 7.2 Muff coupling.

Symbols: d = diameter of shaft

D = outside diameter of sleeve

l = length of sleeve = length of key

b = width of key

h = thickness of key

τ_s = allowable shear stress for shaft

τ_k = allowable shear stress for key

σ_b' = allowable bearing stress for key

η = key way factor = 0.75

P = power in kW

n = speed in rpm

M_t = torque

τ_{sl} = allowable shear stress in sleeve.

Procedure

Step 1: $$\text{Torque, } M_t = \frac{9.55 \times 10^6 P}{n} \text{ N-mm}$$

Step 2: $$\text{Diameter of shaft, } d = \left[\frac{16 M_t}{\pi \eta \tau_s}\right]^{1/3} \quad \text{...(19-49/P 19.6)}$$

Step 3: Outside diameter of sleeve, $D = 2d + 13$ mm ...(19-45 b)

Step 4: Length of key = Length of sleeve, $l = 3.5d$ ...(19-47)

Step 5: Check for shear stress in sleeve.

Consider sleeve as a hollow shaft, $K = \dfrac{d}{D}$

$$\text{Outside dia, } D = \left[\frac{16 M_t}{\pi \tau_{sl}(1 - K^4)}\right]^{1/3} \quad \text{...(19-46)}$$

Find shear stress in sleeve = τ_{sl} = ?

This should be less than or equal to τ_{sl} allowable. Otherwise, sleeve is not safe. Increase sleeve diameter by 3 to 5 mm and check again.

Step 6: Design of key

(a) $$\text{Length of key, } l_k = \frac{l}{2}$$

(In each shaft, the key length = $l/2$ only)

(b) $$\text{Width of key, } b = \frac{2M_t}{d l_K \tau_K} \Rightarrow b = ?$$

(c) Thickness of key, $h = \dfrac{4M_t}{dl_K\sigma'_b} \Rightarrow h = ?$ ...(T 17-1)

Refer Table 7.1, select standard key cross section b and h.

Problem 6: Design a sleeve coupling to transmit 18 kW at 1000 rpm.

Given data: $P = 18$ kW $\quad n = 1000$ rpm.

Select, $\tau_k = 55$ MPa, $\quad \sigma'_b = 110$ MPa.

For CI sleeve, $\tau_{sl} = 10$ MPa.

$\tau_s = 41$ MPa with key way (75% of 55)

Step 1: $M_t = \dfrac{9.55 \times 10^6 P}{n} = \dfrac{9.55 \times 10^6 \times 18}{1000} = 171.9 \times 10^3$ N-mm

Step 2: Diameter of shaft, $d = \left[\dfrac{16M_t}{\pi\tau_s}\right]^{\frac{1}{3}} = \left[\dfrac{16 \times 171.9 \times 10^3}{\pi \times 41 \times 0.85}\right]^{1/3}$

$d = 29.28 \simeq 30$ mm

Step 3: Outside dia of sleeve, $D = 2d + 13 = 69$ mm

Step 4: Length of sleeve = Length of key = $l = 3.5d$

$l = 3.5 \times 28 = 98$ mm

Step 5: Check for shear stress in sleeve

Sleeve dia, $D = \left[\dfrac{16M_t}{\pi\tau_{sl} \cdot (1 - K^4)}\right]^{1/3}$ When, $k = \dfrac{d}{D} = \dfrac{28}{69} = 0.406$

$\therefore \quad \tau_{sl} = \dfrac{16 \times 171.9 \times 10^3}{69^3 \times \pi(1 - 0.406^4)} = 2.74$ MPa

$\tau_{sl} \le \tau_{sl}$ allowable hence safe.

Step 6: Design of key

(a) Length of key in each shaft, $l_k = \dfrac{l}{2} = \dfrac{98}{2} = 49$ mm

(b) Width of key, $b = \dfrac{2M_t}{dl\tau_K}$

$b = \dfrac{2 \times 171.9 \times 10^3}{28 \times 49 \times 55} = 4.56$ mm

(c) Thickness of key, $h = \dfrac{4M_t}{dl\sigma'_b}$

$$h = \frac{4 \times 171.9 \times 10^3}{28 \times 49 \times 110} = 4.56 \text{ mm}$$

Select standard key, $b = 5$ mm

$h = 5$ mm (From Table 7.1).

Problem 7: A sleeve coupling used to transmit a torque of 90 N-m has the following details: Diameter of shaft = 30 mm. Outside diameter of sleeve = 75 mm, length of sleeve = 100 mm, key used = 8 × 7. Find:

(i) Shear stress induced in the shaft,
(ii) Shear stress induced in the sleeve,
(iii) Shear stress induced in the key, and
(iv) Crushing stress in decide in the key.

Given data: Torque $M_t = 90$ Nm $= 90 \times 10^3$ N-mm

$d = 30$ mm, $l = 100$ mm, $D = 75$ mm

$b = 8$ mm, $h = 7$ mm, $\therefore \quad l_k = \frac{100}{2} = 50$ mm.

(i) Shear stress in the shaft

$$d = \left[\frac{16M_t}{\pi\tau_S}\right]^{1/3} \Rightarrow \tau_s = \frac{16M_t}{\pi d^3}$$

$$\tau_s = \frac{16 \times 90 \times 10^3}{\pi \times 30^3} = 16.98 \text{ MPa}$$

(ii) Shear stress in the sleeve

$$D = \left[\frac{16M_t}{\pi\tau_{sl}(1-K^4)}\right]^{\frac{1}{3}} \Rightarrow \tau_{sl} = \frac{16M_t}{\pi D^3(1-K^4)}$$

$$K = \frac{d}{D} = \frac{30}{75} = 0.4$$

$$\tau_{sl} = \frac{16 \times 90 \times 10^3}{\pi \times 75^3(1-0.4^4)} = 1.115 \text{ MPa}$$

(iii) Shear stress in the key

$$b = \frac{2M_t}{dl_K\tau_K} \Rightarrow \tau_k = \frac{2M_t}{dl_K b}$$

$$\tau_k = \frac{2 \times 90 \times 10^3}{30 \times 50 \times 8} = 15 \text{ MPa}$$

(iv) Crushing stress in key

$$h = \frac{4M_t}{dl_K\sigma_b'} \Rightarrow \sigma_b' = \frac{4M_t}{dl_K h}$$

$$\sigma_b' = \frac{4 \times 90 \times 10^3}{30 \times 50 \times 7} = 34.29 \text{ MPa.}$$

2. Marine Type Flange Coupling or Solid Flange Coupling

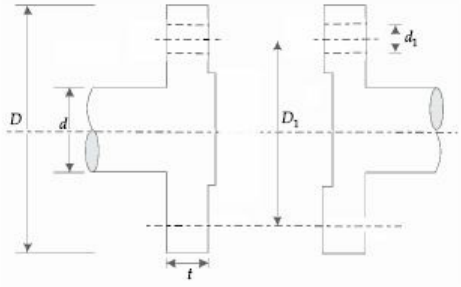


Fig. 7.3 Marine coupling.

...(Fig. 19-2/P 19.15)

(Here the flange is integral with the shaft. Hence, no key and key way)

Symbols: d = diameter or shaft,
D = outside diameter of flange
D_1 = bolt circle diameter
d_1 = bolt diameter
t = thickness of flange
i = no of bolts
τ_s = allowable shear stress for shaft
τ_b = allowable shear stress for bolt
P = power, kW
n = speed, rpm
M_t = torque, N-mm

Procedure ...(Page 19.4)

Step 1: Torque, $M_t = \dfrac{9.55 \times 10^6 P}{n}$

Step 2: Diameter of shaft, $d = \left[\dfrac{16 M_t}{\pi \tau_S}\right]^{1/3}$

Step 3: No. of bolts, $i = \dfrac{d}{30} + 5$ ...(19-15 b/P 19.4)

Step 4: Bolt circle diameter, $D_1 = 1.5d$ ...(19.18)

Step 5: Bolt diameter based on torque capacity of shaft,

$$d_1 = \left[\frac{d^3 \tau_S}{2 i D_1 \tau_b}\right]^{1/2} \quad \text{...(19-16 a)}$$

Select standard bolt diameter from Table 9.2 ...(T 18-7/P 18-16)

Step 6: Outside dia of flange, $D = D_1 + 3d_1$ ...(19-19)

Step 7: Thickness of flange, $t = 0.25d$. ...(19-17)

Problem 8: Design marine type flange coupling to transmit 8.5 kW at 800 rpm. Allowable shear stress for shaft and bolt material may be taken as 60 MPa.

Given data: $P = 8.5$ kW, $n = 800$ rpm, $\tau_s = \tau_b = 60$ MPa.

Step 1: Torque, $M_t = \dfrac{9.55 \times 10^6 \times 8.5}{800} = 101.47 \times 10^3$ N-mm

Step 2: Dia of shaft, $d = \left[\dfrac{16 \times 101.47 \times 10^3}{\pi \times 60}\right]^{\frac{1}{3}} = 20.49 \simeq 21$ mm

Step 3: No. of bolts, $i = \dfrac{d}{30} + 5 = 5.7 \simeq 6$ nos. ...(19-15 b)

Step 4: Bolt circle diameter, $D_1 = 1.5d = 31.5 \simeq 32$ mm (19-18)

Step 5: Bolt diameter based on torque capacity of shaft

$$d_1 = \left[\frac{d^3 \tau_s}{2i D_i \tau_b}\right]^{\frac{1}{2}} = \left[\frac{21^3 \times 60}{2 \times 6 \times 32 \times 60}\right]^{\frac{1}{2}} \quad \text{...(19-16 a)}$$

$= 4.91 \simeq 5$ mm (std dia from Table 9.2) ...(T 18-7)

Step 6: Outside diameter of flange, $D = D_1 + 3d_1 = 47$ mm ...(19-19)

Step 7: Thickness of flange, $t = 0.25d = 5.25 \simeq 6$ mm. ...(19-18)

3. C.I. Flange Coupling or Common Flange Coupling or Protected Type Flange Coupling

Symbols: d = diameter of shaft
D = outside diameter of flange
D_1 = bolt circle diameter
d_1 = bolt diameter
t = thickness of flange
i = no. of bolts
D_2 = diameter of hub
l = length of hub = length of key
b, h = width and thickness of key
τ_s = allowable shear stress for shaft
τ_b = allowable shear tress for bolt
τ_k = allowable shear stress for key
σ_b' = allowable bearing stress for key
P = power, kW
n = speed, rpm.
M_t = torque, N-mm
η = key way factor = 0.75

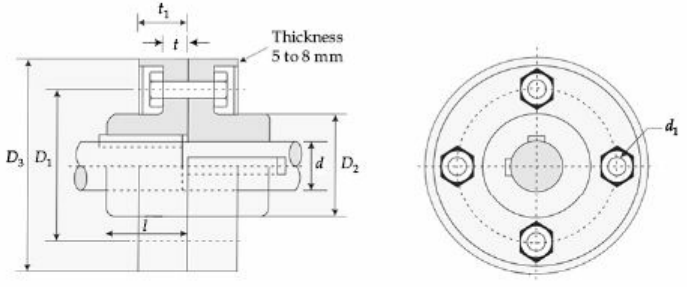


Fig. 7.4 Protective flange coupling.

...(Fig. 19-1/P 19.15)

Procedure: (For both protected type and unprotected type) ...(P 19-3)

Step 1: Torque, $M_t = \dfrac{9.55 \times 10^6 P}{n}$

Step 2: Diameter of shaft, $d = \left[\dfrac{16 M t}{\pi \eta \tau_s}\right]^{\frac{1}{3}}$ ...(19-49)

Step 3: No. of bolts, $i = 0.02\,d + 3$ ...(19-1 b/P 19.3)

Step 4: Bolt circle diameter, $D_1 = 2d + 50$ ...(19-12 b)

Step 5: Bolt diameter, $d_1 = \dfrac{0.5d}{\sqrt{i}}$ ...(19-8)

Select standard bolt diameter from Table 9.2 ...(T 18-7)

Step 6: Check for shear stress induced in bolt

Torque capacity based on bolts, $M_t = i\left(\dfrac{\pi}{4}d_1^2\right)\tau_b \dfrac{D_1}{2}$ ...(19-4)

Find $\tau_b \le \tau_b$ allowable.

Otherwise, select next bigger size bolt from bolt table and repeat check.

Step 7: Design of hub

(a) Hub dia, $D_2 = 1.5\,d + 25$ ...(19-13 b)

(b) Hub length, $l = 1.25\,d + 18.75$ ...(19-14 d)

Step 8: Design of flange

(a) Dia of flange, $D = 2.5d + 75$ ...(19-14 b)

(b) Thickness of flange, $t = 0.25d$ ...(19-18)

Step 9: Check for shear stress in flange

Torque capacity of flange, $M_t = t(\pi D_2)\tau_f \dfrac{D_2}{2}$ ...(19-6)

Find $\tau_f \le \tau_f$ allowable.

Otherwise, increase t by 2 to 3 mm and check again.

Step 10: Design of key

(a) Length of key = Length of Hub = l

(b) Width of key, $b = \dfrac{2M_t}{d\,l\,\tau_k}$

(c) Thickness of key, $h = \dfrac{4M_t}{d\,l\,\sigma_b'}$. Select standard key cross-section from Table 7.1. ...(T 17-1)

Problem 9: Design a rigid flange coupling to transmit 18 kW at 1440 rpm. The allowable shear stress for CI flange is 4 MPa. The shafts, keys and bolts are made of annealed steel having allowable shear stress of 93 MPa. Allowable crushing stress for key = 186 MPa.

Given data: $P = 18$ kW $\quad n = 1440$ rpm.

$\tau_f = 4$ MPa $\quad \tau_s = \tau_k = \tau_b = 93$ MPa

$\sigma_b' = 186$ MPa. Assume key way factor $\eta = 0.75$

Step 1: $$\text{Torque, } M_t = \frac{9.55 \times 10^6 P}{n} = 119.4 \times 10^3 \text{ N-mm}$$

Step 2: $$\text{Diameter of shaft, } d = \left[\frac{16Mt}{\pi\eta\tau_s}\right]^{\frac{1}{3}} \qquad d = \left[\frac{16 \times 119.4 \times 10^3}{\pi \times 0.75 \times 93}\right]^{1/3} = 20.58 \simeq 22 \text{ mm}$$

Step 3: No. of bolts, $i = 0.02d + 3 = 3.44 \simeq 4$ bolts

Step 4: Bolt circle diameter, $D_1 = 2d + 50 = 94$ mm

Step 5: $$\text{Bolt diameter, } d_1 = \frac{0.5d}{\sqrt{i}} = 5.5 \text{ mm}$$

Standard diameter from table, $d_1 = 6$ mm ($M6$ bolt)

Step 6: Check for shear stress in bolt:

$$\text{Torque capacity of bolts, } M_t = i\left(\frac{\pi}{4}d_1^2\right)\tau_b \cdot \frac{D_1}{2}$$

$$119.4 \times 10^3 = 4\left(\frac{\pi}{4}6^2\right)\tau_b \cdot \frac{94}{2}$$

$\therefore$ $\tau_b = 22.46 < \tau_b$ allowable (93 MPa) safe.

Step 7: Design of hub:

(a) $$\text{Hub diameter, } D_2 = 1.5d + 25$$

$$D_2 = 1.5 \times 22 + 25 = 58 \text{ mm}$$

(b) $$\text{Hub length, } l = 1.25d + 18.75 = 46.25 \simeq 47 \text{ mm}$$

Step 8: Design of flange:

(a) Outside diameter of flange, $D = 2.5d + 75 = 130$ mm

(b) $$\text{Flange thickness, } t = 0.25d = 5.5 \simeq 6 \text{ mm}$$

Step 9: Check for shear stress in flange

$$\text{Torque capacity of flange, } \quad M_t = t(\pi D_2)\,\tau_f\left(\frac{D_2}{2}\right)$$

$$119.4 \times 10^3 = 6\,(\pi \times 58)\tau_f \times \left(\frac{58}{2}\right)$$

$\therefore$ Shear stress in flange, $\tau_f = 3.77 < \tau_f$ allowable (4 MPa) safe.

Step 10: Design of key

(a) Length of key = Length of hub $l = 47$ mm

(b) $$\text{Width of key, } b = \frac{2M_t}{dl\tau_k}$$

$$b = \frac{2 \times 119.4 \times 10^3}{22 \times 47 \times 93} = 2.48 \simeq 3 \text{ mm}$$

(c) Thickness of key, $h = \dfrac{4M_t}{d\,l\,\sigma_b'} = \dfrac{4 \times 119.4 \times 10^3}{22 \times 47 \times 186} = 2.48 \simeq 3$ mm

Select standard key from Table 7.1

$$b = 3 \text{ mm}, \quad h = 3 \text{ mm}.$$

Problem 10: Design a protected type flange coupling to transmit power between two shafts 40 mm and 50 mm. The allowable shear stress for shaft and bolts is 60 MPa. The allowable shear stress and bearing stress for key are 54 MPa and 120 MPa respectively. For CI flange, the allowable shear stress is 6 MPa.

Given data: Dia of shafts $d = 40$ mm and 50 mm

(use $d = 40$ mm smaller diameter to find torque capacity and use $d = 50$ mm bigger value for other calculations)

$\tau_s = \tau_b = 60$ MPa. $\quad \tau_k = 54$ MPa $\quad \sigma_b' = 120$ MPa $\quad \tau_f = 6$ MPa

Step 1: Torque capacity based on smaller diameter shaft

$$M_t = \frac{\pi}{16} d^3 \tau_s \eta = \frac{\pi}{16} 40^3 \times 60 \times 0.75 \qquad \ldots(19\text{–}2)$$

$$= 0.5655 \times 10^6 \text{ N- mm (assume key way factor } \eta = 0.75)$$

Step 2: Diameter of shaft, $d = 50$ mm (bigger one)

Step 3: No. of bolts, $i = 0.02d + 3 = 0.02 \times 50 + 3 = 4$ bolts

Step 4: Bolt circle diameter, $D_1 = 2d + 50 = 150$ mm

Step 5: Bolt diameter, $d_1 = \dfrac{0.5d}{\sqrt{i}} = 12.5 \simeq 14$ mm

(Use *M*14 bolt, from Table 9.2)

Step 6: Check for shear stress in bolt

$$M_t = i\left(\frac{\pi}{4} d_1^2\right) \tau_b \left(\frac{D_1}{2}\right) \Rightarrow 0.5655 \times 10^6 = 4\left(\frac{\pi}{4} \times 14^2\right) \tau_b \left(\frac{150}{2}\right)$$

$\therefore \quad \tau_b = 12.25 < 60$ safe. (τ_b allowable)

Step 7: Design of hub

(a) Dia of hub, $D_2 = 1.5d + 25 = 100$ mm

(b) Lenght of hub, $l = 1.25d + 18.75 = 81.25 \simeq 82$ mm

Step 8: Design of flange:

(a) Outside diameter of flange, $D = 2.5d + 75 = 200$ mm

(b) Thickness of flange, $t = 0.25d = 12.5 \simeq 13$ mm

Step 9: Check for shear stress in flange

$$\text{Torque capacity of flange } M_t = t(\pi D_2)\,\tau_f \frac{D_2}{2}$$

$$0.5655 \times 10^6 = 13(\pi \times 100)\,\tau_f \times \frac{100}{2}$$

$\therefore$ Shear stress induced in flange $\tau_f = 3.00 < \tau_{f\ \text{allowable}}$, hence safe.

Step 10: Design of key

(a) Length of key = length of hub $l = 82$ mm

(b) Width of key for $d = 40$ mm shaft

$$b = \frac{2M_t}{dl\tau_k} = 6.38 \simeq 7$$

For $d = 50$ mm shaft

$$b = \frac{2M_t}{dl\tau_k} = 5.11 \simeq 6$$

(c) Thickness of key for $d = 40$ mm shaft

$$h = \frac{4M_t}{dl\sigma_b'} = 5.75 \simeq 6$$

For $d = 50$ mm shaft, $h = \frac{4M_t}{dl\sigma_b'} = 4.597 \simeq 5$ mm

Use standard key 8×7 for both shafts (bigger one).

Problem 11: Design a flange coupling to connect the shafts of a motor and a centrifugal pump for the following specification.

Pump output = 3000 liter/min.
Total head = 20 m
Pump speed = 600 rpm
Pump efficiency = 70%

Select C40 steel shaft ($\sigma_y = 328.6$ MPa), C30 steel for bolts and key ($\sigma_y = 294.2$ MPa). Take factor of safety = 2. Allowable shear stress for CI flange = 15 MPa

Given data: Pump output, $Q = 3000$ lpm

$$1 \text{ liter of water} = 1000 \text{ ml} = 1000 \text{ CC} = \frac{1000}{100^3}\ \text{m}^3$$

$$= 1 \times 10^{-3}\ \text{m}^3$$

$\therefore$ $$Q = \frac{3000 \times 10^{-3}}{60} = 0.05\ \text{m}^3/\text{sec}.$$

Total head $H = 20$ m.

Speed, $n = 600$ rpm.

Pump efficiency, $\eta_p = 0.7$

$\therefore$ $$\text{Power of motor, } P = \frac{wQH}{1000\eta_p}$$

where w = Specific weight of water = 9810 N/m^3

$\therefore$ $$P = \frac{9810 \times 0.05 \times 20}{1000 \times 0.7} = 14.014 \text{ kW} \simeq 14 \text{ kW}$$

For C40 steal shaft

$$\sigma_y = 328.6 \text{ MPa.} \quad FOS = 2$$

$\therefore$ $$\sigma = \frac{\sigma_y}{FOS} = 164.3 \text{ MPa}$$

Assume $\tau = 0.5\sigma = 82.15 \text{ MPa} = \tau_s$

For C30 bolts and keys, $\sigma_y = 294.2$ MPa. FOS = 2

$\therefore$ $$\sigma = \frac{\sigma_y}{FOS} = 147.1 \text{ MPa} = \sigma_b'$$

$$\tau = 0.50\sigma = 73.55 \text{ MPa} = \tau_b = \tau_k$$

Take key way factor for shaft, $\eta = 0.75$

For CI flange $\tau_f = 15$ MPa.

$P = 14$ kW, $n = 600$ rpm.

$\tau_s = 82.15$ MPa, $\eta = 0.75$

$\tau_k = \tau_b = 73.55$ MPa, $\sigma_b' = 147.1$ MPa

$\tau_f = 15$ MPa

(Complete the problem similar to problem No. 9).

Problem 12: A CI flange coupling transmits 7.5 kW at 200 rpm. The shaft diameter is 45 mm. The flanges of coupling are bolted width 4 bolts of $M12 \times 1$ on a bolt circle diameter 250 mm. The web thickness of flange is 20 mm. Hub diameter is 120 mm key used $10 \times 8 \times 36$. Determine:

(i) The shear stress induced in the shaft,
(ii) The shear stress induced in the bolts,
(iii) The shear stress induced in the flange,
(iv) The shear stress induced in the key,
(v) The crushing stress induced in the key.

Given data:

Power, $P = 7.5$ kW

Speed, $n = 200$ rpm

Shaft diameter, $d = 45$ mm. No. of bolts $i = 4$

Bolt diameter, $d_1 = M12 \Rightarrow d_1 = 12$ mm.

Bolt circle diameter, $D_1 = 250$ mm. Hub diameter $D_2 = 120$ mm

Flange thickness, $t = 20$ mm.

key $= 10 \times 8 \times 36 = b \times h \times l$

$$\text{Torque, } M_t = \frac{9.55 \times 10^6 P}{n} = \frac{9.55 \times 10^6 \times 7.5}{200} = 0.358 \times 10^6 \text{ N-mm}$$

(i) Shear stress in the shaft

For shaft $M_t = \dfrac{\pi d^3 \tau_s}{16} \Rightarrow \tau_s = \dfrac{16 \times 0.358 \times 10^6}{\pi \times 45^3} = 20.02$ MPa

(ii) Shear stress in bolts:

For bolts $M_t = i\left(\dfrac{\pi}{4} d_1^2\right) \tau_b \cdot \dfrac{D_1}{2} \Rightarrow \tau_b = \dfrac{4 \times 2 \times 0.358 \times 10^6}{4 \times \pi \times 12 \times 250} = 6.33$ MPa

(iii) Shear stress in flange

For flange $M_t = t(\pi D_2) \tau f \left(\dfrac{D_2}{2}\right) \Rightarrow \tau_f \dfrac{2 \times 0.358 \times 10^6}{20 \times \pi \times 120 \times 120} = 0.79$ MPa

(iv) Shear stress in key

Width of key $b = \dfrac{2Mt}{d l \tau_k} \Rightarrow \tau_k = \dfrac{2 \times 0.358 \times 10^6}{45 \times 36 \times 10} = 44.2$ MPa

(v) Crushing stress in key

Thickness of key $h = \dfrac{4Mt}{d e \sigma_b'} \Rightarrow \sigma_b' = \dfrac{4 \times 0358 \times 10^6}{45 \times 36 \times 8} = 110.49$ MPa.

4. Pin Type or Brush Type Flexible Coupling

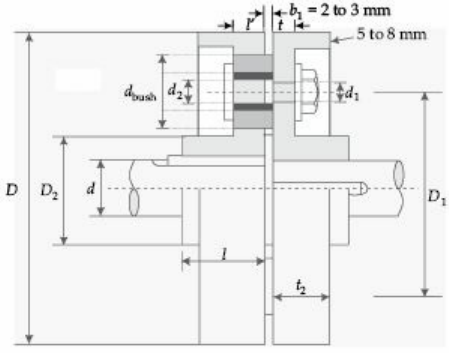


Fig. 7.5 Bush type flexible coupling.

Symbols: d = diameter of shaft ...(Fig. 19-4/P 19.15)

D = outside diameter of flange

D_1 = bolt circle diameter

d_1 = bolt diameter
i = no. of bolts
D_2 = diameter of hub
l = length of hub.
b, h = width and thickness of key.
l' = length of rubber bush
d' = outside dia of rubber bush
d_2 = bigger diameter of pin at bush
τ_s = allowable shear stress for shaft
τ_b = allowable shear stress for pin
σ_b = allowable normal stress for pin
τ_k = allowable shear stress for key
σ_b' = allowable bearing stress for key
p_b = allowable bearing stress for rubber bush.
P = power in kW
n = speed in rpm
M_t = torque, N-mm
η = key way factor = 0.75
F = load at each pin (bolt)
b_1 = gap between flanges = 2 to 3 mm.

Procedure

Step 1: Torque, $M_t = \dfrac{9.55 \times 10^6 P}{n}$

Step 2: Diameter of shaft, $d = \left[\dfrac{16 M_t}{\pi \tau_s \eta}\right]^{1/3}$ ...(19-49/P 19.6)

Step 3: No. of bolts, $i = 0.02d + 3$ ...(19-1 b)

Step 4: Bolt circle diameter, $D_1 = 2d + 50$ ...(19-12 b)

Step 5: Bolt diameter, $d_1 = \dfrac{0.5d}{\sqrt{i}}$ select standard bolt dia from tables. ...(19-8)

Steps 6: Assume bigger dia of pin, $d_2 = 2d_1$

Step 7: Assume 2 mm thick brass bush and on that 10 mm thick rubber bush.

$\therefore$ Outside diameter of rubber bush, $d' = d_2 + 2(2 + 10)$

Step 8: Force at each pin

Torque, $M_t = i\, F \dfrac{D_1}{2} \Rightarrow F = ?$ ...$\left(\dfrac{19-33a}{P19-5}\right)$

Step 9: To find length of rubber bush l'

$$F = p_b (l' d') \quad \text{...(19-33 b)}$$

Find $l' = ?$

p_b = allowable pressure on rubber bush = 1 MPa

Step 10: Check of pin for principal stresses:

(a) Shear stress in pin, $\tau_p = \dfrac{F}{(\pi/4) d_1^2}$ ...(19-34/P 19.5)

(b) Bending stress in pin, $\sigma_p = \dfrac{F(l'/2 + b_1)}{(\pi d_1^3/32)}$ ...(19.35)

(c) Combined or principal stresses in the pin:

(i) Maximum normal stress, $\sigma_{max} = \dfrac{1}{2}\left[\sigma_p + \left(\sigma_p^2 + 4\tau_p^2\right)^{1/2}\right]$

$$\sigma_{max} \leq \sigma_{b\,allowable} \text{ and}$$

(ii) Maximum shear stress, $\tau_{max} = \dfrac{1}{2}(\sigma_p^2 + 4\,\tau_p^2)^{1/2}$, $\tau_{max} \leq \tau_b$ allowable.

Otherwise, the pin is not safe. Increase the bolt diameter in step 5 and repeat then onwards.

Step 11: Design of hub:

(a) Hub diameter, $D_2 = 1.5d + 25$ ...(19-13 b)

(b) Hub length, $l = 1.25d + 18.75$ ...(19-14 d)

Step 12: Design of flange:

(a) Outside diameter of flange, $D = 2.5d + 75$ ...(19-14 b)

(b) Thickness of flange $\simeq$ length of rubber bush.

Step 13: Design of key:

(a) Length of key = Length of hub = l

(b) Width of key, $b = \dfrac{2M_t}{dl\,\tau_k}$ ...(19-50)

(c) Thickness of key, $h = \dfrac{4M_t}{dl\,\sigma_b'}$ ...(19-51)

Ref. Table 7.1 and select standard key cross section. ...(T 17-1)

Problem 13: Design a bush type flanged coupling to transmit 15 kW at 2000 rpm. Draw a neat a sketch of the coupling. Maximum torque to be transmitted is 30% more than mean torque.

Given data: $P = 15$ kW, $n = 2000$ rpm, $M_t = 1.3(M_t)$

Assume $\tau_s = \tau_k = \tau_b = 55$ MPa, $\sigma_b = \sigma_b' = 110$ MPa

For rubber bush $p_b = 1$ MPa.

Key way factor $\eta = 0.75$

Step 1: Torque $M_t = \left(\frac{9.55 \times 10^6 P}{n}\right) \times 1.3 = \frac{9.55 \times 10^6 \times 15 \times 1.3}{2000}$ 93112.5 N-mm

Step 2: Diameter of shaft, $d = \left[\frac{16 M_t}{\pi \eta \tau_s}\right]^{1/3}$

$$= \left[\frac{16 \times 93112.5}{\pi \times 0.75 \times 55}\right]^{1/3} = 22.56 \simeq 23 \text{ mm}$$

Step 3: No. of bolts, $i = 0.02d + 3 = 3.46 \simeq 4$ bolts

Step 4: Bolt circle diameter, $D_1 = 2d + 50 = 96$ mm

Step 5: Bolt diameter, $d_1 = \frac{0.5d}{\sqrt{i}} = 5.75 \simeq 6$ mm

Take standard diameter for bolt = $d_1 = 10$ mm for safety

Step 6: Assume bigger dia of pin, $d_2 = 2d_1 = 20$ (slightly bigger value)

Step 7: Assume 2 mm thick brass bush and 10 mm thick rubber bush, outside dia of rubber bush $d' = d_2 + (2 + 10)$

$$d' = 20 + 2(2 + 10) = 44 \text{ mm}$$

Step 8: Force at each pin, $F = \frac{2M_t}{iD_1} = 484.96$ N

Step 9: But $F = p_b l' d'$

$\therefore$ length of bush, $l' = \frac{F}{d' p_b} = \frac{484.96}{36 \times 1} = 13.47 \simeq 14$ mm

Step 10: Check for stress in pin

(a) Shear stress in pin, $\tau_p = \frac{F}{\left(\frac{\pi}{4} d_1^2\right)}$

$$\tau_p = 6.18 \text{ MPa}$$

(b) Bending stress in pin, $\sigma_b = \frac{F\left(\frac{l'}{2} + b_1\right)}{\frac{\pi}{32} d_1^3}$

$$\sigma_b = \frac{484.96 \times \left(\frac{14}{2} + 2\right)}{\frac{\pi}{32} 10^3} = 44.46 \text{ MPa}$$

(c) Combined stresses

(i) Maximum normal stress

$$s_{max} = \frac{1}{2}\left[\sigma_b + \left(\sigma_b^2 + 4\tau_p^2\right)^{1/2}\right] = 45.3 \text{ MPa}$$

This is less than s_b allowable.

(ii) Maximum shear stress

$$\tau_{max} = \frac{1}{2}\left(\sigma_b^2 + 4\tau_p^2\right)^{\frac{1}{2}} = 23.07 \text{ MPa}$$

This is also less than t_b allowable. Hence the pin is safe.

Step 11: Design of Hub:

(a) Hub diameter, $D_2 = 1.5d + 25$

$$D_2 = 1.5 \times 23 \times 25 = 59.5 \simeq 60 \text{ mm}$$

(b) Hub length, $l = 1.25d + 18.75$

$$l = 47.5 \simeq 48 \text{ mm}$$

Step 12: Design of flange:

(a) Outside diameter of flange, $D = 2.5d + 75$

$$D = 2.5 \times 23 \times 75 = 132.5 \simeq 133 \text{ mm}$$

(b) Thickness of flange, t = length of bush l¢ = 14 mm

Step 13: Design of key:

(a) Length of key, $l = 48$ mm

(b) Width of key, $b = \dfrac{2M_t}{dl\tau_K} = 3.06$ mm

(c) Thickness of key, $h = \dfrac{4M_t}{dl\sigma_b'} = 3.06$ mm

Select standard key, $b = 3$ mm

$h = 3$ mm. (from Table 7.1).

Problem 14: A pin type flexible coupling has six bolts of $M12 \times 1$. The rubber bush has a diameter of 40 mm length of 30 mm. The bolt circle diameter is 180 mm. The allowable shear stress and normal stress for bolt are 60 MPa and 120 MPa respectively. Find the power that can be transmitted by this pin. Take allowable bearing pressure for rubber bush as 0.8 MPa. The shaft diameter = 24 mm and rotates at 900 rpm.

Given data:

$i = 6$, $d_1 = 12$ mm, $d' = 40$ mm

$l' = 30$ mm, $D_1 = 180$ mm, $d = 24$ mm

$\tau_{b\,max} = 60$ MPa, $d_1 = 12$ mm, $\sigma_{b\,max} = 120$ MPa.

$p_b = 0.8$ MPa, $n = 900$ rpm

(1) *Torque capacity of shaft* $M_t = \frac{\pi d^3}{16}\tau_s \eta$

Assume $\tau = 55$ MPa, $\eta = 0.75$ key way factor

$$M_{t_S} = \frac{\pi}{16} 20^3 (55) \times 0.75$$

$$= 64795.35 \text{ N-mm} \quad ...(1)$$

(2) *Torque capacity based on pin (bolt)*

$$M_t = iF \frac{D_1}{2}$$

To find, F from combined stress

(a) Shear stress in pin, $\tau_p = \frac{F}{0.784 d_1^2}$

$$\tau_p = 8.86 \times 10^{-3} \text{ F}$$

(b) Bending stress in pin, $\sigma_b = \frac{F\left(\frac{l'}{2} + b_1\right)}{\frac{\pi}{32} d_1^3}$

$$= \frac{F\left(\frac{30}{2} + 2\right)}{\frac{\pi}{32} \times 12^3} = 0.10 \text{ F}$$

(c) Combined or principal stresses

(i) Max normal stress, $\tau_{b\max} = \frac{1}{2}(\sigma_b + (\sigma_b^2 + 4\tau_p^2)^{1/2})$

$$120 = \frac{1}{2} [0.1 \text{ F} + (0.1 \text{ F})^2 + (8.86 \times 10^{-3} \text{ F})^2)^{1/2}]$$

$$F = 1197.65 \text{ N}$$

(ii) Max shear stress, $\tau_{b\max} = \frac{1}{2}(\sigma_b^2 + 4\tau_p^2)^{1/2}$

$$60 = \frac{1}{2}((0.1 \text{ F})^2 + (8.86 \times 10^{-3} \text{ F})^2)^{1/2}$$

$$F = 1195.32 \text{ N}$$

From rubber bush.

$$F = p_b l' d' = 0.8 \times 30 \times 40 = 960 \text{ N}$$

Taking the least allowable force $F = 960$,

$$M_t = i F \frac{D_1}{2} = 6 \times 960 \times \frac{180}{2} = 518400 \text{ N-mm}$$

$\therefore$ Maximum torque that can be transmitted by this coupling = least of M_t

= 64795.35 Nmm

$$M_t = \frac{9.55 \times 10^6 P}{n}$$

$\therefore$

$$P = \frac{64795.35 \times 900}{9.55 \times 10^6} = 6.11 \text{ kW}$$

EXERCISE

1. Design a rectangular key to transmit 12 kW at 1440 rpm.
2. Find the power capacity a keyed shaft 50 mm diameter fitted with a rectangular key of 8 × 7 × 42. The allowable stresses may be taken as 60 MPa in shear and 110 MPa in crushing.
3. Design a muff coupling to transmit 6.5 kW at 1000 rpm. Assume suitable stress for the shaft, key and muff.
4. Design a protected type CI flange coupling to transmit 9 kW at 1500 rpm. The maximum torque to be transmitted is 20% greater than mean torque. Assume allowable shear stress for shaft, bolts and keys is 72 MPa and allowable crushing stress for key is 150 MPa.
5. Design solid flange coupling to transmit a torque of 900 N-mm
6. Design a pin type flexible coupling to transmit 12.5 kW at 720 rpm.
7. A m.s. shaft has to transmit 75 kW at 600 rpm. The allowable shear stress for shaft meterial is 60 MPa. Angle of twist is not to exceed 0.9°/m length. Design a C.I. Flange couping for this shaft. The allowable shear stress for bolts is 45 MPa, for key is 50 MPa and for CI flage is 15 MPa.

   (**Hint:** Find diameter of shaft $d = \sqrt[3]{\frac{16 M_t}{\pi \tau_s \eta}}$

   and d from, $\frac{M_t}{J} = \frac{G\theta}{l} \Rightarrow d = ?$

   use the bigger value of d for further calculations.)

CHAPTER

8 Power Screws

INTRODUCTION

Screws used for power transmission are known as power screws. They provide a means for obtaining large mechanical advantage, in such applications as screw jacks, *c*-lamps, hand presses land screw on lathe machines, hoisting machines etc.

There are two types of thread forms that are commonly used for power screws, namely, square thread and angular thread. Angular thread includes acme thread and ISO trapezoidal thread. ...(P 18.7)

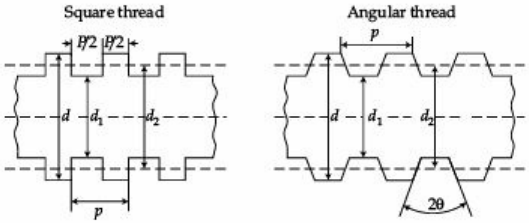


Fig. 8.1

d = outside diameter or nominal diameter of thread, mm
p = pitch distance between two consecutive threads, mm
d_1 = root diameter or inside diameter of thread, mm
d_2 = mean diameter of thread, mm

For standard screw, $d_1 = d - p$

$$d_2 = \frac{d + d_1}{2} = d - \frac{p}{2}$$

For angular thread, 2θ = total V – angle

For acme threaded screw, $2\theta = 29°$

For trapezoidal threaded screw $2\theta = 30°$

l = lead of thread = distance moved in one revolution, mm

= no. of starts × pitch

= $1 \times p$ for single start thread

= $2 \times p$ for double start thread

= $3 \times p$ for triple start thread etc.

α = lead angle or helix angle, degrees

$$\tan \alpha = \frac{l}{\pi d_2} \quad \text{...(18-26/P 18.8)}$$

W = axial load on the screw, N

μ = Coefficient of thread friction, depends on the type of lubricant

μ = 0.07 for machine oil or graphite

= 0.11 for lard oil

= 0.14 for heavy machine oil ...(T 18-4/P 18.8)

μ_c = coefficient of collar friction, depends on material combination of screw and trust collar

μ_c = 0.17 for soft steel screw and C.I. collar ...(T 18-5 a/P 18.9)

= 0.147 for hardened steel screw and C.I. collar

= 0.1 for soft steel screw and bronze collar

= 0.08 for hardened steel screw and bronze collar (starting friction)

D_o = outside diameter of thrust collar, mm

D_i = inside diameter of thrust collar, mm

d_c = mean diameter of thrust collar = $\frac{D_o + D_i}{2}$ mm

M_t = total friction of torque inducing collar friction, N-mm

P = power, in kW

n = speed of screw, rpm

η = efficiency,

l_n = nut length or height of nut, mm

$P_b = \sigma_b'$ = allowable bearing stress in threads

The value of σ'_b depends on the type of application and combination nut and screw material, N/mm^2 ...(T 18-6/P 18.15)

σ'_b = 18 – 24 MPa, for hand press with steel screw and bronze nut,

= 13 – 17 MPa for screw jack with steel screw and C.I. nut.

= 11 – 17 MPa for screw jack with steel screw and bronze nut.

= 4 – 7 MPa for hoisting screw with steel screw and C.I. nut.

= 5 – 10 MPa for hoisting screw with steel screw and bronze nut.

= 1 – 1.5 MPa for lead screw with steel screw and bronze nut.

8.1 SQUARE THREADED SCREWS

(a) Total frictional torque inducing collar friction

$$M_t = W\left[\frac{d_2}{2}\left(\frac{\tan\alpha+\mu}{1-\mu\tan\alpha}\right)+\frac{\mu_c d_c}{2}\right] \quad ...(18\text{-}29/P\ 18.8)$$

(b) Efficiency, $\eta = \dfrac{d_2\tan\alpha}{\left(\dfrac{\tan\alpha+\mu}{1-\mu\tan\alpha}\right)d_2+\mu_c d_c}$ ...(18-33/P 18.10)

(c) Power of motor required to drive the screw

$$P = \frac{M_t \cdot n}{9.55\times10^6}\ \text{kW}$$

(d) Length of nut, $l_n = \dfrac{4Wp}{\sigma'_b\pi(d^2-d_1^2)}$ ...(18-41)

(e) Condition for overhauling

$$\tan\alpha \geq \frac{\mu d_2+\mu_c d_c}{d_2-\mu\mu_c d_c} \quad ...(18\text{-}37)$$

Overhauling is a condition that the load starts moving downward under gravity, without any external load, in power screws. Self locking is opposite of overhauling. The load does not move down under gravity and force has to be applied to move the load down the inclined plane (thread).

If the friction angle is more than lead angle, then the screw is self locking type and if friction angle is less than lead angle, then the screw is known as overhauling type (not self locking).

(f) Compressive stress in the screw

$$\sigma_c = \frac{W}{A_c} = \frac{W}{\left(\dfrac{\pi}{4}d_1^2\right)}\ \text{MPa}$$

(g) Shear stress in the screw

$$\tau_s = \frac{M_t \cdot r_1}{J} \text{ MPa}$$

where $r_1 = \frac{d_1}{2}$ and $J = \frac{\pi}{32} d_1^4$

(h) Combined stresses in the screw

(i) Maximum normal stress

$$\sigma_{max} = \frac{1}{2}[\sigma_c + (\sigma_c^2 + 4\tau_s^2)^{1/2}]$$

(ii) Maximum shear stress

$$\tau_{max} = \frac{1}{2}[\sigma_c^2 + 4\tau_s^2]^{1/2}.$$

8.2 ANGULAR THREAD SCREWS

(a) Total frictional torque including collar friction

$$M_t = W\left[\frac{d_2}{2}\left(\frac{\tan\alpha + \frac{\mu}{\cos\theta}}{1 - \frac{\mu\tan\alpha}{\cos\theta}}\right) + \frac{\mu_c d_c}{2}\right] \quad \text{...(18-30 a/P 18-10)}$$

(b) Efficiency

$$\eta = \frac{d_2 \tan\alpha}{\left(\frac{\tan\alpha + \frac{\mu}{\cos\theta}}{1 - \frac{\mu\tan\theta}{\cos\theta}}\right) d_2 + \mu_c d_c} \quad \text{...(18-32)}$$

(c) Condition for overhauling

$$\tan\alpha \geq \left[\frac{\frac{\mu}{\cos\theta} \cdot d_2 + \mu_c d_c}{d_2 - \frac{\mu\mu_c d_c}{\cos\theta}}\right]$$

All other calculations are same as square threaded screw mentioned in 8.1 above.

PROBLEMS

Problem 1: A power transmission screw having a square thread of 30 × 6 propells a weight of 20 kN at a speed of 3 m/min. The collar has an inside diameter of 30 mm and outside diameter of 60 mm. The coefficient of thread friction is 0.15 and collar friction is 0.20. Determine:

(i) The power of motor required to drive the screw,

(ii) The efficiency,

(iii) Length of bronze nut required if the allowable bearing pressure is 12 MPa, and

(iv) The nature and magnitude of stresses induced in the screw.

Given data: Diameter, $d = 30$ mm

Pitch, $p = 6$ mm

$\therefore$ inside diameter of screw, $d_1 = d - p = 24$ mm

Mean diameter of screw, $d_2 = d - \frac{1}{2} p = 27$ mm

Outside diameter of collar, $D_o = 60$ mm

Inside diameter of collar, $D_i = 30$ mm

$\therefore$ Mean collar diameter, $d_c = \frac{D_o + D_i}{2} = 45$ mm

Load, $W = 20$ kN $= 20 \times 10^3$ N

Velocity, $V = 3$ m/min $= 3000$ mm/min

Coefficient of thread friction $\mu = 0.15$

Coefficient of collar friction, $\mu_c = 0.2$

Allowable bearing pressure for nut, $\sigma_b' = 12$ MPa

Assume single start thread, $\therefore$ lead, $l = 1 \times p = 6$ mm

$$\text{Lead angle } \tan \alpha = \frac{l}{\pi d_2} = \frac{6}{\pi \times 27} = 0.0707$$

$$\text{Speed of screw, } n = \frac{V}{l} = \frac{3000}{6} = 500 \text{ rpm}$$

(Lead is distance moved in one revolution of screw

For $l = 6$ mm, 1 rev.

For $V = 3000$ mm/min, speed $n = \frac{V}{l}$ rpm)

(i) To find power

Total frictional torque including collar friction for square threads

$$M_t = W\left[\frac{d_2}{2}\left(\frac{\tan\alpha+\mu}{1-\mu\tan\alpha}\right)+\frac{\mu_c d_c}{2}\right]$$

$$= 20\times10^3\left[\frac{27}{2}\left(\frac{0.0707+0.15}{1-0.15\times0.0707}\right)+\frac{0.2\times45}{2}\right]$$

$$= 150.03\times10^3 \text{ N-mm} = 150.03 \text{ Nm}$$

$\therefore$ Power, $P = \dfrac{2\pi n M_t}{60000} = \dfrac{2\times\pi\times500\times150.03}{60000} = 7.86$ kW

(ii) Efficiency, $\eta = \dfrac{d_2\tan\alpha}{\left(\dfrac{\tan\alpha+\mu}{1-\mu\tan\alpha}\right)d_2+\mu_c d_c}$

$$= \frac{27\times0.0707}{\left(\dfrac{0.0707+0.15}{1-0.15\times0.0707}\right)27+0.2\times45} = 0.1271 = 12.71\%$$

(iii) Length of nut, $\ln = \dfrac{4Wp}{\sigma_b'\pi(d^2-d_1^2)} = \dfrac{4\times20\times10^3\times6}{12\times\pi(30^2-24^2)} = 39.298$

$\ln \approx 40$ mm.

(iv) Nature and magnitude of stresses

(a) Compressive stress in the screw, $\sigma_c = \dfrac{W}{A_c} = \dfrac{W}{\left(\dfrac{\pi}{4}d_1^2\right)}$

$$\sigma_c = \frac{20\times10^3}{\left(\dfrac{\pi}{4}\times24^2\right)} = 44.21 \text{ MPa}$$

(b) Shear stress due to torque

$$\tau_s = \frac{M_t\cdot r_1}{J} \quad \text{where} \quad r_1 = \frac{d_1}{2} = 12 \text{ mm and}$$

$$J = \frac{\pi d_1^4}{32} = \frac{\pi\times24^4}{32} \text{ mm}^4$$

$$\therefore \quad \tau_s = \frac{150.03 \times 10^3 \times 12}{\left(\frac{\pi \times 24^4}{32}\right)} = 55.27 \text{ MPa}$$

(c) Principal or combined stresses

(i) Maximum normal stress

$$\sigma_{max} = \frac{1}{2}[\sigma_c + (\sigma_c^2 + 4\tau_s^2)^{1/2}]$$

$$= \frac{1}{2}[44.21 + (44.21^2 + 4 \times 55.27^2)^{1/2}] = 81.7 \text{ MPa}$$

(ii) Maximum shear stress

$$\sigma_{max} = \frac{1}{2}(\sigma_c^2 + 4\tau_s^2)^{1/2} = \frac{1}{2}(44.21^2 \times 4 \times 55.27^2)^{1/2}]$$

$$= 59.6 \text{ MPa}$$

Problem 2: A square threaded power screw has a nominal diameter of 44 mm and a pitch of 7 mm with double threads. The load on the screws 6 kN and mean diameter of the thrust washer is 50 mm. Coefficient of friction is 0.12 determine:

(i) Torque required to raise the load

(ii) Torque required to lower the load

(iii) Efficiency and

(iv) Is the screw of self locking type?

Given data: Square thread, diameter, $d = 44$ mm

Pitch, $p = 7$ mm

$\therefore$ Mean diameter of thread, $d_2 = d - \frac{1}{2} P$

$$d_2 = 44 - \frac{7}{2} = 40.5 \text{ mm}$$

Double threads, $\therefore$ lead, $l = 2 \times p = 14$ mm

Load, $W = 6000$ N

Mean diameter of thrust washer, $d_c = 50$ mm

Coefficient of friction, $\mu = \mu_c = 0.12$

$$\text{Load angle } \tan\alpha = \frac{l}{\pi d_2} = \frac{14}{\pi \times 40.5} = 0.11$$

(i) Torque required to raise the load = Total frictional torque including collar friction

$$M_t = W\left[\frac{d_2}{2}\left(\frac{\tan\alpha + \mu}{1 - \mu\tan\alpha}\right) + \frac{\mu_c d_c}{2}\right]$$

$$= 6000\left[\frac{40.5}{2}\left(\frac{0.11 + 0.12}{1 - 0.12 \times 0.11}\right) + \frac{0.12 \times 50}{2}\right]$$

$$= 46.32 \times 10^3 \text{ N-mm.}$$

(ii) Torque required to lower the load

$$M_t' = W\left[\frac{d_2}{2}\left(\frac{\mu - \tan\alpha}{1 + \mu\tan\alpha}\right) + \frac{\mu_c d_c}{2}\right]$$

$$= 6000\left[\frac{40.5}{2}\left(\frac{0.12 - 0.11}{1 + 0.12 \times 0.11}\right) + \frac{0.12 \times 50}{2}\right]$$

$$M_t' = 19.2 \times 10^3 \text{ N-mm}$$

(iii) Overall efficiency, $\eta = \dfrac{d_2 \tan\alpha}{\left(\dfrac{\tan\alpha + \mu}{1 - \mu\tan\alpha}\right) d_2 + \mu_c d_c}$

$$\eta = \frac{40.5 \times 0.11}{\left(\dfrac{0.11 + 0.12}{1 - 0.12 \times 0.11}\right) 40.5 + 0.12 \times 50} = 0.2885 = 28.85\%$$

(iv) Check for self locking

Condition for self locking of square threaded screw:

$$\tan\alpha \le \frac{\mu d_2 + \mu_c d_c}{d_2 - \mu\mu_c d_c}$$

LHS $\tan \alpha = 0.11$

$$\text{RHS} = \frac{0.12 \times 40.5 + 0.12 \times 50}{40.5 - 0.12 \times 0.12 \times 50} = 0.273$$

Since LHS < RHS, the screw is self locking and not overhauling.

Problem 3: The square thread of a screw jack has a specification of 80×16 and is to raise a load of 100 kN. The mean radius of the thrust collar is 65 mm. The coefficient of friction for the threads and collar are 0.1 and 0.12 respectively. Determine:

(i) The torque required to raise the load,

(ii) The overall efficiency, and

(iii) Does the screw overhaul under load. Comment.

Given data: Square thread diameter, $d = 80$ mm

Pitch, $p = 16$ mm

$\therefore$ Mean diameter of screw, $d_2 = d - \frac{1}{2}p$

$$d_2 = 80 - \frac{1}{2} \times 16 = 72 \text{ mm}$$

Mean collar radius, $r_c = 65 \text{ mm} = \frac{d_c}{2}$

$\therefore$ Mean collar diameter, $d_c = 2 \times 65 = 130$ mm

Lead, $W = 100 \text{ kN} = 100 \times 10^3 \text{ N}$

Coefficient of thread friction, $\mu = 0.1$

Collar friction, $\mu_c = 0.12$

Assume single start, $\therefore$ lead, $l = p = 16$ mm

$$\tan \alpha = \frac{l}{\pi d_2} = \frac{16}{\pi \times 72} = 0.0707$$

(i) Torque required to raise the load

$$M_t = W\left[\frac{d_2}{2}\left(\frac{\tan \alpha + \mu}{1 - \mu \tan \alpha}\right) + \frac{\mu_c d_c}{2}\right]$$

$$= 100 \times 10^3 \left[\frac{72}{2} \left(\frac{0.0707 + 0.1}{1 - 0.1 \times 0.0707} \right) + \frac{0.12 \times 130}{2} \right]$$

$$= 1.399 \times 10^6 \text{ N-mm}$$

(ii) Efficiency

$$\eta = \frac{d_2 \tan \alpha}{\left(\frac{\tan \alpha + \mu}{1 - \mu \tan \alpha} \right) d_2 + \mu_c d_c}$$

$$\eta = \frac{72 \times 0.0707}{\left(\frac{0.0707 + 0.1}{1 - 0.1 \times 0.0707} \right) 72 \times 0.12 \times 130} = 0.1819 = 18.19\%$$

(iii) Condition for overhauling of square threads

$$\tan \alpha \geq \frac{\mu d_2 + \mu_c d_c}{d_2 - \mu \mu_c d_c}$$

LHS $\tan \alpha = 0.0707$

$$\text{RHS} = \frac{0.1 \times 72 + 0.12 \times 130}{72 - 0.1 \times 0.12 \times 130} = 0.3237$$

Since LHS is < RHS, the screw does not overhaul under load i.e., the screw is self locking and safe.

Problem 4: The cutter of a broaching machine is pulled by square threaded screw of 50 mm external diameter and 8 mm pitch. The operating nut takes a load of 42 kN of a flat surface of 84 mm external diameter and 56 mm internal diameter coefficient of friction is 0.15 for all contact surfaces. Determine the power required to rotate the operating nut, when the cutting speed is 15 m/min. Also find the efficiency of the screw.

Given data: Square thread, diameter, $d = 50$ mm

Pitch, $p = 8$ mm

Load, $W = 42$ kN $= 42 \times 10^3$ N

Outside dia of collar, $D_o = 84$ mm

Inside dia of collar, $D_i = 56$ mm

$\therefore$ Mean dia of collar, $d_c = \frac{D_o + D_i}{2} = \frac{84 + 56}{2} = 70$ mm

Coefficient of friction, $\mu = \mu_c = 0.15$

Velocity, $V = 15$ m/min $= 15000$ mm/min

Mean diameter of screw, $d_2 = d - \frac{1}{2}p = 46$ mm

Assume single start, $l = p = 8$ mm

$$\tan\alpha = \frac{l}{\pi d_2} = \frac{8}{\pi \times 46} = 0.0554$$

$$\text{Speed of screw, } n = \frac{V}{l} = \frac{15000}{8} = 1875 \text{ rpm}$$

(i) Power

Total frictional torque including collar friction

$$M_t = W\left[\frac{d_2}{2}\left(\frac{\tan\alpha + \mu}{1 - \mu\tan\alpha}\right) + \frac{\mu_c d_c}{2}\right]$$

$$= 42 \times 10^3\left[\frac{46}{2}\left(\frac{0.0554 + 0.15}{1 - 0.15 \times 0.0554}\right) + \frac{0.15 \times 70}{2}\right]$$

$$M_t = 420.579 \times 10^3 \text{ N-mm}$$

But $$M_t = \frac{9.55 \times 10^6 P}{n}$$

$$\text{Power, } P = \frac{M_t \cdot n}{9.55 \times 10^6} = \frac{420.579 \times 10^3 \times 1875}{9.55 \times 10^6}$$

$$P = 82.57 \text{ kW}$$

(ii) Efficiency

$$\eta = \frac{d_2 \tan\alpha}{\left(\frac{\tan\alpha + \mu}{1 - \mu\tan\alpha}\right)d_c + \mu_c d_c}$$

$$\eta = \frac{46 \times 0.0554}{\left(\frac{0.0554 + 0.15}{1 - 0.15 \times 0.0554}\right)46 + 0.15 \times 70} = 0.1272$$

$$\eta = 12.72\%.$$

Problem 5: A power screw has 6 mm pitch and 40 mm diameter. The screw is subjected to an axial load of 6 kN. The length of nut is 12 mm. Determine:

(i) The bearing pressure between the threads,

(ii) The shear stress on the threads, and

(iii) The compressive stress in the screw.

Given data: Square thread diameter, $d = 40$ mm

Pitch, $p = 6$ mm

Load, $W = 6000$ N

Nut length, $l_n = 12$ mm

Inside diameter of thread, $d_1 = d - p = 34$ mm

Mean diameter of thread, $d_2 = d - \frac{1}{2}p = 37$ mm

(i) Bearing pressure

$$\text{Nut length, } ln = \frac{4Wp}{\sigma_b' \pi (d^2 - d_1^2)}$$

$$12 = \frac{4 \times 6000 \times 6}{\sigma_b' \pi (40^2 - 34^2)}$$

$\therefore$ bearing pressure, $\sigma_b' = 8.603$ MPa

(ii) Transverse shear stress on threads

$$\tau = \frac{W}{\pi d_2 t i}$$

t = thickness of thread $= \frac{p}{2} = \frac{6}{2} = 3$ mm

i = no. of threads $= \frac{l_n}{p} = \frac{12}{6} = 2$

$\therefore$ $$\tau = \frac{6000}{\pi \times 37 \times 3 \times 2} = 8.603 \text{ MPa}$$

(iii) Compressive stress in the screw, $\sigma_c = \frac{W}{\left(\frac{\pi}{4} d_1^2\right)}$

$$\sigma_c = \frac{6000}{\frac{\pi}{4} \times 34^2} = 6.61 \text{ MPa.}$$

Problem 6: A sluice gate weighing 500 kN is raised at a speed of 6m/min by two screw rods with square threads 50 × 8 mm. The two screw rods are driven by bevel gears and motor. Determine:

(i) The torque required to raise the gate,

(ii) The speed of rotation of the screw rod assuming the threads are triple start,

(iii) The maximum stresses induced in the screw,

(iv) The efficiency of the screw,

(v) The length of nuts required to support the load taking allowable bearing pressure as 12 MPa,

(vi) Check for overhaul.

Given data: Total weight of gate = 500 kN = 500×10^3 N

No. of screws = 2

$\therefore$ Load on each screw, $W = \dfrac{500 \times 10^3}{2} = 250 \times 10^3$ N

Speed, $V = 6$ m/min = 6000 mm/min.

Square thread, diameter, $d = 50$ mm

pitch, $p = 8$ mm

triple start, $\therefore$ lead, $l = 3p = 24$ mm

mean dia of thread, $d_2 = d - \dfrac{1}{2}p = 46$ mm

inside dia of thread, $d_1 = d - p = 42$ mm

Details of collar not given, hence assume that there is no collar.

Select $\mu = 0.11$ assuming lard oil for lubrication

$$\tan \alpha = \frac{l}{\pi d_2} = \frac{24}{\pi \times 46} = 0.1661$$

$$\text{speed of screw, } n = \frac{V}{l} = \frac{6000}{24} = 250 \text{ rpm}$$

Bearing pressure, $\sigma'_b = 12$ MPa.

Frictional torque in the screw neglecting collar friction,

$$M_t = W \frac{d_2}{2} \left(\frac{\tan \alpha + \mu}{1 - \mu \tan \alpha} \right)$$

$$= 250 \times 10^3 \times \frac{46}{2} \left(\frac{0.1661 + 0.11}{1 - 0.11 \times 0.1661} \right)$$

$$M_t = 1.617 \times 10^6 \text{ N-mm}$$

(i) $\therefore$ Total torque required to raise the total load of 500 kN,

$$M_t = 2 \times M_t = 3.2342 \times 10^6 \text{ N-mm}$$

$$\text{Power of motor, } P = \frac{M_t \cdot n}{9.55 \times 10^6} = \frac{3.2342 \times 10^6 \times 250}{9.55 \times 10^6}$$

$$P = 84.665 \text{ kW}$$

(ii) Speed of rotation of screw, $n = 250$ rpm

(iii) Stresses induced in screw

(a) Compressive stress, $\sigma_c = \dfrac{W}{A_c} = \dfrac{250 \times 10^3}{\left(\dfrac{\pi}{4} \times 42^2 \right)} = 180.45$ MPa

(b) Shear stress in the screw, $\tau_s = \dfrac{M_t \cdot r_1}{J}$

$$\tau_s = \frac{1.617 \times 10^6 \times \dfrac{d_1}{2}}{\left(\dfrac{\pi d_1^4}{32}\right)} = \frac{1.617 \times 10^6 \times \left(\dfrac{42}{2}\right)}{\left(\dfrac{\pi}{32} 42^4\right)} = 111.16 \text{ MPa}$$

(c) Combined stresses

(1) Maximum principal stress

$$\sigma_{max} = \frac{1}{2}[\sigma_c + (\sigma_c + 4\tau_s^2)^{1/2}]$$

$$= \frac{1}{2}[180.45 + (180.45^2 + 4 \times 111.16^2)^{1/2}]$$

$$\sigma_{max} = 233.39 \text{ MPa}$$

(2) Maximum shear stress, $\tau_{max} = \dfrac{1}{2}(\sigma_c^2 + 4\tau_s^2)^{1/2}$

$$= \frac{1}{2}(180.45^2 + 4 \times 111.16^2)^{1/2} = 143.168 \text{ MPa}$$

(iv) Efficiency of the screw, $\eta = \dfrac{d_2 \tan \alpha}{\left(\dfrac{\tan \alpha + \mu}{1 - \mu \tan \alpha}\right) d_2 + \mu_c d_c}$

$$\eta = \frac{46 \times 0.1661}{\left(\dfrac{0.1661 + 0.11}{1 - 0.11 \times 0.1661}\right) 46 + 0} = 0.5906 = 59.06\%$$

(v) Length of nut, $\quad ln = \dfrac{4Wp}{\sigma_b' \pi (d^2 - d_1^2)}$

$$ln = \frac{4 \times 250 \times 10^3 \times 8}{12\pi(50^2 - 42^2)} = 288.32 \text{ mm} \approx 289 \text{ mm}$$

(vi) Check for overhauling

$$\tan \alpha \geq \left(\frac{\mu d_2 + \mu_c d_c}{d_2 - \mu \mu_c d_c}\right)$$

since μ_c and d_c are taken $= 0$

$$\tan \alpha \geq \mu$$

LHS, $\tan \alpha = 0.1661$

RHS $\mu = 0.11$

Since LHS > RHS, the screw overhauls and is not self locking.

Problem 7: A bench vice is operated by a square threaded screw of 24 mm diameter and 5 mm pitch. A force of 180 N is applied at the end of the lever of length 200 mm. The mean diameter of the collar is 30 mm. Coefficients of thread friction and collar friction are 0.12 and 0.15 respectively. Determine the clamping force developed and the efficiency.

Given data: Square thread diameter, $d = 24$ mm

pitch, $p = 5$ mm

$\therefore$ Mean diameter of thread, $d_2 = d - \frac{1}{2}p$

$$d_2 = 24 - \frac{5}{2} = 21.5 \text{ mm}$$

Assume single start lead, $l = p = 5$ mm

Mean collar diameter, $d_c = 30$ mm

$\mu = 0.12, \quad \mu_c = 0.15$

Operating force, $F = 180$ N

Length of lever, $L = 200$ mm

$\therefore$ Applied torque, $M_t = F \cdot L = 180 \times 200 = 36 \times 10^3$ N-mm

$$\tan \alpha = \frac{l}{\pi d_2} = \frac{5}{\pi \times 21.5} = 0.074$$

Total frictional torque

$$M_t = W\left[\frac{d_2}{2}\left(\frac{\tan \alpha + \mu}{1 - \mu \tan \alpha}\right) + \frac{\mu_c d_c}{2}\right]$$

$$36 \times 10^3 = W\left[\frac{21.5}{2}\left(\frac{0.074 + 0.12}{1 - 0.12 \times 0.074}\right) + \frac{0.15 \times 30}{2}\right]$$

$\therefore$ Clamping force, $W = 8267.9$ N

$$\text{Efficiency, } \eta = \frac{d_2 \tan \alpha}{\left(\frac{\tan \alpha + \mu}{1 - \mu \tan \alpha}\right) d_2 + \mu_c d_c}$$

$$= \frac{21.5 \times 0.074}{\left(\frac{0.074 + 0.12}{1 - 0.12 \times 0.074}\right) 21.5 + 0.15 \times 30}$$

$$\eta = 0.1827 = 18.27\%$$

Problem 8: A machine slide weighing 3 kN is elevated by a double start acme threaded screw at a rate of 0.84 m/s. The coefficient of thread friction is 0.15 and collar friction is 0.2. The end of the screw is carried on a thrust collar of 32 mm inside diameter and 58 mm outside the diameter. The screw has a diameter of 44 mm and pitch of 7 mm. If the screw is made of 45 C8 steel (σ_y = 353 MPa), is it strong enough to sustain the load. Also, find the efficiency and check for self locking. What is the power of motor required to drive the screw?

Given data: Weight, $W = 3000$ N

Double start $\therefore$ lead, $l = 2p$

Velocity, $V = 0.84$ m/s = 840 mm/sec.

Coeff. of thread friction, $\mu = 0.15$

Collar friction, $\mu_c = 0.2$

Inside diameter of thrust collar, $D_i = 32$ mm

Outside diameter of thrust collar, $D_o = 58$ mm

$\therefore$ Mean diameter of thrust collar, $d_c = \dfrac{D_i + D_o}{2}$

$$d_c = \frac{32 + 58}{2} = 45 \text{ mm}$$

$\sigma_y = 353$ MPa. Assume FOS = 2.5

$\therefore$ Allowable normal stress, $\sigma_{max} = \dfrac{\sigma_y}{FOS} = \dfrac{353}{2.5} = 141.2$ MPa

Allowable shear stress, $\tau_{max} = 0.5\,\sigma_{max} = 70.6$ MPa

Screw diameter, $d = 44$ mm

pitch, $p = 7$ mm

acme thread, $\therefore$ V-angle $2\theta = 29°$

$\theta = 14.5°$

Mean diameter of thread, $d_2 = d - \dfrac{1}{2}p$

$$d_2 = 44 - \frac{7}{2} = 40.5 \text{ mm}$$

$l = 2p = 2 \times 7 = 14$ mm (double start)

$$\tan\alpha = \frac{l}{\pi d_2} = \frac{14}{\pi \times 40.5} = 0.11$$

Root diameter of thread, $d_1 = d - p = 44 - 7 = 37$ mm

(i) Compressive stress in the screw, $\sigma_c = \dfrac{W}{A_c} = \dfrac{W}{\left(\dfrac{\pi}{4} d_1^2\right)}$

$$\sigma_c = \frac{3000}{\dfrac{\pi}{4} \times 37^2} = 2.79 \text{ MPa}$$

(ii) Shear stress in the screw, $\tau_s = \dfrac{M_t \cdot r_1}{J} = \dfrac{M_t \cdot \dfrac{d_1}{2}}{\left(\dfrac{\pi d_1^4}{32}\right)} = \dfrac{16 M_t}{\pi d_1^3}$

For angular threads, $M_t = W\left[\dfrac{d_2}{2}\left(\dfrac{\tan\alpha + \mu/\cos\theta}{1 - \dfrac{\mu \tan\alpha}{\cos\theta}}\right) + \dfrac{\mu_c d_c}{2}\right]$

$$= 3000\left[\frac{40.5}{2}\left(\frac{0.11 + 0.15/\cos 14.5}{1 - \dfrac{0.11 \times 0.15}{\cos 14.5}}\right) + \frac{0.2 \times 45}{2}\right]$$

$$= 29.874 \times 10^3 \text{ N-mm}$$

$\therefore$ $\tau_s = \dfrac{29.874 \times 10^3 \times 16}{\pi \times 37^3} = 3 \text{ MPa}$

(iii) Combined or principal stresses

(a) Maximum principal stress

$$\sigma_{max} = \frac{1}{2}[\sigma_c + (\sigma_c^2 + 4\tau_s^2)^{1/2}]$$

$$= \frac{1}{2}[2.79 + (2.79^2 + 4 \times 3^2)^{1/2}] = 4.7$$

This is very much less than σ_{max} allowable value of 141.2 MPa. Hence, safe.

(b) Maximum shear stress

$$\tau_{max} = \frac{1}{2}(\sigma_c^2 + 4\tau_s^2)^{1/2} = \frac{1}{2}(2.79^2 + 4 \times 3^2)^{1/2}$$

$$= 3.31 \text{ MPa} < \tau_{allowable} (70.6 \text{ MPa})$$

Hence, the selection is proper.

$$\text{Efficiency, } \eta = \frac{d_2 \tan \alpha}{\left(\frac{\tan \alpha + \frac{\mu}{\cos \theta}}{1 - \frac{\mu \tan \alpha}{\cos \theta}}\right) d_2 + \mu_c d_c}$$

$$\eta = \frac{40.5 \times 0.11}{\left(\frac{0.11 + 0.15/\cos 14.5}{1 - 0.15 \times 0.11/\cos 14.5}\right) 40.5 + 0.2 \times 45}$$

$$= 0.2237 = 22.37\%$$

Check for self locking: Condition for self locking of angular thread

$$\tan \alpha \le \left[\frac{\frac{\mu d_2}{\cos \theta} + \mu_c d_c}{\frac{d_2 - \mu \mu_c d_c}{\cos \theta}}\right]$$

LHS $\tan \alpha = 0.11$

$$\text{RHS} = \left[\frac{\frac{0.15 \times 40.5}{\cos 14.5} + 0.2 \times 45}{40.5 - \frac{0.15 \times 0.2 \times 45}{\cos 14.5}}\right] = 0.391$$

Since LHS < RHS, the screw is self locking.

$$\text{Power of motor, } P = \frac{M_t \cdot n}{9.55 \times 10^6}$$

$$\text{Speed of screw, } n = \frac{V}{l} = \frac{840}{14} = 60 \text{ rpm}$$

$$P = \frac{29.874 \times 10^3 \times 60}{9.55 \times 10^6} = 0.188 \text{ kW}.$$

Problem 9: A power screw for a jack has acme threads of proportion $36 \times 30 \times 6$. Friction coefficient of threads is 0.1 and at the collar is 0.12. Determine the weight that can be lifted by this jack through a human effort of 300 N at the end of lever-span of 500 mm. What is its efficiency and nut length required considering the bearing pressure as 9 MPa? Mean collar diameter = 40 mm.

Given data: Acme thread, $\therefore 2\theta = 29°; \theta = 14.5°$

$d = 36$ mm, $d_1 = 30$ mm, $p = 6$ mm

Force $F = 300$ N

Length of lever, $L = 500$ mm.

$\mu = 0.10$ and $\mu_c = 0.12$

$d_c = 40$ mm

Applied torque, $M_t = FL = 300 \times 500 = 150 \times 10^3$ N-mm

Mean diameter of thread, $d_2 = \frac{d + d_1}{2} = d - \frac{1}{2}p = 33$ mm

Assume single start, $l = p = 6$ mm

$$\tan \alpha = \frac{l}{\pi d_2} = \frac{6}{\pi \times 33} = 0.0579$$

(i) Total frictional torque for angular thread

$$M_t = W\left[\frac{d_2}{2}\left(\frac{\tan \alpha + \mu / \cos \theta}{1 - \mu \tan \alpha / \cos \theta}\right) + \frac{\mu_c d_c}{2}\right]$$

$$150 \times 10^3 = W\left[\frac{33}{2}\left(\frac{0.0579 + 0.1/\cos 14.5}{1 - 0.1 \times 0.0579/\cos 14.5}\right) + \frac{0.12 \times 40}{2}\right]$$

$\therefore$ Load capacity, $W = 29{,}552.94$ N

(ii) Efficiency $\eta = \dfrac{d_2 \tan \alpha}{\left(\dfrac{\tan \alpha + \mu / \cos \theta}{1 - \mu \tan \alpha / \cos \theta}\right) d_2 + \mu_c d_c}$

$$\eta = \frac{33 \times 0.0579}{\left(\dfrac{0.0579 + 0.1/\cos 14.5}{1 - 0.0579 \times 0.1/\cos 14.5}\right) 33 + 0.12 \times 40}$$

$$= 0.1882 = 18.82\%$$

(iii) Length of nut, $l_n = \dfrac{4Wp}{\sigma_b' \pi (d^2 - d_1^2)}$

$$l_n = \frac{4 \times 29{,}552.94 \times 6}{9 \times \pi (36^2 - 30^2)} = 63.35 \approx 64 \text{ mm.}$$

Problem 10: The lead screw of a lathe machine has a single start ISO trapezoidal threads of 30 mm outside diameter and 6 mm pitch. It drives the tool carriage exerting an axial load of 1.5 kN. The thrust collar has inside and outside diameters as 30 mm and 50 mm. The carriage moves at a speed of 12 mm/sec. Find the power required to drive the screw. Take the friction coefficient as 0.14. Also find the efficiency of the drive.

Given data: ISO trapezoidal thread $\therefore 2\theta = 30° \Rightarrow \theta = 15°$

diameter $d = 30$ mm pitch $p = 6$ mm

$\therefore$ Mean diameter of thread, $d_2 = d - \frac{1}{2}p = 27$ mm

Load, $W = 1.5$ kN $= 1500$ N.

Inside diameter of thrust collar, $D_i = 30$ mm

Outside diameter of thrust collar, $D_o = 50$ mm

$\therefore$ Mean collar diameter, $d_c = \frac{D_i + D_o}{2} = 40$ mm.

$\mu = \mu_c = 0.14$

Speed, $V = 12$ mm/sec $= 12 \times 60 = 720$ mm/min

single start, $\therefore l = p = 6$ mm

$$\text{Speed, } n = \frac{V}{l} = \frac{720}{6} = 120 \text{ rpm.}$$

$$\tan \alpha = \frac{l}{\pi d_2} = \frac{6}{\pi \times 27} = 0.0707$$

(i) Total frictional torque including collar friction for angular threads,

(a)
$$M_t = W\left[\frac{d_2}{2}\left(\frac{\tan \alpha + \mu / \cos \theta}{1 - \mu \tan \alpha / \cos \theta}\right) + \frac{\mu_c d_c}{2}\right]$$

$$= 1500\left[\frac{27}{2}\left(\frac{0.0707 + 0.14/\cos 15}{1 - 0.14 \times 0.0707/\cos 15}\right) + \frac{0.14 \times 40}{2}\right]$$

$$M_t = 8611.89 \text{ N-mm.}$$

$\therefore$ Power required, $P = \frac{M_t \cdot n}{9.55 \times 10^6} = \frac{8611.89 \times 120}{9.55 \times 10^6}$

$$P = 0.1082 \text{ kW}$$

(b)
$$\text{Efficiency, } \eta = \frac{d_2 \tan \alpha}{\left(\frac{\tan \alpha + \mu / \cos \theta}{1 - \mu \tan \alpha / \cos \theta}\right) d_2 + \mu_c d_c}$$

$$\eta = \frac{27 \times 0.0707}{\left(\frac{0.0707 + 0.14/\cos 15}{1 - 0.0707 \times 0.14/\cos 15}\right) 27 + 0.14 \times 40}$$

$$\eta = 0.1662 \approx 16.62\%.$$

Problem 11: A triple ISO trapezoidal threaded screw is operated by a torque of 40 N-m at its lower end. The nut is loaded and prevented from turning. The screw has 48 mm outside diameter with 8 mm pitch. The end of the screw is mounted on a thrust ball bearing. Coefficient of thread friction is 0.15. Determine:

(i) The load that could be lifted,

(ii) Whether the screw overhauls. Comment,

(iii) The average bearing pressure between screw and nut threads assuming nut length of 50 mm.

Given data: (i) Triple start, $\therefore \quad l = 3p$

ISO trapezoidal thread, $\therefore \quad 2\theta = 30°$

$$\theta = 15°$$

$$\text{Torque applied, } M_t = 40 \text{ Nm} = 40 \times 10^3 \text{ N-mm}$$

$$\text{dia of screw, } d = 48 \text{ mm}$$

$$\text{pitch, } p = 8 \text{ mm}$$

$\therefore \quad$ Mean dia of screw, $d_2 = d - \frac{1}{2}p = 44$ mm.

$$\mu = 0.15$$

Since a ball bearing is used for thrust collar,

$$\mu_c = 0$$

$$\text{Nut length, } l_n = 50 \text{ mm.}$$

$$\text{lead, } l = 3p = 3 \times 8 = 24 \text{ mm.}$$

$$\text{Lead angle } \tan\alpha = \frac{l}{\pi d_2} = \frac{24}{\pi \times 44} = 0.1736$$

For angular thread, frictional torque

$$M_t = W\left[\frac{d_2}{2}\left(\frac{\tan\alpha + \mu/\cos\theta}{1 - \mu\tan\alpha/\cos\theta}\right) + \frac{\mu_c d_c}{2}\right]$$

$$40 \times 10^3 = W\left[\frac{44}{2}\left(\frac{0.1736 + 0.15/\cos 15}{1 - 0.15 \times 0.1736/\cos 15}\right) + 0\right]$$

$\therefore \quad$ Load that can be lifted, $W = 5379.18$ N

(ii) Condition for hauling of angular thread

$$\tan\alpha \geq \left(\frac{\frac{\mu d_2}{\cos\theta} + \mu_c d_c}{d_2 - \mu\,\mu_c d_c/\cos\theta}\right)$$

$$\text{LHS } \tan\alpha = 0.1736$$

$$\text{RHS} = \left(\frac{0.15 \times 44/\cos 15 + 0}{44 - 0}\right) = 0.1553$$

Since LHS > RHS, the screw overhauls and is not self locking.

(iii) Average bearing pressure,

$$\text{Nut length, } l_n = \frac{4Wp}{\sigma_b' \pi (d^2 - d_1^2)}$$

$$\therefore \quad \sigma_b' = \frac{4 \times 5379.18 \times 8}{50 \times \pi (48^2 - 40^2)} = 1.557 \text{ MPa.}$$

Problem 12: The following data apply to the C-clamp of Fig. 8.2

ISO metric threads
Pitch = 1.75 mm (single-threaded)
Outside diameter = 12 mm
Root diameter = 9.853 mm
Root area = 76.25 mm^2
Coefficient of thread friction = 0.12
Coefficient of collar friction = 0.25
Mean collar radius = 6 mm
Load W = 4000 N
Operator can comfortably exert a force of 80 N at the end of the handle.

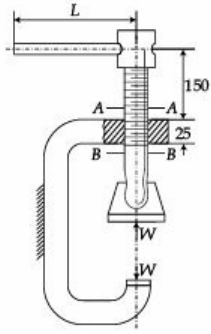


Fig. 8.2

(a) What length of handle, L, is needed?
(b) What is the maximum shear stress in the body of the screw and where does this exist?
(c) What is the bearing pressure P on the threads?

Given data: ISO Metric thread, $2\theta = 60° \Rightarrow \theta = 30°$

Pitch, $p = 1.75$ mm $= l$ for single start

Outside diameter, $d = 12$ mm

Root diameter, $d_1 = 9.853$ mm

$\therefore$ mean diameter of thread, $d_2 = \frac{d + d_1}{2}$ 3

$$d_2 = \frac{12 + 9.853}{2} = 10.9265 \text{ mm}$$

Root area, $A_c = 76.25$ mm^2

$\mu = 0.12, \quad \mu_c = 0.25$

$$\text{Mean collar radius} = \frac{d_c}{2} = 6 \text{ mm} \Rightarrow d_c = 12 \text{ mm}$$

Load, $W = 4000$ N

Operating force, $F = 80$ N.

$$\tan \alpha = \frac{l}{\pi d_2} = \frac{1.75}{\pi \times 10.9265} = 0.05098 \approx 0.051$$

(i) To find the length of the handle, $L = \dfrac{M_t}{F}$

Total frictional torque including collar friction,

$$M_t = W\left[\frac{d_2}{2}\left(\frac{\tan\alpha + \mu/\cos\theta}{1 - \mu\tan\alpha/\cos\theta}\right) + \frac{\mu_c d_c}{2}\right]$$

$$= 4000\left[\frac{10.9265}{2}\left(\frac{0.051 + 0.12/\cos 30}{1 - 0.12 \times 0.051/\cos 30}\right) + \frac{0.25 \times 12}{2}\right]$$

$$M_t = 10172.03 \text{ N-mm}$$

$\therefore$ Length of handle $\quad L = \dfrac{10172.03}{80} = 127.15 \approx 128 \text{ mm}$

(ii) Section AA: Just above the nut

The screw is subjected to torque and bending

(a) Bending stress $\sigma = \dfrac{M_b C}{I}$

M_b = bending moment = $F \cdot L = 80 \times 150$

$M_b = 12000 \text{ N mm}$

$$C = \frac{d_1}{2} = \frac{9.853}{2} = 4.9265 \text{ mm}$$

$$I = \frac{\pi}{64} d_1^4 = \frac{\pi}{64} \times 9.853^4 = 462.64 \text{ mm}^4$$

$\therefore \quad \sigma = \dfrac{12000 \times 4.9265}{462.64} = 127.78 \text{ MPa}$

(b) Shear stress due to torque, $\tau = \dfrac{M_t \cdot r_1}{J}$

$$\tau = \frac{10{,}172.03 \times 4.9265}{\left(\frac{\pi}{32} \times 9.853^4\right)} = 54.16 \text{ MPa}$$

(c) $\therefore$ Maximum shear stress, $\tau_{max} = \dfrac{1}{2}(\sigma^2 + 4\tau^2)^{1/2}$

$$= \frac{1}{2}(127.78^2 + 4 \times 54.16^2)^{1/2} = 83.76 \text{ MPa}$$

(iii) At Section BB:

Section BB below the nut is subjected to torque and direct compressive load. The torque is only frictional torque at collar, since it is beyond the screw and nut portion.

(a) Compressive stress, $\sigma = \frac{W}{A_c} = \frac{4000}{76.25} = 52.46$ MPa

(b) Shear stress, $\tau = \frac{M_t \cdot r_1}{J}$

M_t = frictional torque at collar only

$$= \frac{W \cdot \mu_c d_c}{2} = \frac{4000 \times 0.25 \times 12}{2} = 6000 \text{ N-mm}$$

$$\therefore \quad \tau = \frac{6000 \times 4.9265}{\left(\frac{\pi}{32} \times 9.853^4\right)} = 31.95 \text{ MPa}$$

(c) $\therefore$ Maximum shear stress, $\tau_{max} = \frac{1}{2}(\sigma^2 + 4\,\tau^2)^{1/2}$

$$= \frac{1}{2}(52.46^2 + 4 \times 31.95^2)^{1/2}$$

$$\tau_{max} = 41.335 \text{ MPa}$$

Hence from these two values of τ_{max}, the maximum shear stress, $\tau_{max} = 83.76$ MPa and it occurs at AA above the nut.

(iv) To find bearing pressure on thread,

Consider nut length $l_n = \frac{4\,Wp}{\sigma_b' \pi \left(d^2 - d_1^2\right)}$

$$\therefore \quad \sigma_b' = \frac{4 \times 4000 \times 1.75}{25 \times \pi \times \left(12^2 - 9.853^2\right)} = 7.598 \text{ MPa.}$$

8.3 SCREW JACK

Figure 8.3 shows a common screw jack used for automobiles.

W = load to be lifted
d = outside diameter or nominal diameter of screw
p = pitch
d_h = diameter of handle
l_h = length of han111111111dle.
D = outside diameter of nut

D_i, D_o = inside and outside the diameter of thrust collar.

d_c = mean diameter of the thrust collar = $\frac{D_i + D_o}{2}$

l_n = nut length.

t = thickness of body.

σ_{max} = allowable normal stress in the screw

τ_{max} = allowable shear stress in the screw

σ_b = allowable bending stress in the handle

σ'_b = allowable bearing pressure in the threads

τ_n = allowable shear stress in the nut

μ = coefficient of thread friction

μ_c = coefficient of collar friction

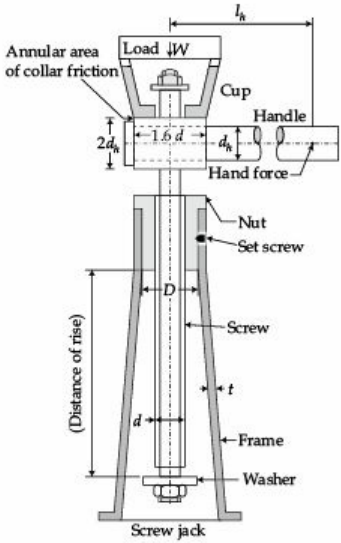


Fig. 8.3

Design Procedure

Step 1: Design of screw

(a) Considering the screw under axial compression

$$\sigma_c = \frac{W}{A_c} \Rightarrow \therefore \text{ core area, } A_c = \frac{W}{\sigma_c}$$

σ_c = allowable compressive stress in screw = σ_{max}

Based on A_c select standard core dia of screw = d_1

(Ref. Table 8.1 for selection ...(T 18-8/P 18.20)

(b) Considering the screw as a column loaded in axial compression, when fully extended, Rankine's equation for columns

$$\sigma = \frac{W}{A_c}\left[1 + a\left(\frac{l_e}{k}\right)^2\right] \quad \text{...(2-107/P 2.18)}$$

σ = allowable stress in the screw = σ_{max}

$$A_c = \frac{\pi}{4} d_1^2$$

a = Rankine's constant = $\frac{1}{7500}$ for steels

l_e = equivalent length of the column

Screw jack is a column with one end fixed at nut and other end is free where the load is sitting.

$\therefore$ Equivalent length, $l_e = 2 \times$ (lift or extension)

$$K = \text{radius of gyration} = \sqrt{\frac{I}{A}} = \sqrt{\frac{\left(\frac{\pi}{64} d_1^4\right)}{\left(\frac{\pi}{4} d_1^2\right)}} = \frac{d_1}{4}$$

Substitute and find core diameter $= d_1 = ?$

Select the bigger of these two and select standard square threaded screw based on core diameter $= d_1$. From Table 8.1. i.e., select d, d_1, p, and A_c. ...(T 18-8/P 18.20)

Step 2: Check for principal stresses in screw

(a) Compressive stress in the screw, $\sigma_c = \dfrac{W}{A_c}$

(b) Shear stress in the screw, $\tau_s = \dfrac{M_{ts} \cdot r_1}{J}$

where M_{ts} = frictional torque in the screw only

$$= W \frac{d_2}{2}\left(\frac{\tan\alpha + \mu}{1 - \mu \tan\alpha}\right)$$

$$d_2 = d - \frac{1}{2}p, \quad \tan\alpha = \frac{l}{\pi d_2}$$

(l = lead $= 1 \times p$ assuming single start)

$$r_1 = \frac{d_1}{2}$$

$$J = \frac{\pi}{32} d_1^4$$

Find $\tau_s = ?$

(c) Combined or principal stresses.

(i) Maximum normal stress

$$\sigma_{max} = \frac{1}{2}[\sigma_c + (\sigma_c^2 + 4\tau_s^2)^{1/2}]$$

This should be $< \sigma_{allowable}$

(ii) Maximum shear stress, $\tau_{max} = \dfrac{1}{2}(\sigma_c^2 + 4\tau_s^2)^{1/2}$

This should be $< \tau_{allowable}$

If any of these combined stresses are found to be more than allowable stresses, then the screw is not safe. Then select next bigger diameter screw and check again.

Step 3: Design of screw head and collar

Assume pin head diameter, $D_o = 1.6d$ = outside the diameter of the collar

pin diameter, $D_i = 0.5d$ = Inside the diameter of the collar.

$\therefore$ Mean diameter of the collar, $d_c = \dfrac{D_i + D_o}{2}$

Step 4: Design of nut:

Assuming bronze for nut,

(a) Length of nut, $l_n = \dfrac{4Wp}{\sigma_b' \pi(d^2 - d_1^2)}$

Find nut length = l_n

(b) Tensile stress in the nut $= \sigma_{nut} = \dfrac{W}{\frac{\pi}{4}(D^2 - d^2)}$

Find outside dia of the nut = D

Assume, σ_b' = 15 MPa and

σ_{nut} = 40 to 50 MPa.

Step 5: Frictional torque

Total frictional torque including the collar friction $M_t = W\left[\dfrac{d_2}{2}\left(\dfrac{\tan\alpha + \mu}{1 - \mu\tan\alpha}\right) + \dfrac{\mu_c d_c}{2}\right]$

...(18-29/P 18.8)

Step 6: Design of handle

(a) Length of handle

Assume that a man can apply a force of, F = 250 to 350 N at the end of the handle,

length of the handle, $l_h = \dfrac{M_t}{F}$

(b) To find the diameter of the handle

The handle is subjected to bending moment

(It is like cantilever subjected to load at end)

$$\text{WKT } \frac{M_b}{I} = \frac{\sigma_b}{c}$$

where M_b = bending moment $= Fl_h = M_t$ itself

I = moment of inertia for handle, $I = \dfrac{\pi d_h^4}{64}$

$C = \dfrac{d_h}{2}$, σ_b = allowable bending stress.

Select C30 steel for handle, σ_y = 294.2 MPa

Assume, FOS = 2.5

$$\therefore \quad \sigma_b = \frac{\sigma_y}{FOS} = \frac{294.2}{2.5} = 117.68 \text{ MPa}$$

Substitute and find the diameter of handle d_h = ?

Step 7: Efficiency, $$\eta = \frac{d_2 \tan \alpha}{\left(\frac{\tan \alpha + \mu}{1 - \mu \tan \alpha}\right) d_2 + \mu_c d_c}$$ ...(18-33/P 18.10)

Step 8: Self locking condition

The screw jack to be of self locking type, hence check for self locking.

$$\tan \alpha < \frac{\mu d_2 + \mu_c d_c}{d_2 - \mu \mu_c d_c}.$$ ...(18-37/P 18.10)

Problem 13: Select the thread proportions of a screw rod of a screw press to sustain an axial compressive load of 40 kN for an supported length of 300 mm. The allowable stresses for screw are 120 MPa in compression and 72 MPa in shear.

Given data: Load, W = 40,000 N

Lift or extension = 300 mm

σ_{max} = 120 MPa, τ_{max} = 72 MPa

Step 1: Consider the screw under compression

$$\sigma = \frac{W}{A_c}$$

$$\therefore \quad A_c = \frac{40{,}000}{120} = 333.33$$

From tables of square threads normal series (Table 8.1) ...(T 18-8/P 18.20)

select standard screw, d = 26 mm

d_1 = 21 mm

p = 5 mm

A_c = 346 mm^2

Step 2: Consider the screw as column, loaded in axial compression

For columns

Rankine's formula for columns, $$\sigma = \frac{W}{A_c}\left[1 + a\left(\frac{l_e}{k}\right)^2\right]$$ ...(2-107/P 2.18)

$$a = \frac{1}{7500}, \quad A_c = \frac{\pi}{4} d_1^2, \quad \sigma = 120 \text{ MPa}$$

$$l_e = 2 \times \text{lift} = 2 \times 300 = 600 \text{ mm}$$

$$K = \sqrt{\frac{I}{A}} = \frac{d_1}{4}$$

$$120 = \frac{40000}{\frac{\pi}{4} d_1^2}\left[1 + \frac{1}{7500}\left(\frac{600}{\frac{d_1}{4}}\right)^2\right]$$

$$120 = \frac{50.929 \times 10^3}{d_1^2}\left[1 + \frac{768}{d_1^2}\right]$$

$$120 = \frac{50.929 \times 10^3}{d_1^2} = \left[\frac{d_1^2 + 768}{d_1^2}\right]$$

$$d_1^4 - 424.41 d_1^2 - 325.95 \times 10^3 = 0$$

$$d_1^2 = 821.287 \Rightarrow d_1 = 28.66 \text{ mm}$$

Selecting standard screw based on d_1 from Table 8.1 ...(T 18-8)

$$d = 36 \text{ mm}$$

$$d_1 = 30 \text{ mm}$$

$$p = 6 \text{ mm}$$

$$A_c = 707 \text{ mm}^2$$

Select the bigger one of the above two,

$$d = 36$$

$$d_1 = 30 \quad \text{and} \quad p = 6 \text{ mm}, A_c = 707 \text{ mm}^2$$

Step 3: Check for principal stresses in screw

(a) Compressive stress in screw, $\sigma_c = \frac{W}{A_c} = \frac{40{,}000}{707}$

$$\sigma_c = 56.58 \text{ MPa}$$

(b) Shear stress in the screw

$$\tau_s = \frac{M_{ts} r_1}{J}$$

M_{ts} = Frictional torque in screw $= \frac{W d_2}{2}\left(\frac{\tan\alpha + \mu}{1 - \mu \tan\alpha}\right)$

$$d_2 = d - \frac{1}{2}p = 36 - \frac{6}{2} = 33 \text{ mm}$$

$$\tan\alpha = \frac{l}{\pi d_2} \text{ assume single start, } l = p = 6 \text{ mm}$$

$$\therefore \quad \tan\alpha = \frac{6}{\pi \times 33} = 0.0579$$

Assume, $\mu = 0.11$

$$\therefore \quad M_{ts} = 40{,}000 \times \frac{33}{2}\left(\frac{0.0579 + 0.11}{1 - 0.11 \times 0.0579}\right)$$

$$M_{ts} = 111.524 \times 10^3 \text{ N-mm}$$

$$r_1 = \frac{d_1}{2} = \frac{30}{2} = 15 \text{ mm}$$

$$J = \frac{\pi d_1^4}{32} = \frac{\pi \times 30^4}{32} = 79{,}521.56 \text{ mm}^4$$

$$\therefore \quad \tau_s = \frac{111.524 \times 10^3 \times 15}{79{,}521.56} = 21.04 \text{ MPa}$$

(c) Combined or principal stresses

(i) Maximum normal stress

$$\sigma_{max} = \frac{1}{2}[\sigma_c + (\sigma_c^2 + 4t_s^2)^{1/2}]$$

$$= \frac{1}{2}[56.58 + (56.58^2 + 4 \times 21.04^2)^{1/2}]$$

$$= 63.54 \text{ MPa} < \sigma_{allowable} (120) \text{ safe.}$$

(ii) Maximum shear stress

$$\tau_{max} = \frac{1}{2}(\sigma_c^2 + 4\tau_s^2)^{1/2}$$

$$= \frac{1}{2}[56.58^2 + 4(21.04)^2]^{1/2}$$

$$= 35.26 \text{ MPa} < \tau_{allowable} (75) \text{ safe}$$

Hence use the screw

$$d = 36 \text{ mm}$$

$$d_1 = 30 \text{ mm}$$

$$p = 6 \text{ mm.}$$

Problem 14: Design a screw jack for a load capacity of 90 kN and to have a list of 360 mm.

Given data: $W = 90 \text{ kN} = 90 \times 10^3 \text{ N}$

lift or extension = 360 mm

Selecting 40 C8 (C40) steel for screw, C30 steel for handle, *CI* collar, bronze nut

For screw, $\sigma_y = 328.6$ MPa from Tables

Use factor of safety of 4 for screw, since human life at risk.

Allowable compressive stress, $\sigma_{max} = \frac{\sigma_y}{FOS} = \frac{328.6}{4} = 82.15$ MPa

Allowable shear stress, $\tau_{max} = 0.5\sigma = 0.5 \times 82.15 = 41.08$ MPa

For handle, $\sigma_y = 294.2$ for C30 steel, FOS = 2.5

Allowable bending stress, $\sigma_b = \dfrac{294.2}{2.5} = 117.68$ MPa

For bronze nut,

allowable bearing pressure, $\sigma_b' = 15$ MPa

allowable tensile stress, $\sigma_t = 50$ MPa

select coefficient of thread friction, $\mu = 0.11$ ...(T 18-4)

collar friction, $\mu_c = 0.147$ ...(T 18-5 a)

Step 1: Design of the screw

(a) For the screw under compression

$$\sigma_c = \frac{W}{A_c} \Rightarrow Ac = \frac{W}{\sigma_c} = \frac{90{,}000}{82.15} = 1095.56 \text{ mm}^2$$

From square thread Table 8.1 select standard screw based on $A_c = 1095.56$

$d = 48$ mm

$d_1 = 40$ mm ...(T 18-8/P 18.20)

$p = 8$ mm

(b) For the screw as column loaded axially

Rankine's formula for columns

$$\sigma = \frac{W}{A_c}\left[1 + a\left(\frac{l_e}{k}\right)^2\right] \quad \text{...(2-107/P 2.18)}$$

$\sigma = 82.15$ MPa, $W = 90{,}000$ N, $A_c = \dfrac{\pi}{4} d_1^2$

a = Rankine's constant $= \dfrac{1}{7500}$

l_e = equivalent length of column = 2 × lift

$= 2 \times 360 = 720$ mm

k = radius of gyration $= \sqrt{\dfrac{I}{A}} = \dfrac{d_1}{4}$

$$82.15 = \frac{90{,}000}{\left(\frac{\pi}{4} d_1^2\right)}\left[1 + \frac{1}{7500}\left(\frac{720}{\frac{d_1}{4}}\right)^2\right]$$

$$82.15 = \frac{114.59 \times 10^3}{d_1^{\,2}}\left[1 + \frac{1105.92}{d_1^{\,2}}\right]$$

$$d_1^4 - 1394.89 d_1^2 - 1.543 \times 10^6 = 0$$

$\therefore$ $$d_1^2 = 2127.03 \text{ mm}^2$$

$\therefore$ $$d_1 = 46.065 \text{ mm}$$

From Table 8.1, select standard screw based on d_1 ...(T 18-8)

$$d = 60 \text{ mm}$$

$$d_1 = 51 \text{ mm}$$

$$p = 9 \text{ mm}$$

$$A_c = 2043 \text{ mm}^2$$

Recommended screw size

$$d = 60 \text{ mm}$$

$$d_1 = 51 \text{ mm}$$

$$p = 9 \text{ mm}$$

$$A_c = 2043 \text{ mm (bigger one)}$$

Step 2: Check for principal stresses in screw

(a) Compressive stress, $\sigma_c = \dfrac{W}{A_c} = \dfrac{90{,}000}{2043} = 44.05$ MPa

(b) Shear stress in the screw, $\tau_s = \dfrac{M_{ts}\, r_1}{J}$

where $$M_{ts} = W \frac{d_2}{2}\left(\frac{\tan\alpha + \mu}{1 - \mu \tan\alpha}\right)$$

$$d_2 = d - \frac{1}{2}p = 60 - \frac{9}{2} = 55.5$$

$$\tan\alpha = \frac{l}{\pi d_2} \quad (l = p = 9 \text{ mm})$$

$$= \frac{9}{\pi \times 55.5} = 0.0516$$

$\therefore$ $$M_{ts} = 90{,}000 \times \frac{55.5}{2}\left[\frac{0.0516 + 0.11}{1 - 0.0516 \times 0.11}\right]$$

$$= 405.9 \times 10^3 \text{ N-mm}$$

$$r_1 = \frac{d_1}{2} = \frac{51}{2} = 25.5 \text{ mm}$$

$$J = \frac{\pi}{32} d_1^4 = \frac{\pi}{32} 51^4 = 664.17 \times 10^3 \text{ mm}^4$$

$$\therefore \quad \tau_s = \frac{\left(405.9 \times 10^3\right) \times 25.5}{664.17 \times 10^3} = 15.58 \text{ MPa.}$$

(c) Combined or principal stresses

(i) Maximum normal stress

$$\sigma_{max} = \frac{1}{2}[\sigma_c + (\sigma_c^2 + 4\tau_s^2)^{1/2}]$$

$$= \frac{1}{2}[44.05 + \{(44.05)^2 + 4(15.58)^2\}^{1/2}] = 49.0 \text{ MPa}$$

$$< \sigma_{allowable} \ (82.15) \text{ safe}$$

(ii) $$\tau_{max} = \frac{1}{2}(\sigma_c^2 + 4\tau_s^2)^{1/2}$$

$$= \frac{1}{2}[44.05^2 + 4(15.58)^2]^{1/2} = 26.98 \text{ MPa}$$

$$< \tau_{allowable} \ (41.08) \text{ safe}$$

Step 3: Design of the screw head and the collar

Assume the outside diameter of the collar = Diameter of the screw head, $D_o = 1.6d$

$$D_o = 1.6 \times 60 = 96 \text{ mm}$$

Inside diameter of collar = Diameter of pin $D_i = 0.5\ d$

$$D_i = 0.5 \times 60 = 30 \text{ mm}$$

$\therefore$ Mean diameter of thrust collar, $d_c = \dfrac{D_o + D_i}{2} = 63$ mm

Step 4: Design of the nut

(a) Length of the nut, $l_n = \dfrac{4Wp}{\sigma_b' \pi (d^2 - d_1^2)}$ ...(18-41)

$$l_n = \frac{4 \times 90{,}000 \times 9}{15 \times \pi (60^2 - 51^2)} = 68.82 \simeq 69 \text{ mm}$$

(b) To find outside diameter the nut

$$\text{Tensile stress in nut, } \sigma_t = \frac{W}{\frac{\pi}{4}(D^2 - d^2)}$$

$$50 = \frac{90000}{\frac{\pi}{4}(D^2 - 60^2)} \Rightarrow D = 76.76 \simeq 77 \text{ mm}$$

Step 5: Total frictional torque including collar friction

$$M_t = W\left[\frac{d_2}{2}\left(\frac{\tan\alpha+\mu}{1-\mu\tan\alpha}\right)+\frac{\mu_c d_c}{2}\right] \quad ...(18\text{-}29)$$

$$= 90{,}000\left[\frac{55.5}{2}\left(\frac{0.0516+0.11}{1-0.11\times0.0516}\right)+\frac{0.147\times63}{2}\right]$$

$$= 822.65\times10^3 \text{ N-mm}$$

Step 6: Design of the handle

(a) Length of the handle, $l_h = \dfrac{M_t}{F}$

F = operating force = 300 N (assumed)

$$\therefore \quad l_h = \frac{822.65\times10^3}{300} = 2742.2 \simeq 2743 \text{ mm}$$

(b) For the handle in bending, $\dfrac{M_b}{I} = \dfrac{\sigma_b}{c}$

where M_b = bending moment = $Fl_h = M_t$

$$I = \frac{\pi d_h^4}{64}, \quad C = \frac{d_h}{2}, \quad \sigma_b = 117.68 \text{ MPa}$$

$$\frac{822.65\times10^3}{\left(\frac{\pi}{64}d_h^4\right)} = \frac{117.68}{\left(\frac{d_h}{2}\right)} \Rightarrow d_h = 41.45 \simeq 42 \text{ mm}$$

Step 7: Efficiency, $\eta = \dfrac{d_2\tan\alpha}{\left(\dfrac{\tan\alpha+\mu}{1-\mu\tan\alpha}\right)d_2+\mu_c d_c}$ ...(18.33)

$$\eta = \frac{55.5\times0.0516}{\left(\frac{0.0516+0.11}{1-0.11\times0.0516}\right)55.5+0.147\times63} = 0.1567 = 15.67\%$$

Step 8: Check for self locking

For self locking $\tan\alpha < \dfrac{\mu d_2+\mu_c d_c}{d_2-\mu\mu_c d_c}$ ...(18.37)

LHS: $\tan\alpha = 0.0516$

$$\text{RHS} = \frac{\mu d_2+\mu_c d_c}{d_2-\mu\mu_c d_c} = \frac{0.11\times55.5+0.147\times63}{55.5-0.11\times0.147\times63} = 0.282$$

$\because$ LHS < RHS, the screw is self locking and safe.

Problem 15: The maximum load on the nut and screw of a screw clamp is 50 kN. Design the screw clamp considering the following data:

Allowable compressive stress = 120 MPa.
Bearing pressure in the threads = 17 MPa
Allowable bending stress in the handle = 80 MPa
Coeff. of friction = 0.1
Mean diameter of thrust collar = 40 mm

Given data: Load, $W = 50 \times 10^3$ N

$\sigma_{c_{all}} = 120$ MPa

$\sigma_b = 80$ MPa

$\sigma_b' = 17$ MPa

$\mu = \mu_c = 0.1$

$d_e = 40$ mm

Step 1: Design of screw

$$\text{Compressive stress, } \sigma_c = \frac{W}{A_c} \Rightarrow 120 = \frac{50{,}000}{A_c}$$

$\therefore$ $A_c = 416.67$ mm^2

Refer Table 8.1, select standard screw based on A_c

$d = 30$ mm

$d_1 = 24$ mm

$p = 6$ mm

$A_c = 452$ mm^2

Step 2: Design of nut

$$\text{Nut length, } l_n = \frac{4Wp}{\sigma_b' \pi (d^2 - d_1^2)} \quad \text{...(18-41)}$$

$$l_n = \frac{4 \times 50{,}000 \times 6}{17 \times \pi (30^2 - 24^2)} = 69.35 \simeq 70 \text{ mm.}$$

Step 3: Design of handle

Total frictional torque including collar friction

$$M_t = W\left[\frac{d_2}{2}\left(\frac{\tan\alpha + \mu}{1 - \mu\tan\alpha}\right) + \frac{\mu_c d_c}{2}\right] \quad \text{...(18-29)}$$

assuming single start $l = p = 1 \times 6 = 6$ mm

$$d_2 = d - \frac{1}{2}p = 30 - \frac{6}{2} = 27 \text{ mm}$$

$$\tan\alpha = \frac{l}{\pi d_2} = \frac{6}{\pi \times 27} = 0.07074$$

$$M_t = 50 \times 10^3 \left[\frac{27}{2} \left(\frac{0.07074 + 0.1}{1 - 0.1 \times 0.07074} \right) + \frac{0.1 \times 40}{2} \right]$$

$$= 216.07 \times 10^3 \text{ N-mm}$$

But, $M_t = F \cdot l_h$

Assuming the operating force as $F = 300$ N

$$l_h = \frac{216.07 \times 10^3}{300} = 720.24 \text{ mm} \simeq 721 \text{ mm}$$

For the lever in bending

$$\frac{M_b}{I} = \frac{\sigma_b}{C}$$

$$M_b = F \cdot l_n = M_t = 216.07 \times 10^3 \text{ N-mm}$$

$$I = \frac{\pi d_h^4}{64}, \quad C = \frac{d_h}{2}, \quad \sigma_b = 80 \text{ MPa}$$

$$\frac{216.07 \times 10^3}{\left(\frac{\pi}{64} d_h^4\right)} = \frac{80}{\left(\frac{d_h}{2}\right)} \Rightarrow d_h = 30.19 \simeq 31 \text{ mm}.$$

Table 8.1 Basic dimensions (mm) for square threads (normal series)

Nom dia	Major		Minor	pitch	e	r	h_2	b	h_1	a	H	Core Area
	Bolt d	Nut D	d_1	P								mm^2
22	22	22.5	17									227
24	24	24.5	19									284
26	26	26.5	21	5	2.5	0.25	2	0.5	2.5	0.25	2.75	346
28	28	28.5	23									415
30	30	30.5	24									452
32	32	32.5	26									531
(34)	34	34.5	28	6	3	0.25	2.5	0.5	3	0.25	3.25	616
36	36	36.5	30									707
(38)	38	38.5	31									755
40	40	40.5	33									855
(42)	42	42.5	35	7	3.5	0.25	3	0.5	3.5	0.25	3.75	962
44	44	44.5	37									1075
(46)	46	46.5	38									1134
48	48	48.5	40									1256
50	50	50.5	42	8	4	0.25	3.5	0.5	4	0.25	4.25	1385
52	52	52.5	44									1521

Contd.

Table 8.1 (Contd.)

Nom dia	Major		Minor d_1	pitch P	e	r	h_2	b	h_1	a	H	Core Area mm^2
	Bolt d	Nut D										
55	55	55.5	46									1662
(58)	58	58.5	49									1886
60	60	60.5	51	9	4.5	0.25	4	0.5	4.5	0.25	4.75	2043
(62)	62	62.5	53									2206
65	65	65.5	55									2376
(68)	68	68.5	58									2642
70	70	70.5	60									2827
(72)	72	72.5	62									3019
75	75	75.5	65	10	5	0.25	4.5	0.5	5	0.25	5.25	3318
(78)	78	78.5	68									3632
80	80	80.5	70									3848
(82)	82	82.5	72									4072
85	85	85.5	73									4185
(88)	88	88.5	76									4536
90	90	90.5	78									4778
(92)	92	92.5	80	12	6	0.25	5.5	0.5	6	0.25	6.25	5027
95	95	95.5	83									5411
(98)	98	98.5	86									5809

EXERCISE

1. A machine weighing 30 kN is to be raised by a two start square thread screw of 50 mm diameter, 8 mm pitch screw jack at a maximum speed of 720 mm/min. The coefficient of thread friction is 0.2 and the collar friction is 0.1. The thrust collar of the screw has inside and outside diameters of 30 mm and 60 mm.
2. A 100 kN screw jack with a maximum extension of 200 mm has single start square threads. Allowable compressive stress is 35 MPa, allowable bearing pressure on the threads is 17.5 MPa. Find the:
   (i) Size of screw,
   (ii) Size of collar and nut,
   (iii) Torque required to raise the load, and
   (iv) Efficiency of the screw jack.

3. A screw for the transmission of power is to propel the load of 28 kN at a speed of 20 m/min. The quadruple square threaded screw is to have pitch of 20 mm. The root diameter is 40 mm. The mean radius of collar is 45 mm. Coefficient of friction is 0.1 for both. Determine the:
   (i) Power of motor required to drive the screw,
   (ii) Efficiency,
   (iii) Resultant stresses in the screw.
4. Select the thread proportions of a screw rod for a screw press to sustain an axial load of 30 kN and for an extension of 300 mm. The material for screw has an yield stress of 300 MPa. Use factor of safety of 2.
5. A screw press is to exert a force of 60 kN with an applied torque of 550 N-m. The unsupported length of screw is 0.25 m. A thrust bearing is used as collar. The screw is single start square thread and is made of steel having normal stress of 88 MPa. Design the screw and check your design.
6. The lead screw of a lathe machine is acme thread of 30 × 6. It drives the carriage at a speed of 3 m/mm against a cutting force of 9 kN. The coefficient of thread friction is 0.12 and collar friction is 0.15. The mean collar diameter is 42 mm. Find the:
   (i) Power of motor required to drive the screw,
   (ii) Efficiency,
   (iii) Nature and magnitude of stresses induced,
   (iv) Length of bronze nut required taking allowable bearing pressure as 1.5 MPa.
7. A power screw has 7 mm pitch, 40 mm diameter acme threads. The screw is subjected to an axial thrust of 7.5 kN. The nut length is 18 mm. Determine the frictional torque in the screw, if the coefficient of friction is 0.15. Also find the stresses in the screw and bearing pressured induced in the nut.
8. A machine slide weighing 6 kN is elevated by a trapezoidal threaded screw 33 mm diameter and 6 mm pitch at a rate of 1.2 m/min. The coefficient of friction is 0.18. Calculate the power of motor required and the efficiency, if the mean collar diameter is 42 mm.
9. In a large gate value used to a high pressure water line, the gate weighs 6 kN and the friction force due to water pressure is 2 kN. The outside diameter of the trapezoidal threaded screw is 38 mm and the pitch is 8 mm. The outside and inside diameters of thrust washer are 75 mm and 25 mm respectively. Coefficient of thread friction is 0.15 and collar friction is 0.25. Determine the:
   (i) Torque required to raise the gate,
   (ii) Efficiency.

CHAPTER

9 Bolted Joints

Bolted joints, also known as threaded fasteners, are commonly seen on machines and structures. Bolts hold two or more parts together. These are used to fasten the parts of the machine or structure that calls for frequent assembling and disassembling. Some examples of fasteners are bolts, nuts, studs, screws etc. The most commonly used type of thread form is ISO metric thread for fasteners.

ISO metric threads are designated by the letter *M* followed by diameter × pitch. For coarse series, it is generally designated by *M* following by diameter. A fine series screw of 8 mm dia 1 mm pitch is designated by *M*8 × 1. A coarse series screw of diameter 8 mm is designated by *M*8.

Screws are available in 3 types of series—fine series, normal series and coarse series—depending on the pitch. Ref Table 9.2 for ISO screw sizes. (T 18-7/P 18.16)

9.1 STRESSES IN SCREWS

The stresses induced in bolts due to static loading are as follows:

F = axial tensile load in bolt, N

d = diameter of screw (major diameter), mm

d_1 = minor diameter or core diameter, mm

d_2 = mean diameter of screw = $\frac{d + d_1}{2}$

A_c = core area mm^2

(i) Tensile stress, $\sigma = \dfrac{F}{A_C} = \dfrac{F}{(\pi/4)\, d_1^2}$

(ii) Due to tightening, the bolt is subjected to torque

Torsional shear stress due to torque, $\tau = \dfrac{M_t \cdot r_1}{J}$

$$r_1 = \frac{d_1}{2}, \qquad J = \frac{\pi d_1^4}{32}$$

(iii) Also due to tightening, the threads experience direct shear stress.

Direct shear stress, $\tau = \dfrac{F}{\pi d_1 i t}$

i = number of threads in engagement
t = width of section of thread of root.

(iv) Crushing or bending stress between threads

$$\sigma_c = \frac{F}{(\pi/4)(d^2 - d_1^2) i}$$

(v) Combined or principle stress in the screw.

(a) Maximum principal stress

$$\sigma_{max} = \frac{1}{2}[\sigma + (\sigma^2 + 4\tau^2)^{1/2}]$$

(b) Maximum shear stress

$$\tau_{max} = \frac{1}{2}(\sigma^2 + 4\tau^2)^{1/2}$$

9.2 STRESSES DUE TO THE INITIAL TIGHTENING LOAD AND EXTERNAL FORCE

(i) Initial load or tightening load on the bolt

$F_i = 2804.69\, d$ for joint without gasket (metal to metal joint)

$F_i = 1402.34\, d$ for gasket joint. (18-5a/P 18.2)

where d = external diameter of screw (18-6a)

$$\simeq \frac{d_1}{0.8}, \quad d_1 = \text{root diameter.}$$

(ii) Axial load per belt due to external load

$$F_a = \frac{F}{i}$$

F = external load = Area cylinder × pressure

$$F = \frac{\pi}{4} D_i^2 \times P_f$$

D_i = inside diameter of cylinder, mm
P_f = internal pressure, MPa.
i = no. of bolts

Find load on bolt, $F_f = KFa + F_i$ (18-4/P 18.2)
where K = Constant

$$K = \left[\frac{\frac{E_b A_b}{L}}{\frac{E_b A_b}{L} + \frac{E_g A_g}{l_g}} \right] \quad \text{(18-4 a/P 18.2)}$$

E_b = modulus of elasticity of bolt, MPa.
E_g = modulus of elasticity of gasket, MPa
A_b = area of cross section bolt, mm^2
A_g = area of cross section of gasket, mm^2
L = length of bolt in contact, mm
l_g = thickness of gasket, mm

If $F_a > F_f$ then take $F_f = F_a$

F_{all} = allowable load on bolt = $A_c \cdot \sigma_{bolt}$

$$F_{all} = \frac{\pi}{4} d_1^2 \cdot \sigma_{bolt}$$

For safe design $F_{all} \geq F_f$
Find d_1. Select suitable bolt based on d_1 from Table 9.2. (T 18-7)
Table 9.1 gives value of K for the combination of various materials.

Table 9.1 Values of K (T 18-1/P 18-15)

Type of joint	*K*
Soft packing with studs	1.00
Soft packing with through bolts	0.75
Asbestos	0.60
Soft copper gasket with long bolts	0.50
Hard copper gasket with bolts	0.25
Metal to metal joint	0.00

PROBLEMS

Problem 1: Determine the size of the bolt subject to a tensile load of 30 kN. The allowable tensile stress for bolt material may be taken as 120 MPa.

Given data: Load, $F = 30{,}000$ N

Stress, $\sigma = 120$ MPa

$\therefore$ Core area, $A_c = \dfrac{F}{\sigma} = \dfrac{30{,}000}{120} = 250 \text{ mm}^2$

Select standard bolt from Table 9.2 based on A_c (T 18-7)

Select $M20 \times 2$ bolt for $A_c = 258 \text{ mm}^2$.

Problem 2: Determine the load carrying capacity of the following bolts taking allowable tensile stress as 120 MPa. (i) $M30 \times 2$ (ii) $M16$ (iii) $M30$.

Given data: $\sigma = 120$ MPa.

Find $F = ?$

(i) For $M30 \times 2$ bolt, from Table 9.2, fine series. (T 18-7)

Area, $A_c = 621 \text{ mm}^2$

$\therefore$ Load capacity, $F = \sigma \cdot A_c = 120 \times 621 = 74{,}520$ N

(ii) For $M16$ bolt, from Table 9.2 coarse since

$A_c = 157 \text{ mm}^2$

$\therefore$ $F = 120 \times 157 = 18{,}840$ N

(iii) For $M30$ bolt, $A_c = 561 \text{ mm}^2$

$F = 120 \times 561 = 67{,}320$ N.

Problem 3: A bolt carries a tensile load of 8 kN and the tightening load is 3 kN. It is made of steel having allowable tensile stress of 120 MPa. Find its size. A soft copper gasket is used.

Given data: $F_t = F_a = 8000$ N

$F_i = 3000$ N

$\sigma = 120$ MPa. (T 18-1)

For soft copper gasket from Table 9.1,

$K = 0.5$

$\therefore$ Find load on bolt, $F_f = KF_a + F_i = 0.5 \times 8000 + 3000 = 7000$ N

Take F_f at least $= F_a$

$\therefore$ $F_f = 8000$ N

But, $F_f = A_c \times \sigma$

$8000 = A_c \times 120 \Rightarrow A_c = 66.66 \text{ mm}^2$

Select standard bolt $M12$ from Table 9.2 (T 18-7)

(Corresponding to $A_c \geq 66.66$)

Problem 4: The cylinder head of a steam engine is subjected to a steam pressure of 0.7 N/mm^2. It is held in position by means of 12 bolts. A soft copper gasket is used to make the joint leak proof. The effective diameter of the cylinder is 300 mm. Find the size of the bolt so that the stress in the bolt is not to exceed 100 N/mm^2.

Given data: Pressure, $P_f = 0.7$ N/mm^2

No. of bolts, $i = 12$

Soft copper gasket from Table 9.1, $K = 0.5$ (T 18-1)

Diameter of the cylinder, $D_i = 300$ mm

Allowable stress, $\sigma = 100$ N/mm^2

(i) Initial load on bolt

$$F_i = 1402.34d \quad (18\text{-}6\ a)$$

$$= 1402.34\left(\frac{d_1}{0.8}\right)$$

$$F_i = 1752.93d_i$$

(ii) Total load, $F = \frac{\pi}{4} D_i^2 P_f$

$$F = \frac{\pi}{4} 300^2 \times 0.7 = 49480.08 \text{ N}$$

$\therefore$ Axial load on each bolt, $F_a = \frac{F}{i}$

$$F_a = \frac{49480.08}{12} = 4123.34 \text{ N}$$

(iii) Final load on bolt, $F_f = KF_a + F_i$ (18-4)

$$F_f = 0.5 \times 4123.34 + 1752.93d_i$$

Equate this to allowable capacity

$$F_{all} = \left(\frac{\pi}{4} d_1^2\right) \cdot \sigma = \frac{\pi}{4} d_i^2 \times 100$$

Equating, $\frac{\pi}{4} d_1^2 \times 100 = 0.5 \times 4123.34 + 1752.93d_i$

$$78.54d_1^2 - 1752.93d_1 - 2061.6 = 0$$

$$d_1 = 23.44 \text{ mm}$$

Select standard screw from Table 9.2 based on

$d_1 = 23.44$ i.e. at $d_1 = 25.706$

select *M*30 screw.

Problem 5: A cover plate is bolted on to the flanged end of a pressure vessel through 6 bolts. The inner diameter of the pressure vessel is 200 mm and is subjected to an internal pressure of 10 MPa. Selecting carbon steel C40 (σ_y = 328.6 MPa) as the material for the bolts determine the size of the bolts, considering initial tension for the following cases:

(i) Metal to metal joints,

(iii) A copper gasket.

Given data: No. of bolts = $i = 6$

Inside diameter of the pressure vessel, $D_i = 200$ mm

Internal pressure, $P_f = 10$ MPa.

$\sigma_y = 328.6$ MPa

Assume factor of safety, FOS = 2.5

$$\therefore \quad \text{Allowable stress in bolt, } \sigma = \frac{\sigma_y}{\text{FOS}} = \frac{328.6}{2.5}$$

$$\sigma = 131.44 \text{ MPa.}$$

(i) Metal to metal contact

$K = 0$ from Table 9.1 (T 18-1)

$$\text{Total axial force, } F = \frac{\pi}{4} D_i^2 P_f$$

$$= \frac{\pi}{4} 200^2 \times 10 = 314.16 \times 10^3 \text{ N}$$

$$\therefore \quad \text{Load on each bolt, } F_a = \frac{F}{i} = \frac{314.16 \times 10^3}{6}$$

$$F_a = 52.36 \times 10^3 \text{ N}$$

$F_i = 2804.69d$ for metal to metal contact

$$F_i = 2804.69\left(\frac{d_1}{0.8}\right)$$

$$F_f = KF_a \times F_i$$

$$F_f = 0 + 2804.69\left(\frac{d_1}{0.8}\right)$$

$$\left(\frac{\pi}{4} d_1^2\right)\sigma = 2804.69\left(\frac{d_1}{0.8}\right)$$

$$\frac{\pi}{4} d_1^2 \times 131.44 = 2804.69\left(\frac{d_1}{0.8}\right)$$

$$d_1 = 33.96 \text{ mm}$$

Select standard screw *M*42 From Table 9.2 (T 18-7)

(Corresponding to $d_1 > 33.96$).

(ii) For copper gasket take $K = 0.5$ (T 18-1)

$$F_i = 1402.34d$$

$$F_f = KF_a + F_i$$

$$\left(\frac{\pi}{4}d_1^2\right)\sigma = 0.5 \times 52.36 \times 10^3 + 1402.34\left(\frac{d_1}{0.8}\right)$$

$$\frac{\pi}{4}d_1^2 \times 131.44 = 26180 + 1752.92d_1$$

$$103.23d_1^2 - 1752.92d_1 - 26180 = 0$$

$$d_1 = 26.54 \text{ mm}$$

Select standard screw *M*36 from Table 9.2. (T 18-7)

Problem 6: A steel bolt of *M*20 is used to connect two plates of each 16 mm thick. A soft copper of gasket of 3 mm thick is used in between the plates for the joint to be leak proof. The outside and inside diameters of gasket are 50 mm and 22 mm respectively. Take modulus of elasticity of bolt material as 200 MPa and for gasket material as 120 MPa. The bolt is subject to an axial load of 15 kN. Determine the stress induced in the bolt.

Given data: Screw size = *M*20

From Table 9.2, area of bolt = $A_b = 245 \text{ mm}^2$ (T 18-7)

Outside dia of gasket, $d_{og} = 50$ mm

Inside dia of gasket, $d_{ig} = 22$ mm

Thickness of gasket, $l_g = 3$ mm.

$$E_b = 200 \text{ GPa} = 200 \times 10^3 \text{ MPa}$$

$$E_g = 120 \text{ GPa} = 120 \times 10^3 \text{ MPa}$$

Length of bolt, $l_b = 16 + 3 + 16 = 35$ mm

(Bolt length should cover 2 plates each 16 mm and gasket thickness = 3 mm)

Axial load, $F_a = 15000$ N

$$\text{Area of gasket, } A_g = \frac{\pi}{4}(50^2 - 22^2) = 1583.36 \text{ mm}^2$$

Initial or tightening load, $F_i = 1402.34d$

$$F_i = 1402.34 \times 20 = 28.04 \times 10^3 \text{ N}$$

Final load on bolt, $F_f = KF_a + F_i$ (18-4)

$$K = \left[\frac{\frac{E_b A_b}{l_b}}{\frac{E_b A_b}{l_b} + \frac{E_g A_g}{l_g}}\right] \quad (18\text{-}4\text{ a})$$

$$K = \frac{\left(\frac{200 \times 10^3 \times 245}{35}\right)}{\left(\frac{200 \times 10^3 \times 245}{35}\right) + \left(\frac{120 \times 10^3 \times 1583.36}{3}\right)} = 0.0216$$

$$F_f = 0.0216 \times 15 \times 10^3 + 28.04 \times 10^3$$

$$F_f = 28364 \text{ N}$$

$$\text{Stress} = \frac{F_f}{A_b} = \frac{28364}{245} = 115.77 \text{ MPa.}$$

9.3 IMPACT LOADING

$$\text{Energy absorbed, } U = \frac{F \cdot \delta}{2}$$

F = maximum load under impact

δ = maximum instantaneous deflection under impact,

$$\therefore \quad \text{Stress, } \sigma = \frac{F}{A}$$

A = area of cross section.

Problem 7: A $M20 \times 2$ steel bolt of 100 mm long is subjected to impact load. The energy absorbed by the bolt is 2 N-m.

(i) Determine the stress in the shank of the bolt if, there is no threaded portion between the nut and the bolt head.

(ii) Determine the stress in the shank if, the entire length of the bolt is threaded.

Given data: Screw = $M20 \times 2$

$d = 20$ mm, $P = 2$ mm

From Table 9.2, For $M20 \times 2$ screw, $A_c = 258$ mm^2 (T 18-7)

Length, $l = 100$ mm Assume $E = 206.8 \times 10^3$ MPa for steel.

Energy, $U = 2$ N-m $= 2 \times 10^3$ N-mm

$$U = \frac{F\delta}{2}$$

$$\text{WKT } \delta = \frac{Fl}{AE}$$

$$\therefore \quad U = \frac{F}{2}\left(\frac{Fl}{AE}\right)$$

(i) No threaded portion between nut and bolt head.

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 20^2 = 341.16 \text{ mm}^2$$

(Full dia is available to resist load)

$$2000 = \frac{F}{2} \cdot \left(\frac{F \times 100}{314.16 \times 206.8 \times 10^3} \right) \Rightarrow F = 50.98 \times 10^3 \text{ N}$$

$$\therefore \quad \text{Impact stress, } \sigma = \frac{F}{A} = \frac{50.98 \times 10^3}{314.16} = 162.26 \text{ MPa}$$

(ii) It the entire length of the bolt is threaded.
Then the core area of the thread, $A_c = 258$ from table

$$U = \frac{F}{2} \left(\frac{Fl}{AE} \right)$$

$$2000 = \frac{F}{2} \left(\frac{F \times 100}{258 \times 206.8 \times 10^3} \right) \Rightarrow F = 46.197 \times 10^3 \text{ N}$$

$$\therefore \quad \text{Impact stress, } \sigma = \frac{F}{A} = \frac{46.197 \times 10^3}{258} = 179.06 \text{ MPa.}$$

9.4 ECCENTRIC LOADING: TYPE-I

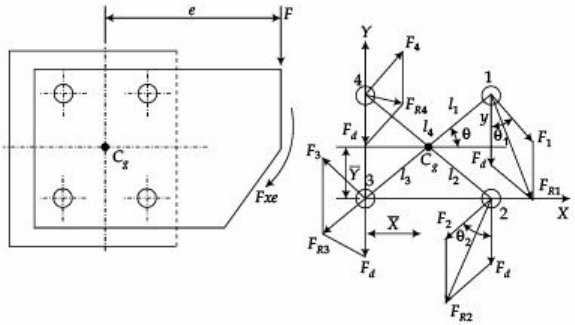


Fig. 9.1

Figure 9.1 shows an eccentrically loaded bolted joint. The load is eccentric because it acts $||^{l}$ to and away from C_g axis

F = force
C_g = center of gravity
e = eccentricity, is the distance from C_g to force
1, 2, 3, etc. are bolts.
Join C_g and bolt centre 1, 2, 3 etc.
l_1 = distance from C_g to bolt 1,
l_2 = distance from C_g to bolt 2,
l_3 = distance from C_g to bolt 3, etc.
$\overline{X}$ and $\overline{Y}$ = distance to C_g from reference axes
F_d = direct load on each bolt acts parallel to given force F
F_1, F_2, F_3, etc. = normal forces on bolts 1, 2, 3, etc. act perpendicular to l_1, l_2, l_3, etc.
F_{R_1} = resultant force on bolt 1,
F_{R_2} = resultant force on bolt 2, etc.
θ_1 = angle between F_d and F_1
θ_2 = angle between F_d and F_2 etc.
τ = allowable shear stress for bolt material.

Design Procedure

Step 1: Find the position of centre of gravity C_g.

$$\overline{X} = \frac{X_1 + X_2 + X_3 + \ldots}{\text{number of bolts}}$$

$$\overline{Y} = \frac{Y_1 + Y_2 + Y_3 + \ldots}{\text{number of bolts}}$$

where $X_1, X_2, X_3 \ldots$ are horizontal distances from reference y-axis to bolts 1, 2, 3, etc. and Y_1, Y_2, Y_3 etc. are vertical distances from reference x-axis to bolts 1, 2, 3, etc.

Step 2: Find distances l_1, l_2, l_3, etc.

l_1, l_2, l_3, etc. are diagonals for some triangle. Locate the triangle and find the length

For example, $l_1 = \sqrt{x^2 + y^2}$

Find eccentricity e.

Draw the F_d and F_n for all bolts. Draw resultant. Farthest bolt from C_g (whose l is more) having θ less than 90°is heavily loaded bolt.

Find cos θ for that bolt

For example, $\cos \theta_1 = \frac{x}{l_1}$

Step 3: Direct load per bolt $F_d = \frac{F}{i}$ (This acts parallel to F)

Step 4: Normal load on heavily loaded bolt

$$F_n = \frac{Fel_n}{l_1^2 + l_2^2 + l_3^2 + \ldots}$$

For bolt 1, $F_1 = \frac{Fel_1}{l_1^2 + l_2^2 + l_3^2 + \ldots}$

For bolt 2, $F_2 = \frac{Fel_2}{l_1^2 + l_2^2 + l_3^2 + \ldots} = \frac{F_1}{l_1} \times l_2$

$$F_3 = \frac{Fel_3}{l_1^2 + l_2^2 + l_3^2 \ldots} = \frac{F_1}{l_1} \times l_3 \text{ or } \frac{F_2}{l_2} \cdot l_3 \text{ etc.}$$

These normal loads act perpendicular to l_1, l_2, l_3 etc.

Step 4: Resultant force on heavily loaded bolt

$$F_R = [F_d^2 + F_n^2 + 2F_dF_n \cos \theta_n]^{1/2}$$

Step 6: Area of bolt $A_c = \frac{F_R}{\text{stress}}$

Refer Table 9.2, select standard bolt based on A_c. (T 18-7)

Problem 8: A bracket is bolted as shown in Fig. 9.2. All the bolts are identical and have yield strength of 400 MPa. Determine the size of bolts assuming the factor of safety as 3.

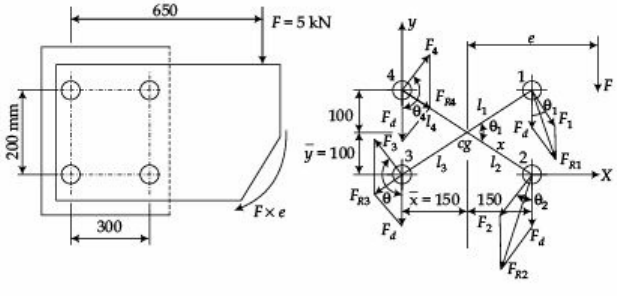


Fig. 9.2

Given data: $F = 5000$ N

$\sigma_y = 400$ MPa, FOS = 3

$\therefore$ Allowable tensile stress $\sigma = \dfrac{\sigma_y}{\text{FOS}} = \dfrac{400}{3} = 133.33$ MPa

Assume allowable shear stress $\tau = 0.5\,\sigma = 66.67$ MPa.

Step 1: To locate C_g of bolts

$$\overline{X} = \frac{X_1 + X_2 + X_3 + \ldots}{i} = \frac{300 + 300 + 0 + 0}{4} = 150 \text{ mm.}$$

$$\overline{Y} = \frac{Y_1 + Y_2 + Y_3 + \ldots}{i} = \frac{200 + 0 + 0 + 200}{4} = 100 \text{ mm.}$$

Locate C_g taking 150 mm horizontal and 100 mm vertial from O. (centre of rivet 3)

Step 2: e = distance from C_g to force

$= 650 - \overline{X} = 650 - 150 = 500$ mm

$$l_1 = \sqrt{x^2 + y^2} = \sqrt{150^2 + 100^2} = 180.28$$

$$l_2 = \sqrt{150^2 + 100^2} = 180.28$$

$$l_3 = \sqrt{150^2 + 100^2} = 180.28$$

$$l_4 = \sqrt{150^2 + 100^2} = 180.28$$

(By virtue of symmetry $l_1 = l_2 = l_3 = l_4$ here)

Draw F_d parallel to F at all bolts.

Draw F_1 perpendicular to l_1, F_2 perpendicular to l_2 etc. such that the direction of bending moment created by $F_1, l_1, F_2, l_2, F_3\, l_3$ etc. will be the same as created by the external moment F_e.

Here F_e is clockwise about C_g. Hence F_1, F_2, F_3 etc. are so drawn to give clockwise moments about C_g.

It is observed that for bolts 1 and 2, the angle θ is less than 90° and for bolts 3 and 4, θ is > 90°. Hence consider bolts 1 and 2, and among these, farthest bolt from C_g is heavily loaded. Since $l_1 = l_2$

$$\cos\theta_1 = \frac{x}{l_1} = \frac{150}{180.28} = 0.832$$

Step 3: Direct load/bolt $F_d = \frac{F}{i} = \frac{5000}{4} = 1250$ N

Step 4: Normal load on heavily loaded bolt

$$F_1 = \frac{Fel_1}{l_1^2 + l_2^2 + l_3^3 + \ldots} = \frac{5000 \times 500 \times 180.28}{180.28^2 + 180.28^2 + 180.28^2 + 180.28^2}$$

$\therefore$ $F_1 = 3466.88$ N.

Step 5: Resultant load on heavily loaded bolt

$$F_R = (F_d^2 + F_1^2 + 2\, Fd\, F_1 \cos\theta_1)^{1/2}$$

$$= (1250^2 + 3466.88^2 + 2 \times 1250 \times 3466.88 \times 0.832)^{1/2}$$

$$F_R = 4559.92 \text{ N}$$

Step 6: $\therefore$ Area of bolt $A_c = \frac{F_R}{\text{Stress}} = \frac{4559.92}{66.67} = 68.4$ mm^2

Ref. Table 9.2 select standard bolt *M*12 (T 18-7)

(For $A_c \geq 68.4$ i.e. at $A_c = 84.3$).

Problem 9: For an eccentrically loaded bolted bracket shown in Fig. 9.3, determine the diameter of bolt required taking allowable shear stress in bolt as 90 MPa.

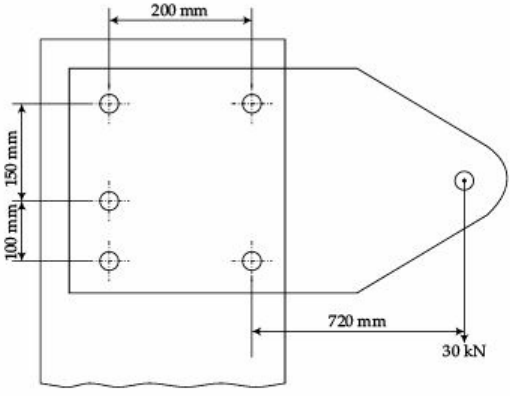


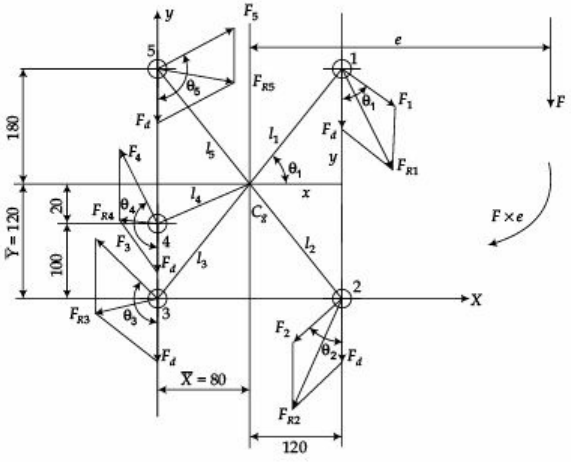
Fig. 9.3

Given data: Load, $F = 30{,}000$ N

Number of bolts = 5 (from figure)

Allowable shear stress in bolts, $\tau = 90$ MPa.

Draw the given bolts separately and mark the C_g, l_1, l_2, e, F_d and F_n as shown.



Step 1: To locate C_g:

$$\overline{X} = \frac{X_1 + X_2 + X_3 + \ldots}{i} = \frac{200 + 200 + 0 + 0 + 0}{5} = 80 \text{ mm}$$

$$\overline{Y} = \frac{Y_1 + Y_2 + Y_3 + \ldots}{i} + \frac{250 + 0 + 0 + 100 + 250}{5} = 120 \text{ mm}$$

Find x and distances to other rivets from C_g.

Step 2: $e = 720 + x = 720 + 120 = 840$ mm.

Draw l_1, l_2, l_3, etc. F_d, F_1, F_2 etc. as shown in above figure.

$$l_1 = \sqrt{x^2 + y^2} = (120^2 + 180^2)^{1/2} = 216.33 \text{ mm}$$

$$l_2 = (120^2 + 120^2)^{1/2} = 169.71 \text{ mm}$$

$$l_3 = (80^2 + 120^2)^{1/2} = 144.22 \text{ mm}$$

$$l_4 = (80^2 + 20^2)^{1/2} = 82.46 \text{ mm}$$

$$l_5 = (80^2 + 180^2)^{1/2} = 196.98 \text{ mm}$$

It is seen from the above figure that for bolts 1 and 2, angles between F_d and F_n are less than 90°, and $l_1 > l_2$ farthest bolt from C_g having $\theta < 90$ i.e. bolt '1' is heavily loaded.

$$\therefore \quad \cos\theta = \frac{x}{l_1} = \frac{120}{216.33} = 0.555$$

Step 3: Direct load/bolt $F_d = \dfrac{F}{i} = \dfrac{30{,}000}{5} = 6000$ N

(This acts parallel to F)

Step 4: Normal load on heavily loaded bolt '1'

$$F_1 = \frac{Fel_1}{\Sigma l^2} = \frac{30000 \times 840 \times 216.33}{(216.33^2 + 169.71^2 + 144.22^2 + 82.46^2 + 196.98^2)}$$

$$= 38391.\ \text{N}$$

Step 5: Resultant load on heavily loaded bolt, '1',

$$F_{R_1} = (F_d^2 + F_1^2 + 2F_dF_1 \cos\theta)^{1/2}$$

$$= (6000^2 + 38391^2 + 2 \times 6000 \times 38391 \times 0.555)^{1/2}$$

$$F_{R_1} = 42018.48 \text{ N}$$

Step 6: $\therefore$ Area of bolt, $A_c = \dfrac{F_{R_1}}{\text{Stress}} = \dfrac{42018.48}{90} = 466.88 \text{ mm}^2$

From Table 9.2 select standard bolt *M*30.

Problem 10: Determine the safe load carried by a bolted bracket shown in Fig. 9.4. The bolts used are *M* 24 and allowable shear stress for bolt material is 60 MPa.

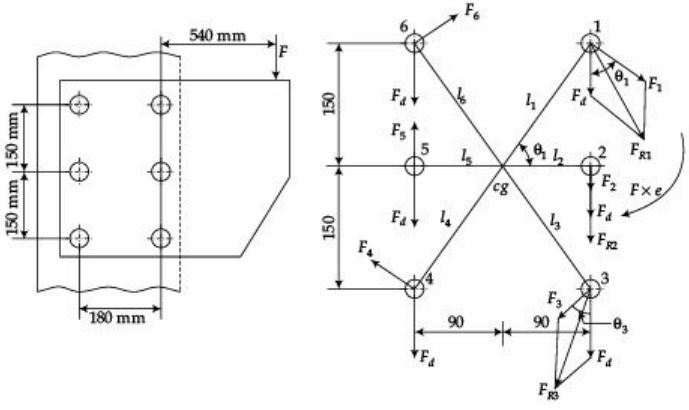


Fig. 9.4

Given data: Size of bolt = *M*24

From Table 9.2 area of bolt, $A_c = 353 \text{ mm}^2$ (T 18-7)

Allowable shear stress in bolt, $\tau = 60$ MPa

Step 1: Since the arrangement of bolts is symmetrical the C_g lies in the middle.

$$\bar{X} = \frac{180}{2} = 90 \text{ mm}$$

$$\bar{Y} = 150 \text{ mm}$$

Step 2:

$$l_1 = l_3 = l_4 = l_6 = \sqrt{90^2 + 150^2} = 174.93 \text{ mm}$$

$$l_2 = l_5 = 90 \text{ mm}$$

It is seen that for bolts 1, 2, and 3, the angle θ is < 90 and bolt 1 is the farthest (or bolt 3, same), hence bolt 1 or bolt 3 is heavily loaded

$$\cos \theta_1 = \frac{x}{l_1} = \frac{90}{174.93} = 0.515$$

$$e = 540 + 90 = 630 \text{ mm}$$

Step 3: Direct load/bolt $F_d = \frac{F}{i} = \frac{F}{6} = 0.1667\,F$

Step 4: Normal load on heavily loaded bolt,

$$F_1 = \frac{Fel_1}{\Sigma l^2} = \frac{F \times 630 \times 174.93}{4 \times 174.93^2 + 2 \times 90^2}$$

$$F_1 = 0.795F$$

Step 5: Resultant load on heavily loaded bolt

$$F_{R_1} = (F_d^2 + F_1^2 + 2F_dF_1 \cos \theta)^{1/2}$$

$$= [(0.1667F)^2 + (0.795F)^2 + 2(0.1667F)(0.795F)0.515]^{1/2}$$

$$F_{R_1} = 0.8924F$$

Step 5: Stress,

$$\sigma_c = \frac{F_R}{A_c} = \frac{0.8924\,F}{353}$$

$\therefore$

$$F = \frac{A_c \cdot \sigma_c}{0.8924} = \frac{353 \times 60}{0.8924} = 23.735 \times 10^3 \text{ N}$$

Capacity, $F = 23.735 \times 10^3$ N.

Problem 11: Determine the size of the bolt required for a bracket bolted as shown in Fig. 9.5. The allowable shear stress for bolt material is 72 MPa.

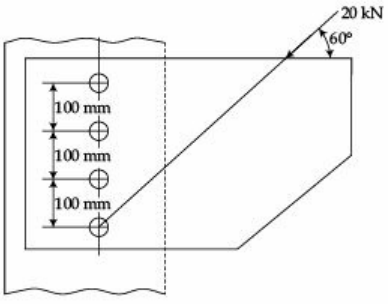


Fig. 9.5

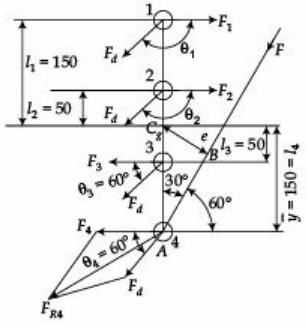


Fig. 9.6

$$F = 20 \text{ kN} = 20 \times 10^3 \text{ N}$$

$$\tau = 72 \text{ MPa.}$$

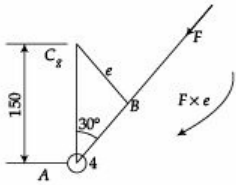
$$\bar{Y} = \frac{300}{2} = 150 \text{ mm (symmetric)}$$

Step 1: $l_1 = l_4 = 150$ mm

$l_2 = l_3 = 50$ mm

Step 2: From triangle ABC_g

$$\sin 30 = \frac{e}{150} \Rightarrow e = 150 \sin 30 = 75 \text{ mm}$$



Direct load on each bolt is acting parallel to given F, i.e. 60° downward with the horizontal.

Normal loads act perpendicularly to l_1, l_2, l_3, etc CW since external moment $F \times e$ is CW about C_g.

It is seen that for bolts 3 and 4, θ is less than 90° and bolt 4 is the farthest from C_g, bolt 4 is heavily loaded

$$\cos \theta_4 = \cos 60 = 0.5$$

Step 3: Direct load per bolt $= F_d = \frac{F}{i} = \frac{20{,}000}{4}$

$$F_d = 5{,}000 \text{ N}$$

Step 4: Normal load on the heavily loaded bolt (4)

$$F_4 = \frac{Fel_4}{\Sigma l^2} = \frac{20{,}000 \times 75 \times 150}{2(150^2 + 50^2)} = 4500 \text{ N}$$

Step 5: Resultant load on the heavily loaded bolt.

$$F_{R_4} = (F_d^2 + F_4^2 + 2 F_d F_4 \cos \theta_4)^{1/2}$$
$$= (5000^2 + 4500^2 + 2 \times 5000 \times 4500 \times 0.5)^{1/2}$$
$$F_{R_4} = 8231.04 \text{ N}$$

Step 6: Area of the bolt $A = \frac{F_R}{\text{Stress}} = \frac{8231.04}{72}$

$$= 114.32 \text{ mm}^2$$

Select the standard bolt = *M*16 (from Table 9.2).

Problem 12: Figure 9.7 shows a bolted joint. The bolt size is *M*36 and allowable shear stress in the bolts is 75 MPa. Find the safe load that can be carried by the joint.

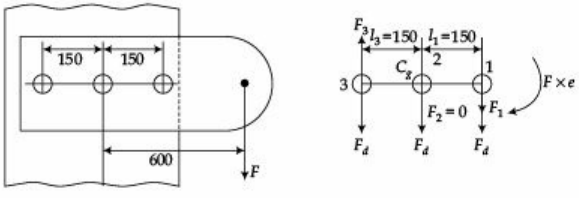


Fig. 9.7

Given data: $\tau = 75$ MPa.

Bolt = *M*36, ∴ From table 9.2, area $A_c = 817 \text{ mm}^2$

Step 1: Centre of gravity

Due to symmetry, $\bar{X} = 150$ mm and $\bar{Y} = 0$. This coincides with bolt (2).

Step 2: $l_1 = l_3 = 150$ mm $l_2 = 0$

It is seen from Fig that $e = 600$ mm

For bolt (1) F_d and F_1 both act downwards.

∴ $\theta = 0 \quad \cos \theta = 1$

bolt 1 is farthest from C_g and $\theta < 90$, hence is heavily loaded

Step 3: Direct load on each bolt, $F_d = \frac{F}{i} = \frac{F}{3}$

Step 4: Normal load on heavily loaded bolt 1

$$F_1 = \frac{Fel_1}{\Sigma l^2} = \frac{F \times 600 \times 150}{150^2 + 0^2 + 150^2} = 2F$$

Step 5: Resultant load on the heavily loaded bolt, '1'

$$F_{R_1} = F_d + F_1 \quad (\because \theta = 0)$$

$$F_{R_1} = \frac{F}{3} + 2F = \frac{7F}{3}$$

Step 6: Area of bolt, $A_c = \dfrac{F_R}{\text{Stress}}$

$$817 = \frac{\left(\frac{7}{3}F\right)}{75} \Rightarrow \text{load capacity } F = 26.26 \times 10^3 \text{ N}$$

$$F = 26.26 \times 10^3 \text{ N}.$$

9.5 ECCENTRIC LOADING: TYPE – II

Rectangular base

Figure 9.8 shows a typical wall bracket with load acting parallel to bolts and the base is rectangular

F = applied load.

e = distance from rocking edge to force

l_1, l_2, l_3, etc., distances from rocking edge to bolts 1, 2, 3, etc.

F_d = direct load per bolt, acts parallel to F

F_1, F_2, F_3, etc., normal forces acting on bolts 1, 2, 3, etc., act perpendicular to l_1 l_2 l_3, etc. (i.e. along the axis of bolts)

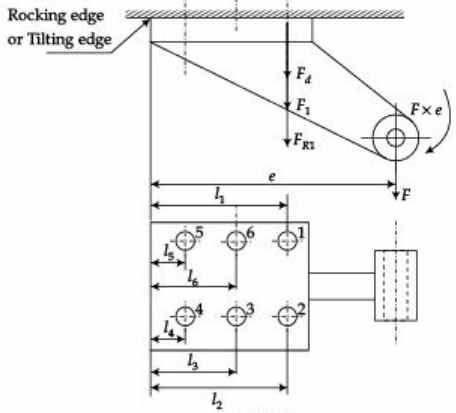


Fig. 9.8

F_R = resultant force on the bolt

σ = allowable tensile stress in the bolts.

A_c = core area of the bolt.

Procedure

Step 1: Find l_1, l_2, l_3, etc.

Step 2: Direct load/bolt $F_d = \dfrac{F}{i}$

Step 3: Normal load on the farthest bolt, which is heavily loaded, $F_1 = \dfrac{Fel_1}{\Sigma l^2}$

Step 4: Resultant load on the heavily loaded bolt

$$F_R = F_d + F_1$$

Step 5: $\therefore$ Core area of the bolt, $A_c = \dfrac{F_R}{\sigma}$

Select the standard bolt from Table 9.2 based on A_c. (T 18-7)

Problem 13: Figure 9.9 shows a wall bracket required to carry a load of 60 kN. The normal yield stress for bolt material is 360 MPa. Taking factor of safety as 2.5, determine the suitable diameter of bolt.

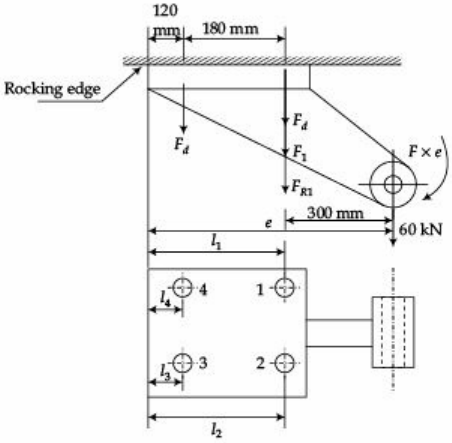


Fig. 9.9

Given data: $F = 60{,}000$ N

$\sigma_y = 360$ MPa

FOS = 2.5

$\therefore$ Allowable stress, $\sigma = \dfrac{\sigma_y}{\text{FOS}}$

$$\sigma = \frac{360}{2.5} = 144 \text{ MPa}$$

Number of bolts, $i = 4$

Step 1: $l_1 = l_2 = 120 + 180 = 300$ mm

$l_3 = l_4 = 120$ mm

$e = 120 + 180 + 300 = 600$ mm

Step 2: Direct load/bolt $F_d = \dfrac{F}{i} = \dfrac{60{,}000}{4} = 15{,}000$ N.

This acts parallel to F, vertical downward on all the bolts.

Step 3: Normal load on bolt, '1', (heavily loaded bolt, which is farthest from rocking edge)

$$F_1 = \frac{Fel_1}{\Sigma l^2} = \frac{60{,}000 \times 600 \times 300}{2 \times (300^2 + 120^2)} = 51.724 \times 10^3 \text{ N}$$

This acts along axis downward at bolt 1.

Step 4: Resultant load on the heavily loaded bolt, '1',

$$F_R = F_d + F_1 = 15{,}000 + 51.724 \times 10^3 \text{ N}$$

$$F_R = 66.724 \times 10^3 \text{ N}$$

Step 5: Core area of the bolt, $A_c = \dfrac{F_R}{\sigma} = \dfrac{66.724 \times 10^3}{144}$

$$A_c = 463.36 \text{ mm}^2 \qquad \text{(T 18-7)}$$

From Table 9.2 select the standard bolt *M*30.

Problem 14: Figure 9.10 shows the outline of a drilling machine, secured by 6 bolts. The maximum permissible tensile stress for bolts is 90 MPa. Find the safe value of the drilling thrust, if the bolts used are *M*30.

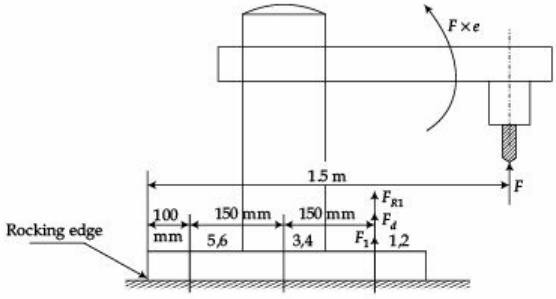


Fig. 9.10

Given data: F = ? bolt diameter = $M30$

From Table 9.2 core area, $A_c = 561\ mm^2$

Allowable tensile stress, $\sigma = 90$ MPa

number of bolts, $i = 6$, eccentricity $e = 1500$ mm

Step 1: Distances from rocking edge to bolts 1, 2, 3, etc.

$$l_1 = l_2 = 100 + 150 + 150 = 400 \text{ mm}$$
$$l_3 = l_4 = 100 + 150 = 250 \text{ mm}$$
$$l_5 = l_6 = 100 \text{ mm}$$

Step 2: Direct load/bolt $F_d = \dfrac{F}{i} = \dfrac{F}{6} = 0.1667F$

This acts upward, parallel to F.

Step 3: Normal load on the heavily loaded bolt (farthest bolt from rocking edge, bolt 1 or 2).

$$F_1 = \frac{Fel_1}{\Sigma l^2}$$

$$F_1 = \frac{F \times 1500 \times 400}{2\,(400^2 + 250^2 + 100^2)} = 1.2903F$$

This also acts upward to give ccw moment (since $F \times e$ is ccw)

Step 4: Resultant load on the heavily loaded bolt

$$F_{R_1} = F_d + F_1 = 0.1667F + 1.2903F$$
$$= 1.4570F$$

Step 5: Area of the bolt, $A_c = \dfrac{F_R}{\sigma} = \dfrac{1.4570F}{90}$

$$561 = \frac{1.4570F}{90}$$

$\therefore$ Load capacity, $F = 34.654 \times 10^3$ N.

9.6 CIRCULAR BASE – ECCENTRICALLY LOADED BRACKETS

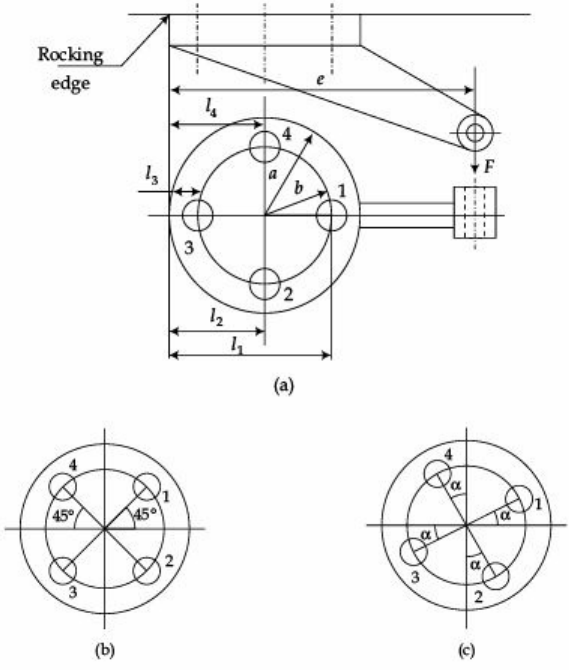


Fig. 9.11 (Fig. 18-5/P 18.15)

Figure 9.11 shows an eccentrically loaded bracket with circular base. The base diameter is $2a$. Pitch circle of bolts is $2b$.

e = distance from rocking edge to force

l_1, l_2, l_3, etc. distance from rocking edge to bolts 1, 2, 3, etc.

Case (a): Bolt no. 1 is along the horizontal line and is farthest from rocking edge and hence is heavily loaded.

$$\text{Normal load, } F_1 = \frac{Fel_1}{l_1^2 + l_2^2 + l_3^3 + \ldots}$$

$$l_1 = a + b$$

$$l_2 = l_4 = a$$

$$l_3 = a - b$$

Substituting, we get

$$F_1 = \frac{2Fe(a+b)}{(2a^2 + b^2)i} \qquad \text{(18-49/P 18.13)}$$

i = number of bolts.

Case (b): The bolts are arranged at 45° to horizontal line, bolts 1 and 2 are heavily loaded

$$l_1 = a + b \cos 45 = l_2$$

$$l_3 = l_4 = a - b \cos 45$$

$$\cos 45° = \frac{1}{\sqrt{2}}$$

Substituting in above equation.

$$F_1 = \frac{2Fe[a + b \cos (180/i)]}{(2a^2 + b^2)i} \qquad \text{(18-50)}$$

(Here $\alpha = 180°/4 = 45°$)

Case (C): When one bolt is heavily loaded inclined at an angle α from the horizontal. (bolt No. = 1)

$$l_1 = a + b \cos \alpha$$

$$l_2 = a + b \sin \alpha$$

$$l_3 = a - b \cos \alpha$$

$$l_4 = a - b \sin \alpha$$

Substituting in F_1 equation, we get

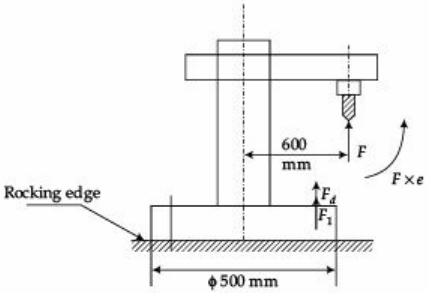
$$F_1 = \frac{2Fe[a + b \cos \alpha]}{(2a^2 + b^2)i} \qquad \text{(18-48)}$$

In all the 3 cases above, direct load/bolt

$$F_d = \frac{F}{i} \text{ acts along axis}$$

If the load is compressive then there will be no direct load ($F_d = 0$).

Problem 15: Figure 9.12 shows the base of a column drilling machine. The drilling force is estimated to be 9 kN. The base is firmly fixed using 4 bolts. The allowable tensile stress for bolt material is 90 MPa. Determine the diameter of the bolt.



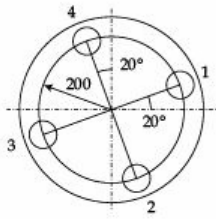


Fig. 9.12

Given data: $F = 9000$ N $\quad e = 600 + \frac{500}{2} = 850$ mm

$2a = 500$ mm $\quad a = 250$ mm

$b = 200$ mm $\quad \sigma = 90$ MPa

$i = 4$ bolts $\quad \alpha = 20°$

Bolts 1 is heavily loaded. (This is case *c*)

1. Direct load/bolts $F_d = \frac{F}{i} = \frac{9000}{4} = 2250$ N

   (acts upward parellel to given load *F*)

2. Normal load, $F_1 = \frac{2Fe(a + b\cos\alpha)}{(2a^2 + b^2)i}$

$$F_1 = \frac{2 \times 9000 \times 850(250 + 200\cos 20)}{(2 \times 250^2 + 200^2)4}$$

$$F_1 = 10152.2 \text{ N.}$$

   (This also acts upward since the moment $F \times e$ is ccw about rocking edge)

3. Resultant load on bolt 1

$$F_R = F_d + F_1$$

$$F_R = 2250 + 10152.2 = 12402.2 \text{ N}$$

4. Area of bolt, $A = \frac{F_R}{\sigma} = \frac{12402.2}{90} = 137.8$

   Select standard bolt *M*16 from Table 9.2. (T 18-7)

Problem 16: Figure 9.13 shows a pillar crane with circular base of diameter 1 m. It is firmly grouted with 8 bolts on pitch circle of diameter 700 mm. Find the suitable size of bolt taking tensile stress as 120 MPa. The crane has to lift a maximum load of 60 kN.

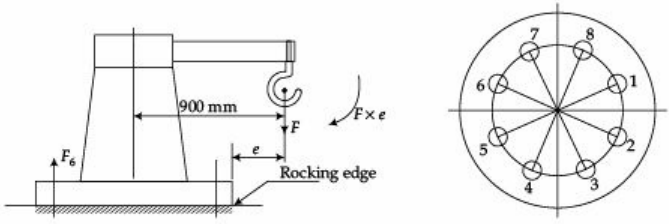


Fig. 9.13

Given data: Diameter of based $2a = 1000$ mm $\Rightarrow a = 500$ mm.

Pitch circle diameter $2b = 700$ mm $\Rightarrow b = 350$ mm

No. of bolts, $i = 8$

Allowable tensile stress, $\sigma = 120$ MPa.

Load, $F = 60{,}000$ N.

1. Since the load is acting downward, the load is taken only by round base and no direct load on bolts ($F_d = 0$).
2. Since bolts 5 and 6 are symmetrically placed both are subjected to maximum stress (case *b*) (farthest bolt from rocking edge)

$$F_5 = F_6 = \frac{Fe(a + b\cos 180/i)}{(2a^2 + b^2)i} \qquad (18\text{-}50)$$

e = distance from rocking edge to load

$e = 900 - 500 = 400$ mm

$$F_5 = \frac{60{,}000 \times 400 \times (500 + 300\cos 180/8)}{(2 \times 500^2 + 300^2)8}$$

$$F_5 = 3951.68 \text{ N}$$

3. $\therefore$ Resultant load on bolt 5 $= 3951.68$ N

4. $\therefore$ Area of bolt $\quad A_c = \dfrac{F_R}{\sigma} = \dfrac{3951.68}{120}$

$$A_c = 32.93 \text{ mm}^2$$

Select standard bolt *M*8 from Table 9.2.

9.7 ECCENTRIC LOADING: TYPE – III

Load acting perpendicular to bolt axis (rectangular base).

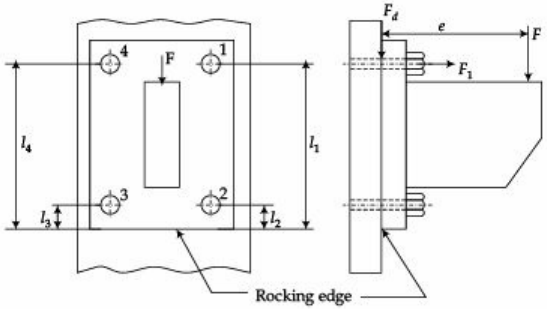


Fig. 9.14

Figure 9.14 shows a bolted bracket subjected to eccentric load acting perpendecular to axis of bolts.

l_1, l_2 l_3, etc. distances from rocking edge to bolts 1, 2, 3, etc.

F_d = direct load/bolt acting parallel to F

F_1 = normal load on bolt 1, acting along axis of bolt

τ_{max} = allowable shear stress in bolts

σ_{max} = allowable normal stress in bolts

e = eccentricity = distance from rocking edge to force

i = number of bolts

This eccentric load has two effects

(1) direct shear load F_d and

(2) tensile load F_1

Procedure

Step 1: Find l_1, l_2, l_3, etc.

Step 2: Direct load/bolt $F_d = \dfrac{F}{i}$

Step 3: Shear stress induced due to direct load

$$\tau = \frac{F_d}{A_c}$$

Step 4: Normal load on the farthest bolt, '1', which is heavily loaded

$$F_1 = \frac{Fel_1}{\Sigma l^2}$$

Step 5: Tensile stress induced due to the normal load

$$\sigma = \frac{F_1}{A_c}$$

Step 6: Combined stresses

(i) Maximum normal stress

$$\sigma_{max} = \frac{1}{2}[\sigma + (\sigma^2 + 4\tau^2)^{1/2}]$$

Find A_c and select dia of bolt from Table 9.2.

(ii) Maximum shear stress.

$$\tau_{max} = \frac{1}{2}[\sigma^2 + 4\tau^2]^{1/2}$$

Find A_c. and select diameter of bolt from Table 9.2.

Recommend the bigger diameter of bolt.

If bolt size is given, then select A_c and find F from both combined stress equations. Recommend the smaller value of force F as maximum capacity.

Problem 17: Figure 9.15 shows a bracket subjected to a transverse load of 45 kN. The allowable normal and shear stress for the bolt material are 135 MPa and 90 MPa respectively. Determine the suitable diameter of the bolts to be used.

Given data: Load, $F = 45 \times 10^3$ N

$e = 725$ mm

$\sigma_{max} = 135$ MPa, $\tau_{max} = 90$ MPa

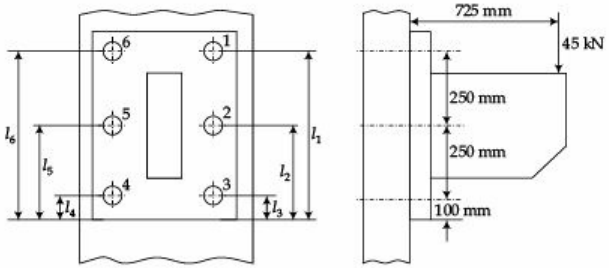


Fig. 9.15

Step 1:

$$l_1 = l_6 = 100 + 250 + 250 = 600 \text{ mm}$$
$$l_2 = l_5 = 100 + 250 = 350 \text{ mm}$$
$$l_3 = l_4 = 100 \text{ mm}$$

Step 2: Direct load/bolt $F_d = \frac{F}{i} = \frac{45{,}000}{6} = 7500$ N

Step 3: Shear stress due to direct load, $\tau = \frac{F_d}{A_c} = \frac{7500}{A_c}$ MPa

Step 4: Normal load on the farthest bolt which is heavily loaded

$$F_1 = \frac{Fel_1}{\Sigma l^2} = \frac{45{,}000 \times 725 \times 600}{2(600^2 + 350^2 + 100^2)} = 19873.1 \text{ N}$$

Step 5: Tensile stress due to normal load, $\sigma = \frac{F_1}{A_c}$

$$\sigma = \frac{19873.1}{A_c} \text{ MPa}$$

Step 6: Combined stress

(i) Maximum normal stress, $\sigma_{max} = \frac{1}{2}[\sigma + (\sigma^2 + 4\tau^2)^{1/2}]$

$$135 = \frac{1}{2}\left[\frac{19873.1}{A_c} + \left\{\left(\frac{19873.1}{A_c}\right)^2 + 4\left(\frac{7500}{A_c}\right)^2\right\}^{1/2}\right]$$

$\therefore$ $A_c = 165.82 \text{ mm}^2$

From Table 9.2, select standard diameter of the bolt = *M*20

(ii) Maximum shear stress, $\tau_{max} = \frac{1}{2}(\sigma^2 + 4\tau^2)^{1/2}$

$$90 = \frac{1}{2}\left[\left(\frac{19873.1}{A_c}\right)^2 + 4\left(\frac{7500}{A_c}\right)^2\right]^{1/2}$$

$\therefore$ $A_c = 138.32 \text{ mm}^2$ $\therefore$ Standard diameter from table = *M*18

Recommended diameter of bolt = *M* 20.

Problem 18: For the eccentrically loaded bracket using *M*20 bolts shown in Fig. 9.16, calculate the maximum load that can be applied if the allowable tensile and shear stresses in the bolt are 90 MPa and 52 MPa respectively.

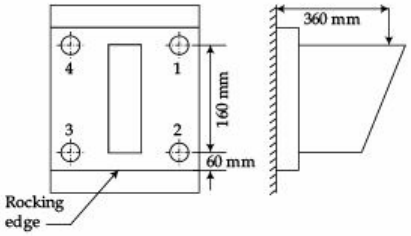


Fig. 9.16

Given data: Bolt diameter = $M20$

For $M20$ bolt, from Table 9.2, Area $A_c = 245 \text{ mm}^2$

$$\sigma_{max} = 90 \text{ MPa} \quad \tau_{max} = 52 \text{ MPa} \quad e = 360 \text{ mm}$$

Step 1:

$$l_1 = l_4 = 60 + 160 = 220 \text{ mm}$$

$$l_2 = l_3 = 60 \text{ mm}$$

number of bolts $i = 4$

Step 2: Direct load/bolt $F_d = \dfrac{F}{i} = \dfrac{F}{4} = 0.25F$

Step 3: Shear stress, $\tau = \dfrac{F_d}{A} = \dfrac{0.25F}{A_c}$

Step 4: Normal stress on the farthest bolt that is heavily loaded,

$$F_1 = \frac{Fel_1}{\Sigma l^2} = \frac{F \times 360 \times 220}{2(220^2 + 60^2)} = 0.7615F$$

Step 5: Tensile stress, $\sigma = \dfrac{F_1}{A} = \dfrac{0.7615F}{A_c}$

Step 6: Combined stress,

(i) Maximum normal stress, $\sigma_{max} = \dfrac{1}{2}[\sigma + (\sigma^2 + 4\tau^2)^{1/2}]$

$$90 = \frac{1}{2}\left[\frac{0.7615F}{A_c} + \left\{\left(\frac{0.7615F}{A_c}\right)^2 + 4\left(\frac{0.25F}{A_c}\right)^2\right\}^{1/2}\right]$$

$$90 = \frac{0.836F}{A_c}$$

$$\therefore \quad F = \frac{90 \times A_c}{0.836} = \frac{90 \times 245}{0.836} = 26.367 \times 10^3 \text{ N}$$

(ii) Maximum shear stress, $\tau_{max} = \frac{1}{2}(\sigma^2 + 4\tau^2)^{1/2}$

$$52 = \frac{1}{2}\left[\left(\frac{0.7615F}{A_c}\right)^2 + 4\left(\frac{0.25F}{A_c}\right)^2\right]^{1/2}$$

$$52 = \frac{0.4555F}{A_c}$$

$$\therefore \quad F = \frac{52A_c}{0.4555} = \frac{52 \times 245}{0.4555} = 27.969 \times 10^3 \text{ N}$$

Recommended capacity, $F = 26.367 \times 10^3$ N (smaller value).

Circular base joint

The conditions are same as discussed earlier in Fig. 9.11*a*,*b*, and *c*.

When one bolt is lying on horizontal axis (Fig. 9.11*a*)

$$F_1 = \frac{2Fe(a+b)}{(2a^2+b^2)i} \qquad \text{(18-49/P 18.13)}$$

When two bolts are equally stressed (Fig. 9.11*b*)

$$F_1 = \frac{2Fe[a+b\cos(180/i)]}{(2a^2+b^2)i} \qquad \text{(18-50)}$$

When one bolt is experiencing maximum stress (Fig. 9.11*c*)

$$F_1 = \frac{2Fe(a+b\cos\alpha)}{(2a^2+b^2)i} \qquad \text{(18-48)}$$

The Procedure of bolt selection is the same as discussed for rectangular base.

Problem 19: A flanged bearing is shown in Fig. 9.17 is fastened to a frame by five bolts spaced equally on bolt circle of 300 mm diameter. The diameter of the flange is 400 mm. Load is 40 kN. Taking the safe tensile stress for bolt material as 60 MPa, determine the size of bolts.

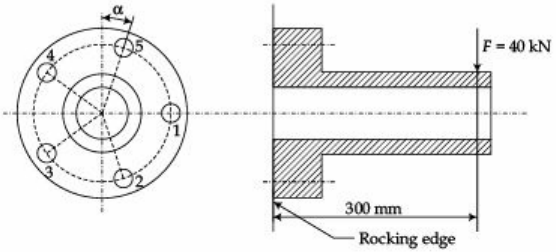


Fig. 9.17

Given data: $F = 40{,}000$ N $\quad e = 300$ mm $\quad \sigma_{max} = 60$ MPa

$i = 5$ bolts $\quad 2a = 400 \quad 2b = 300$ mm

Bolt 5 is farthest from rocking edge and is heavily loaded

$$\text{Angle between bolts} = \frac{360}{5} = 72°$$

$\therefore$ Between bolts 1 and 5, angle $\theta = 72°$

$\therefore \quad \alpha = 90 - 72 = 18°$

Step 1: Direct load/bolt $F_d = \dfrac{F}{i} = \dfrac{40{,}000}{5} = 8000$ N

Step 2: $\therefore$ Shear stress, $\tau = \dfrac{F_d}{A_c} = \dfrac{8000}{A_c}$

Step 3: Normal load on the heavily loaded bolt

$$F_5 = \frac{2Fe(a + b\cos\alpha)}{(2a^2 + b^2)i} \qquad (18\text{-}48)$$

$$= \frac{2 \times 40{,}000 \times 300\,(200 + 150\cos 18)}{(2 \times 200^2 \times 150^2)\,5} = 16046.45 \text{ N}$$

Step 4: Tensile stress, $\sigma = \dfrac{F_5}{A_c} = \dfrac{16046.45}{A_c}$

Step 5: Maximum normal stress

$$\sigma_{max} = \frac{1}{2}[\sigma + (\sigma^2 + 4\tau^2)^{1/2}]$$

$$60 = \frac{1}{2}\left[\frac{16046.45}{A_c} + \left\{\left(\frac{16046.45}{A_c}\right)^2 + 4\left(\frac{8000}{A_c}\right)^2\right\}^{1/2}\right]$$

$\therefore \quad A_c = 322.56 \text{ mm}^2$

From Table 9.2, select standard bolt diameter M 24 mm.

Problem 20: Figure 9.18 shows a flanged bolted joint. The flange diameter is 500 mm and the bolt circle diameter is 350 mm. Determine the size of the bolt, if the yield stress for bolt material is 353 MPa. Use factor of safety as 2.5.

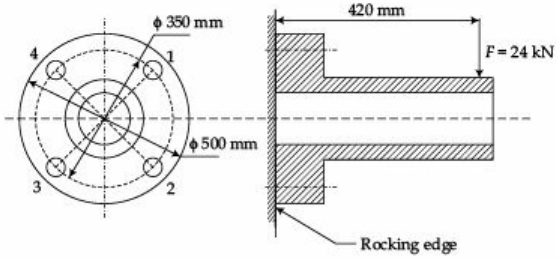


Fig. 9.18

Given data: $F = 24 \times 10^3$ N $\quad i = 4$ bolts $\quad e = 420$ mm

$2a = 500 \text{ mm} \Rightarrow a = 250 \text{ mm}$

$2b = 350 \text{ mm} \Rightarrow b = 175 \text{ mm}$

$\sigma_y = 353$ MPa $\quad$ FOS = 2.5

$\therefore \quad$ Allowable tensile stress, $\sigma = \dfrac{\sigma_y}{\text{FOS}}$

$$\sigma = \sigma_{max} = \frac{353}{2.5} = 141.2 \text{ MPa}$$

Assume allowable shear stress, $\tau_{max} = 0.5\sigma_{max}$

$$\tau_{max} = 0.5 \times 1412 = 70.6 \text{ MPa}$$

(1) Direct load/bolt $F_d = \dfrac{F}{i} = \dfrac{24 \times 10^3}{4} = 6000$ N

(2) Shear stress, $\tau = \dfrac{F_d}{A_c} = \dfrac{6000}{A_c}$

(3) Normal load on heavily loaded bolt
(Farthest bolt from rocking edge is bolt 1 or 4)
Two bolts are equally stressed. (Case b)

$$\therefore \quad F_1 = \frac{2Fe[a + b\cos(180/i)]}{(2a^2 + b^2)i} \tag{18-50}$$

$$F_1 = \frac{2 \times 24 \times 10^3 \times 420\,[250 + 175\cos(180/4)]}{(2 \times 250^2 + 175^2)4}$$

$$= 12.104 \times 10^3 \text{ N}$$

4. $\therefore$ Tensile stress, $\sigma = \frac{F_1}{A} = \frac{12.104 \times 10^3}{A_c}$

5. Combined stresses

(i) Maximum normal stress

$$\sigma_{max} = \frac{1}{2}[\sigma + (\sigma^2 + 4\tau^2)^{1/2}]$$

$$141.2 = \frac{1}{2}\left[\frac{12.104 \times 10^3}{A_c} + \left\{\left(\frac{12.104 \times 10^3}{A_c}\right)^2 + 4\left(\frac{6000}{A_c}\right)^2\right\}^{1/2}\right]$$

$$A_c = 103.26 \text{ mm}^2$$

(ii) Maximum shear stress, $\tau_{max} = \frac{1}{2}(\sigma^2 + 4\tau^2)^{1/2}$

$$70.6 = \frac{1}{2}\left[\left(\frac{12.104 \times 10^3}{A_c}\right)^2 + 4\left(\frac{6000}{A_c}\right)^2\right]^{1/2}$$

$\therefore \quad A_c = 120.89 \text{ mm}^2$

$\therefore$ Recommended area, $A_c = 120.89 \text{ mm}^2$ (bigger one)

Refer table 9.2, select standard bolt *M*16.

Problem 21: Figure 9.19 shows a circular flanged bolted joint carrying a transverse load of 30 kN. Find suitable diameter of the bolt taking allowable tensile and shear stresses in bolt as 150 MPa and 90 MPa respectively.

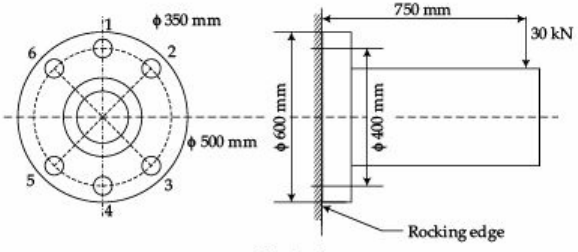


Fig. 9.19

Given data: $F = 30 \times 10^3$ N, $\sigma_{max} = 150$ MPa $\tau_{max} = 90$ MPa

$e = 750$ mm, $2a = 600$ mm $\Rightarrow a = 300$ mm

$2b = 400$ mm $\Rightarrow b = 200$ mm

$i = 6$ bolts, bolt 1 is heavily loaded (farthest bolt from rocking egde) Case: a

Step 1: Direct load/bolt $F_d = \dfrac{F}{i} = \dfrac{30{,}000}{6} = 5000$ N

Step 2: Shear stress, $\tau = \dfrac{F_d}{A_c} = \dfrac{5000}{A_c}$

Step 3: Normal load on bolt 1 $F_i = \dfrac{2Fe\,(a+b)}{(2a^2+b^2)i}$ (18-49)

$$= \frac{2 \times 30{,}000 \times 750 \times (250 + 200)}{[2 \times 250^2 \times 200^2] \times 6} = 20.455 \times 10^3 \text{ N}$$

Step 4: Tensile stress, $\sigma = \dfrac{F_1}{A_c} = \dfrac{20.455 \times 10^3}{A_c}$

Step 5: Combined stresses

(i) Maximum normal stress, $\sigma_{max} = \dfrac{1}{2}[\sigma + (\sigma^2 + 4\tau^2)^{1/2}]$

$$150 = \frac{1}{2}\left[\frac{20.455 \times 10^3}{A_c} + \left\{\left(\frac{20.455 \times 10^3}{A_c}\right)^2 + 4\left(\frac{5000}{A_c}\right)^2\right\}^{1/2}\right]$$

$\therefore$ $A_c = 144.08$ mm^2

(ii) Maximum shear stress, $\tau_{max} = \dfrac{1}{2}(\sigma^2 + 4\tau^2)^{\frac{1}{2}}$

$$90 = \frac{1}{2}\left[\left(\frac{20.455 \times 10^3}{A_c}\right)^2 + 4\left(\frac{5000}{A_c}\right)^2\right]^{1/2}$$

$A_c = 126.49$ mm^2

Recommending the bigger area = 144.08 mm^2

From Table 9.2 select standard bolt = *M* 16

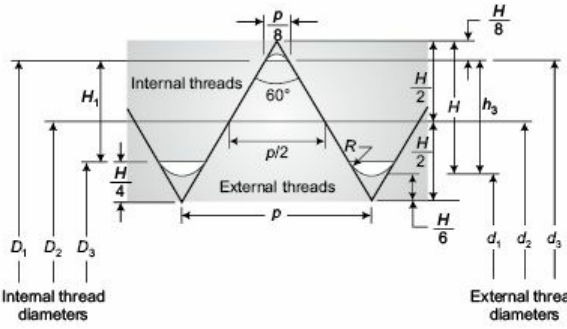


Table 9.2 Basic dimensions (mm) for ISO metric screw threads (fine series)

Designation	Nominal or major dia	Pitch	Pitch diameter	Minor diameter		Tensile stress area
	d_1, D_1	p	d_2, D_2	External (d_3)	Internal (D_3)	N/mm²
M4 × 0.5	4	0.5	3.67	3.39	3.46	9.8
M5 × 0.5	5	0.5	4.67	4.4	4.46	16.1
M6 × 0.75	6	0.75	5.5	5.1	5.2	22.0
M8 × 1.0	8	1.0	7.38	6.77	6.92	39.2
M10 × 1.25	10	1.25	9.188	8.466	8.647	61.2
M10 × 1	10	1.00	9.350	8.773	8.917	64.5
M12 × 1.5	12	1.50	11.026	10.130	10.376	88.1
M12 × 1.25	12	1.25	11.188	10.466	10.647	92.1
M16 × 1.5	16	1.50	15.026	14.160	14.376	167
M16 × 1	16	1.00	15.350	14.773	14.917	178
M20 × 2	20	2.00	18.701	17.546	17.825	258
M20 × 1.5	20	1.50	19.026	18.160	18.376	272
M24 × 2	24	2.00	22.701	21.546	21.835	384
M24 × 1.5	24	1.50	23.026	22.160	22.376	401
M30 × 3	30	3.00	28.051	26.319	26.752	581
M30 × 2	30	2.00	28.701	27.546	27.835	621
M36 × 3	36	3.00	34.051	32.319	32.752	865
M36 × 2	36	2.00	34.701	33.546	33.835	915
M42 × 4	42	4.00	39.402	37.093	37.670	1150
M42 × 3	42	3.00	40.051	38.319	38.752	1210
M48 × 4	48	4.00	45.402	43.093	43.670	1540
M48 × 3	48	3.00	46.051	44.319	44.752	1600

Basic dimensions (mm) for ISO metric screw threads (Coarse series)

Designation	Nominal or major dia	Pitch	Pitch diameter	Minor diameter		Tensile stress area
	d_1, D_1	p	d_2, D_2	External (d_3)	Internal (D_3)	N/mm^2
M4	4	0.70	3.545	3.141	3.242	8.78
M5	5	0.80	4.480	4.019	4.134	14.20
M6	6	1.00	5.350	4.773	4.917	20.10
M8	8	1.25	7.188	6.466	6.647	36.60
M10	10	1.5	9.026	8.160	8.376	58.00
M12	12	1.75	10.863	9.853	10.106	84.30
M16	16	2.00	14.701	13.546	13.835	157
M20	20	2.50	18.376	16.933	17.294	245
M24	24	3.00	22.051	20.319	20.752	353
M30	30	3.50	27.727	25.706	26.211	561
M36	36	4.00	33.402	31.093	31.129	817
M42	42	4.50	39.077	36.479	37.129	1120
M48	48	5.00	44.752	41.866	42.587	1470
M56	56	5.50	52.428	49.252	50.046	2030
M64	64	6.00	60.103	56.639	57.505	2680
M72	72	6.00	68.103	64.639	65.505	3460
M80	80	6.00	76.103	72.639	73.505	4340
M90	90	6.00	86.103	82.639	83.505	5590
M100	100	6.00	96.103	92.639	93.505	7000

EXERCISE

1. A flat cover plate is used to close the flanged end of a pressure vessel of internal diameter 300 mm. The internal pressure is 2.5 MPa. A hard copper gasket is used to make the joint leak proof. Eight bolts are used to fasten the cover plate. Select C30 steal (σ_y = 294.2 MPa) as a material for bolt and factor of safety of 3. Find the diameter of the bolts.
2. Find the strength of a *M*36 bolts if the allowable tensile stress for bolt material is 120 MPa.
3. A pillar crane is fastened to the foundation by 12 bolts spaced equally on a bolt circle of 1.8 m diameter. The diameter of pillar flange is 2.1 m. Determine the size of bolts when a load of 30 kN acts at a radius of 0.8 m. The allowable stress for bolt material is 100 MPa.
4. A bolt in a steel structure is subject to a tensile load of 9 kN and the initial tightening load on bolt is 5 kN. Determine the size of the bolt taking allowable stress as 80 MPa for the bolt.
5. Figure 9.20 shows a bolted joint. Determine the size of bolts required taking allowable tensile stress in bolt as 120 MPa.

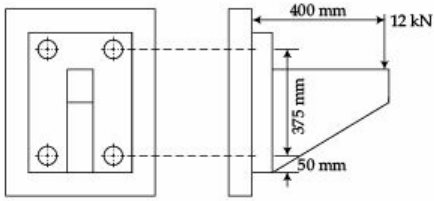


Fig. 9.20

6. A bracket is fixed to a wall by means of 5 bolts as shown in Fig. 9.21. Determine the load carrying capacity of the bracket, if the allowable stresses in bolt are 100 MPa in tension and 60 MPa in shear.

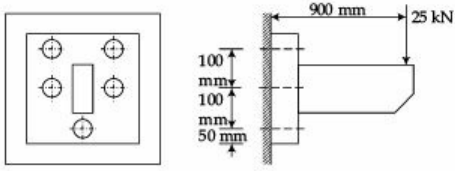


Fig. 9.21

7. Find the size of bolts for a bracket loaded as shown in Fig. 9.22.

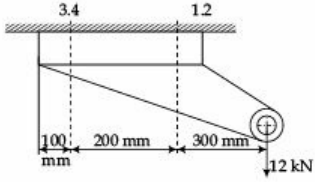


Fig. 9.22

8. Find the dia of the bolt for a bracket loaded as shown in Fig. 9.23. The allowable shear stress for bolt material is 60 MPa.

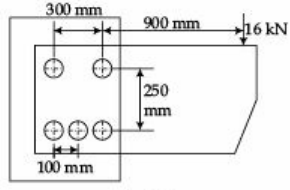


Fig. 9.23

9. Figure 9.24 shows an eccentrically loaded bolted joint. Find the maximum value of eccentricity, 'e', if the allowable shear stress for the material is 75 MPa. The bolts used are $M24$.

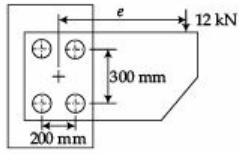


Fig. 9.24

10. Determine the size of bolts for the bracket shown in Fig. 9.25, if bolts are so arranged around the bolt circle diameter of 500 mm, when the 4 bolts are:
    (i) Not equally spaced ($\alpha = 30°$)
    (ii) Equally spaced ($\alpha = 45°$)

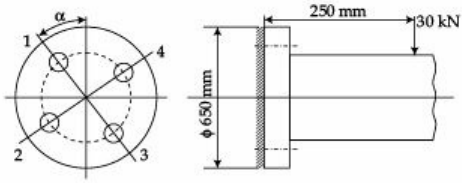


Fig. 9.25

11. A pulley bracket is supported by 4 bolts 2 at A and 2 at B as shown in Fig. 9.26. Determine the size of bolt using allowable tensile stress in bolts as 40 Mpa.
    (**Hint:** For direct load

$$F_c = 1000 \text{ N}, \quad F_p = T_1 + T_2 = 20{,}000 \text{ N}$$

Total $\quad F = F_c + F_p = 1000 + 20{,}000$

$\therefore \quad F = 21{,}000 \text{ N}$

$$F_d = \frac{F}{i} = \frac{21{,}000}{4} = 5250 \text{ N}.$$

For $\quad F_1 = \dfrac{(F_c e_1 + F_p e_2) l_1}{\Sigma l^2}$

$e_1 = 300, \qquad e_2 = 375)$

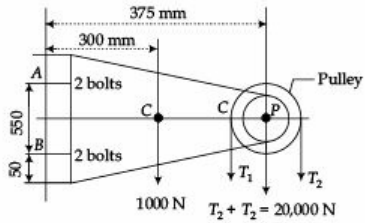


Fig. 9.26

CHAPTER

10 Mechanical Joints

These joints include pin joint, knuckle joint, cotter joint and turn buckle or coupler. These joints basically join two rods and support axial load.

The other types of mechanical joints are riveted joints and welded joints that are discussed in separate chapters.

10.1 TAPER PIN JOINT

This type of joint is used connect a shaft and a sleeve or pipe or two pipes. This joint can support an axial load or can be used for transmitting torque or both. Figure 10.1 shows some typical types of pin joints.

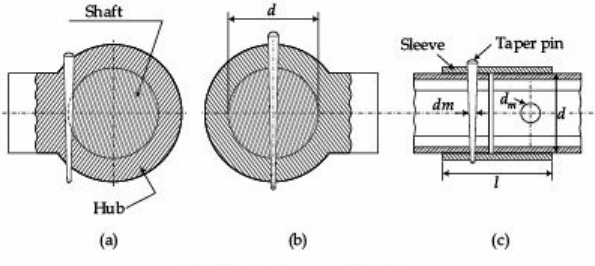


Fig. 10.1 Tapper pin joints.

(Fig. 17-6, 17-7/P 17.8)

Fig. 10.1(a) is an example of use of taper pin as a key,

Fig. 10.1(b) is an example of a shaft and sleeve joined using a taper pin, and

Fig. 10.1(c) shows the joining two hollow tubes using as sleeve and taper pins.

Procedure: The pin is subjected to double shear

$$\text{Shear stress, } \tau = \frac{F}{2A}$$

$$A = \text{mean area of taper pin} = \frac{\pi}{4} d_m^2$$

$$d_m = \text{mean diameter of pin}$$

F is the tangential force acting at the periphery of the shaft. This may be direct pull or push (axial load) or may be due to torque,

$$\text{Torque, } M_t = 2F\left(\frac{d}{2}\right) = F \cdot d$$

where d = dia of shaft.

Problem 1: A shaft of 60 mm diameter is connected to a steel pipe of inside diameter 60 mm by a taper pin. The assembly is subjected to a tensile load of 30 kN. Find the suitable diameter of the pin taking allowable shear stress as 60 MPa.

Given data: Shaft diameter, $d = 60$ mm.

Load, $F = 30 \times 10^3$ N

Shear stress, $\tau = 60$ MPa.

For the pin in double shear, $\tau = \dfrac{F}{2A}$

$$60 = \frac{30{,}000}{2\left(\frac{\pi}{4} d_m^{\,2}\right)}$$

$\therefore$ Mean diameter of pin, $d_m = 17.84 \simeq 18$ mm.

Problem 2: A which shaft of 50 mm diameter is fixed to a sleeve, by means of a taper pin of mean diameter 10 mm which rotates at 1200 rpm. Taking the allowable shear stress as 75 MPa, determine the torque capacity of the pinned joint.

Given data: Diameter of shaft, $d = 50$ mm.

Mean diameter of pin, $d_m = 10$ mm.

Allowable shear stress, $\tau = 75$ MPa.

Speed, $n = 1200$ rpm.

For the pin in double shear, $\tau = \dfrac{F}{2A}$

$$F = \tau(2A) = 75\left(2\frac{\pi}{4}d_m^2\right) = 75 \times 2 \times \frac{\pi}{4} \times 10^2$$

$$F = 11.78 \times 10^3 \text{ N.}$$

$\therefore$ Torque capacity, $M_t = (F \cdot d) = 11.78 \times 10^3 \times 50$

$$M_t = 589 \times 10^3 \text{ N-mm} = 589 \text{ N-m}$$

$\therefore$ Power, $P = \dfrac{2\pi n M_t}{60{,}000} = \dfrac{2 \times \pi \times 1200 \times 589}{60{,}000} = 74.02 \text{ kW.}$

Problem 3: A lever 0.5 m long is pinned to a shaft of diameter 40 mm using a taper pin. If a force of 300 N is applied at the end of the lever, taking allowable shear stress as 90 MPa, find suitable diameter of the taper pin.

Given data: Length of lever, $L = 500$ mm.

Diameter of shaft, $d = 40$ mm.

Force applied, $F = 300$ N

Allowable shear stress, $\tau = 90$ MPa.

Applied torque, $M_t = F \cdot L = 300 \times 500 = 150{,}000$ N-mm

Tangential force acting at the shaft that causes shearing of pin

$$F_\tau = \frac{M_t}{\left(\frac{d}{2}\right)} = \frac{150{,}000}{\left(\frac{40}{2}\right)} = 7500 \text{ N}$$

For the pin in double shear, $\tau = \dfrac{F_\tau}{2A} \Rightarrow 90 = \dfrac{7500}{2\left(\frac{\pi}{4}d_m^2\right)}$

$\therefore$ Pin diameter $= 7.28 \simeq 8$ mm.

10.2 KNUCKLE JOINT

Knuckle joint is shown in Fig. 10.2. This is used to connect two round rods subjected to axial tensile load. The main parts of knuckle joints are fork, eye and a knuckle pin. The end of one rod is made *U*-shaped called fork, and the end of other rod is round in shape, called eye. The knuckle pin is placed inside the hole of the fork and the eye. This cannot support any compressive load.

Symbols: F = tensile load on rods, N

d = diameter of rods, mm

d_1 = diameter at rod end, mm

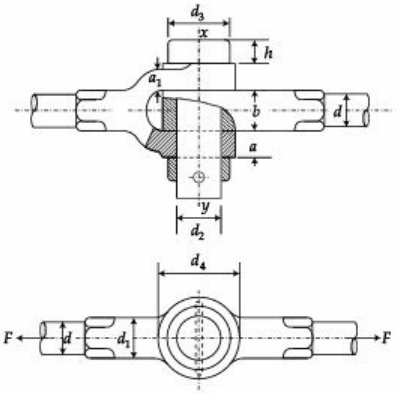


Fig. 10.2 Knuckle joint.

d_2 = diameter of knuckle pin, mm ...(Fig. 17-9/P 17.9)
d_3 = diameter of pin-head, mm
d_4 = diameter eye, mm
a = thickness of fork on each side, mm
b = thickness of eye, mm
h = thickness of pin head, mm
σ_t = allowable tensile stress, MPa
τ = allowable shear stress, MPa
σ_c = allowable crushing or bearing stress, MPa.

Procedure

Step 1: Design of rods

Consider rods under tensile load.

$$\text{Axial stress, } \sigma_t = \frac{F}{\left(\frac{\pi d^2}{4}\right)} = \frac{4F}{\pi d^2} \quad \text{...(17-43/P 17.5)}$$

Find d = ?

Step 2: Rod end

Assume diameter of the rod end, $d_1 = 1.2d$

Step 3: Design of knuckle pin:

For the pin under double shear,

Shear stress in the pin, $$\tau = \frac{F}{2\left(\frac{\pi d_2^2}{4}\right)}$$

$$\tau = \frac{2F}{\pi d_2^2} \quad ...(17\text{-}53)$$

Find pin dia d_2 = ?

Assume pin head diameter, $d_3 = 2d_2$

and pin head thickness, $h = 0.5d_2$

Step 4: Design of the fork

Compressive stress in the fork, $$\sigma_c = \frac{F}{2ad_2} \quad ...(17\text{-}52)$$

Find thickness of the fork a.

Step 5: Design of eye

(a) Compressive stress in the eye, $$\sigma_c = \frac{F}{d_2 b} \quad ...(17\text{-}51)$$

Find thickness of the eye b.

(b) Shear stress in the eye, $$\tau = \frac{F}{b(d_4 - d_2)} \quad ...(17\text{-}48)$$

Find diameter of the eye, d_4 = ?

Problem 4: Design and sketch the assembly of a knuckle joint to connect two mild steel rods, subjected to a tensile load of 90 kN. The material selected for the joint has the following stresses: allowable tensile stress σ_t = 90 MPa, allowable shear stress τ = 60 MPa and allowable crushing stress σ_c = 150 MPa.

Given data: Load, $F = 90 \times 10^3$ N

σ_t = 90 MPa

τ = 60 MPa

σ_c = 150 MPa

Step 1: Design of rods

For the rods in tension $$\sigma_t = \frac{4F}{\pi d^2} \quad ...(17\text{-}46)$$

$$90 = \frac{4 \times 90 \times 10^3}{\pi d^2}$$

$\therefore$ Dia of rods, $d = 35.68 \simeq 36$ mm.

Step 2: Assume diameter at the rod end, $d_1 = 1.2d$

$$d_1 = 1.2 \times 36 = 43.2 \simeq 44 \text{ mm.}$$

Step 3: Design of the knuckle pin

For the pin in double shear, shear stress, $\tau = \dfrac{2F}{\pi d_2^2}$ ...(17-53)

$$60 = \frac{2 \times 90 \times 10^3}{\pi d_2^2}$$

$\therefore$ Diameter of the knuckle pin, $d_2 = 30.9 \simeq 31$ mm.

Assume diameter of the pin head, $d_3 = 2d_2 = 62$ mm

Pin head thickness, $h = 0.5d_2 = 15.5 = 16$ mm.

Step 4: Design of the fork

Compressive stress in the fork, $\sigma_c = \dfrac{F}{2ad_2}$ ...(17-52)

$$150 = \frac{90{,}000}{2 \times a \times 31}$$

$\therefore$ Thickness of the fork $a = 9.67 \simeq 10$ mm

Step 5: Design of the eye

(a) Compressive stress in the eye, $\sigma_c = \dfrac{F}{d_2 b}$ ...(17-51)

$$150 = \frac{90{,}000}{31 \times b}$$

$\therefore$ Thickness of the eye, $b = 19.35 \simeq 20$ mm

(b) Shear stress in the eye, $\tau = \dfrac{F}{b(d_4 - d_2)}$ ...(17-48)

$$60 = \frac{90 \times 10^3}{20(d_4 - 31)}$$

$\therefore$ Diameter of the eye, $d_4 = 106$ mm.

Problem 5: Design a knuckle joint to connect two round rods of 42 mm diameter. The ultimate strength of rods in tension is 420 MPa. Ultimate compressive and shear stresses for the material are 510 MPa and 306 MPa, respectively. Take factor of safety as 6.

Given data: Diameter of the rods, $d = 42$ mm

σ_{ul} for the rods = 420 MPa.

FOS = 6

$\therefore$ Allowable tensile stress in the rods, $\sigma_t = \dfrac{\sigma_u}{FOS}$

$$\sigma_t = \frac{420}{6} = 70 \text{ MPa.}$$

Ultimate compressive stress $\sigma_{uc} = 510$ MPa.

$\therefore$ $$\sigma_c = \frac{\sigma_{uc}}{FOS} = \frac{510}{6} = 85 \text{ MPa.}$$

Ulimate shear stress $\tau_u = 306$ MPa.

$\therefore$ $$\tau = \frac{\tau_u}{FOS} = \frac{306}{6} = 51 \text{ MPa.}$$

Step 1: To find force capacity of the rods

$$F = \frac{\pi}{4} d^2 \sigma_t = \frac{\pi}{4} \times 42^2 \times 70 = 96.98 \times 10^3 \text{ N}$$

The joint may be designed to have equal strength in all areas. Hence, take $F = 96.98 \times 10^3$ N, for the design other elements.

Step 2: Assume diameter at the rod end, $d_1 = 1.2d = 50.4 \simeq 51$ mm

Step 3: Design of the knuckle pin

Shear stress in the pin, $\tau = \dfrac{2F}{\pi d_2^2}$

$$51 = \frac{2 \times 96.98 \times 10^3}{\pi d_2^2}$$

$\therefore$ Diameter of the pin, $d_2 = 34.79 \simeq 35$ mm

Assume pin head diameter, $d_3 = 2d_2 = 70$ mm

Pin head thickness, $h = 0.5d_2 = 17.5 \simeq 18$ mm

Step 4: Design of the fork

Compressive stress in the fork, $\sigma_c = \dfrac{F}{2ad_2}$

$$85 = \frac{96.98 \times 10^3}{2a \times 35}$$

$\therefore$ Thickness of the fork, $a = 16.299 \simeq 17$ mm

Step 5: Design of the eye

(a) Compressive stress in the eye, $\sigma_c = \dfrac{F}{d_2 b}$

$$85 = \frac{96.98 \times 10^3}{35 \times b}$$

$\therefore$ Thickness of the eye, $b = 32.598 \simeq 33$ mm

(b) Shear stress in the eye, $\tau = \dfrac{F}{b(d_4 - d_2)}$

$$51 = \frac{96.98 \times 10^3}{33(d_4 - 35)}$$

$\therefore$ Diameter of the eye, $d_4 = 92.62 \simeq 93$ mm.

10.3 COTTER JOINT

Cotter joint is used to connect two round rods that are subjected to axial tensile or compressive load. Similar to knuckle joint, cotter joint has three parts viz., the sleeve or socket, the spigot and the cotter. The spigot is a solid round member that is kept inside the sleeve. Cotter is a rectangular member that is driven through the slot provided in both sleeve and spigot. The end of one rod is spigot and the end of other rod is the sleeve. Cotter takes the tensile load and collar absorbs the compressive load. A typical cotter joint is shown in Fig. 10.3.

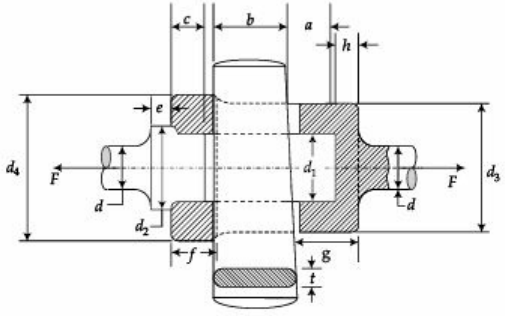


Fig. 10.3 Cotter joint (Socket spigot type).

(Fig. 17-10/P 17.9)

Symbols: F = axial load, tensile or compressive, N
d = diameter of rods, mm

d_1 = diameter of spigot, mm
d_2 = diameter of collar, mm
d_3 and d_4 = outside diameters of sleeve or socket, mm
a = width at rod end, mm
b = width of collar, mm
c = width at socket end, mm
e = thickness of collar, mm
t = thickness of cotter, mm
σ = allowable axial stress in rods MPa
τ = allowable shear stress, MPa
σ_c = allowable crushing stress, MPa

Procedure

Step 1: Design of rods

For the rods under axial load,

$$\text{axial stress in the rods, } \sigma = \frac{4F}{\pi d^2} \qquad \text{...(17-62/P 17.6)}$$

Find diameter of the rods d = ?

Step 2: Design of the spigot and the cotter. (To find d_1 and t)

(a) Crushing strength of the cotter, $F = d_1 t\, \sigma_c$ ...(17-69)

Find $d_1 t$ = ? ...(i)

(b) Axial stress across the slot of the rod

$$\sigma = \frac{4F}{\pi d_1^2 - 4d_1 t} \qquad \text{...(17-63)}$$

Substitute for $(d_1 t)$ from (i) and find d_1 = ?

Dia of the spigot d_1 = ?

Then substitute d_1 in (i) and find t = ?

Thickness of the cotter t = ?

Step 3: Design of the cotter

Strength of the cotter in double shear $F = 2bt\tau$ ...(17-65)

Find width of the cotter b = ?

Step 4: Design of the collar

(a) Bearing stress in the collar, $\sigma_c = \dfrac{4F}{\pi(d_2^2 - d_1^2)}$ ...(17-68)

Find collar diameter, d_2 = ?

(b) Shear stress in the collar, $\tau = \dfrac{F}{\pi d_1 e}$ ...(17-72)

Find thickness of the collar, e = ?

Step 5: Design of the sleeve

(a) Axial stress across the slot of the sleeve

$$\sigma = \frac{4F}{\pi(d_3^2 - d_1^2) - 4t(d_3 - d_1)} \quad ...(17\text{-}64)$$

Find outside diameter of the sleeve at back side d_3 = ?

(b) Crushing strength of the socket

$$F = (d_4 - d_1)t \cdot \sigma_c \quad ...(17\text{-}70)$$

Find outside diameter of the socket at the front d_4 = ?

Step 6: Design of the rod end

Shear stress at the rod end due to double shear

$$\tau = \frac{F}{2ad_1} \quad ...(17\text{-}66)$$

Find width at the rod end a = ?

Step 7: Design of the socket end

Shear stress at the socket end, $\tau = \dfrac{F}{2c(d_4 - d_1)}$ ...(17-67)

Find width at the socket end c = ?

Problem 6: Design a socket and spigot type cotter joint to connect two round rods subjected to a steady load of 120 kN. The material used for the joint is C40 steel having yield stress of 324 MPa. Take factor of safety as 4 for tension, 6 for shear and 3 for crushing based on the yield stress.

Given data: Load, $F = 120 \times 10^3$ N

Yield stress, $\sigma_y = 324$ MPa

FOS = 4 for tension (σ_t)

= 6 for shear (τ) and

= 3 for crushing (σ_c)

$\therefore$ $\sigma_t = \dfrac{\sigma_y}{\text{FOS}} = \dfrac{324}{4} = 81$ MPa

$\tau = \dfrac{\sigma_y}{\text{FOS}} = \dfrac{324}{6} = 54$ MPa

$\sigma_c = \dfrac{\sigma_y}{\text{FOS}} = \dfrac{324}{3} = 108$ MPa.

Step 1: Design of the rods

$$\text{Axial stress in the rods, } \sigma_t = \frac{4F}{\pi d^2}$$

$$81 = \frac{4 \times 120 \times 10^3}{\pi d^2}$$

$\therefore$ Diameter of the rods, $d = 43.43 \simeq 44$ mm

Step 2: Design of the spigot and cotter

(a) Crushing strength of the cotter, $F = d_1 t \sigma_c$

$$\therefore \quad d_1 t = \frac{F}{\sigma_c} = \frac{120 \times 10^3}{108} = 1111.11$$

(b) Axial stress across the slot of the rod

$$\sigma = \frac{4F}{\pi d_1^2 - 4 d_1 t}$$

$$81 = \frac{4 \times 120 \times 10^3}{\pi d_1^2 - 4(1111.11)}$$

$\therefore$ Diameter of the spigot, $d_1 = 57.45 \simeq 58$ mm

$\therefore$ Thickness of the cotter, $t = \frac{d_1 t}{d_1} = \frac{1111.11}{58} = 19.15 \simeq 20$ mm

Step 3: Design of the cotter

Strength of the cotter in double shear, $F = 2bt\tau$

$$120 \times 10^3 = 2 \times b \times 20 \times 54$$

$\therefore$ Width of the cotter, $b = 55.56 \simeq 56$ mm

Step 4: Design of the collar

(a) Bearing stress in the collar, $\sigma_c = \frac{4F}{\pi(d_2^2 - d_1^2)}$

$$108 = \frac{4 \times 120 \times 10^3}{\pi(d_2^2 - 58^2)}$$

$\therefore$ Diameter of the collar, $d_2 = 69.13 \simeq 70$ mm

(b) Shear stress in the collar, $\tau = \frac{F}{\pi d_1 e}$

$$54 = \frac{120 \times 10^3}{\pi \times 58 \times e}$$

$\therefore$ Thickness of the collar, $e = 12.19 \simeq 13$ mm

Step 5: Design of the sleeve

(a) Axial stress across the slot of the sleeve

$$\sigma = \frac{4F}{\pi(d_3^2 - d_1^2) - 4t(d_3 - d_1)}$$

$$81 = \frac{4 \times 120 \times 10^3}{\pi(d_3^2 - 58^2) - 4 \times 20(d_3 - 58)}$$

$$\pi(d_3^2 - 58^2) - 4 \times 20d_3 + 4 \times 20 \times 58 = \frac{4 \times 120 \times 10^3}{81}$$

$$\pi d_3^2 - 80d_3 - 11854.24 = 0$$

$$d_3 = 75.47 \simeq 76 \text{ mm}$$

(b) Crushing strength of the socket, $F = (d_4 - d_1)t\sigma_c$

$$120 \times 10^3 = (d_4 - 58)\ 20 \times 108$$

$$\therefore \quad d_4 = 113.56 \simeq 114 \text{ mm.}$$

Step 6: Design of the rod end

$$\text{Shear stress at the rod end, } \tau = \frac{F}{2ad_1}$$

$$54 = \frac{120 \times 10^3}{2(a)58}$$

$\therefore$ Width at the rod end, $a = 19.16 \simeq 20$ mm

Step 7: Design of the socket end

$$\text{Shear stress at the socket end, } \tau = \frac{F}{2c(d_4 - d_1)}$$

$$54 = \frac{120 \times 10^3}{2c(114 - 58)}$$

$\therefore$ Width at the socket end $c = 19.84 \simeq 20$ mm.

Problem 7: Design a cotter joint to connect two round rods of diameter 30 mm. The following stresses may be used.

Tensile stress = 90 MPa.

Shear stress = 60 MPa.

Crushing stress = 150 MPa.

Given data: Diameter of the rods $d = 30$ mm

Allowable tensile stress $\sigma = 90$ MPa.

Allowable shear stress $\tau = 60$ MPa

Allowable crushing stress $\sigma_c = 150$ MPa.

Step 1: Find force capacity of rods in tension.

Design the other parts to have the same strength

Axial stress in the rods, $\sigma = \dfrac{4F}{\pi d^2}$

$$F = \sigma \frac{\pi}{4} d^2 = 90 \times \left(\frac{\pi}{4}\right) 30^2 = 63.617 \times 10^3 \text{ N}$$

Step 2: Design of the cotter and spigot

(a) Crushing strength of the cotter $F = d_1 t \sigma_c$

$$\therefore \quad d_1 t = \frac{F}{\sigma_c} = \frac{63.617 \times 10^3}{150} = 424.12 \qquad \text{..(i)}$$

(b) Axial stress across the slot of the rod

$$\sigma = \frac{4F}{\pi d_1^2 - 4 d_1 t}$$

Substituting for $d_1 t$ from above,

$$90 = \frac{4 \times 63.617 \times 10^3}{\pi d_1^2 - 4 \times 424.12}$$

$\therefore$ dia of the spigot $d_1 = 37.95 \simeq 38$ mm

Substituting in $d_1 t$, $\quad t = \dfrac{d_1 t}{d_1} = \dfrac{424.12}{38} = 11.16 \simeq 12$

Thickness of the cotter $\quad t = 12$ mm

Step 3: Design of the cotter

Strength of the cotter in double shear, $F = 2bt\tau$

$\therefore$ Width of the cotter, $b = \dfrac{F}{2t\tau} = \dfrac{63.617 \times 10^3}{2 \times 12 \times 60} = 44.18 \simeq 45$ mm

Step 4: Design of the collar

(a) Bearing stress in the collar, $\sigma_c = \dfrac{4F}{\pi(d_2^2 - d_1^2)}$

$$150 = \frac{4 \times 63.617 \times 10^3}{\pi(d_2^2 - 38^2)}$$

$\therefore$ dia of collar, $d_2 = 44.54 \simeq 45$ mm

(b) Shear stress in the collar

$$\tau = \frac{F}{\pi d_1 e}$$

$$60 = \frac{63.617 \times 10^3}{\pi \times 38 \times e}$$

$\therefore$ Thickness of the collar, $e = 8.88 \simeq 9$ mm

Step 5: Design of the the sleeve

(a) Axial stress across the slot of the sleeve

$$\sigma = \frac{4F}{\pi(d_3^2 - d_1^2) - 4t(d_3 - d_1)}$$

$$90 = \frac{4 \times 63.617 \times 10^3}{\pi(d_3^2 - 38^2) - 4 \times 12(d_3 - 38)}$$

$$\pi d_3^2 - \pi \times 38^2 - 48d_3 + 48 \times 38 = \frac{4 \times 63.617 \times 10^3}{90}$$

$$\pi d_3^2 - 48d_3 - 5539.88 = 0$$

$$d_3 = 50.32 \simeq 51 \text{ mm.}$$

(b) Crushing strength of socket, $F = (d_4 - d_1)t\sigma_c$

$$63.617 \times 10^3 = (d_4 - 38)12 \times 150$$

$\therefore$ $d_4 = 73.34 \simeq 74$ mm

Step 6: Design of rod end

Shear stress in the rod end, $\tau = \frac{F}{2ad_1}$

$$60 = \frac{63.617 \times 10^3}{2 \times a \times 38}$$

$\therefore$ $a = 13.95 \simeq 14$ mm

Step 7: Design of the socket end

$$\text{Shear stress in the socket end, } \tau = \frac{F}{2c(d_4 - d_1)}$$

$$60 = \frac{63.617 \times 10^3}{2c(74 - 38)}$$

$$c = 14.73 \simeq 15 \text{ mm}.$$

10.4 TURN BUCKLE OR COUPLER

Figure 10.4 shows a turn buckle. This is used to connect threaded ends of two rods and support a tensile load. The main parts of a turn buckle are two screw rods one with right hand threads and the other with a left hand threads—and the couple that consists of two nuts one at each end with RH and LH threads to suit the threaded rods. By rotating the coupler (nut) the screw rods will either move closer or go apart depending on the direction of rotation.

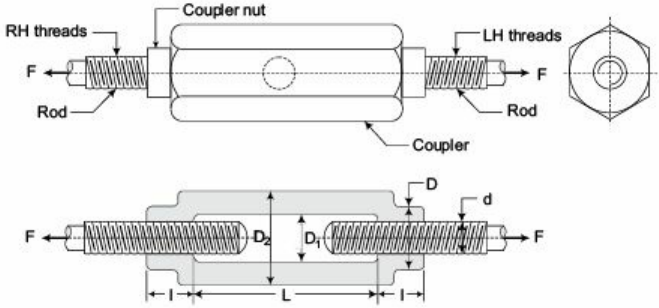


Fig. 10.4 Turn buckle.

...(Fig. 17-12/P 17.9)

Symbols:

F = axial load, N
d = diameter of screw rods
D_1 = Insider diameter of coupler
D_2 = outside diameter of couple
D = outside diameter of nut portion.
l = length of nut portion on each side
L = length of releaved portion of nut.
σ_t = allowable tensile stress
τ = allowable shear stress.

Procedure

Step 1: Design of the screw rods

Strength of the rods based on core diameter

$$F = \frac{\pi}{4} d_c^2 \cdot \sigma_t \qquad \text{...(17-75/P 17.7)}$$

Find core diameter, d_c = ? select standard screw from table 9.2 (Metric thread) d = ?

...(T 18-7)

Step 2: Design of the nut

Resistance of screwed portion of the coupler at each end against shearing.

$$F = \pi l d \tau \qquad \text{...(17-76)}$$

Find nut length, l = ?

But for practical considerations,

$$\text{adopt } l = \text{at least } d \text{ for steel nuts} \qquad \text{...(17-77)}$$

and $l = 1.5d$ for *CI* nuts

Take the bigger one.

Step 3: Strength of the coupler at the nut portion in tension

$$F = \frac{\pi}{4} (D^2 - d^2)\sigma_t \qquad \text{...(17-78)}$$

Find the outside diameter of the nut, D = ?

Step 4: Assume inside dia of coupler $D_1 = 1.2d$

(Relief for the threads)

Step 5: Strength of the coupler at the middle portion

$$F = \frac{\pi}{4} (D_2^2 - D_1^2)\sigma_t \qquad \text{...(17-79)}$$

Find the outside diameter, D_2 = ?

Step 6: Length of coupler, $L = 4d$

Total length including nuts = $l + 4d + l = 6d$, since $l = d$. ...(17-80)

Problem 8: Design a turn buckle to connect two threaded ends of rods and to support a tensile load of 60 kN. The yield stress for the material used is 300 MPa.

Given data: Load, $F = 60 \times 10^3$ N

Yield stress, $\sigma_y = 300$ MPa

Assume factor of safety = FOS = 2.5

$\therefore$ Allowable tensile stress, $\sigma_t = \dfrac{\sigma_y}{\text{FOS}}$

$$\sigma_t = \frac{300}{2.5} = 120 \text{ MPa}$$

Assume allowable shear stress, $\tau = 0.5\sigma$

$$\tau_t = 0.5 \times 120 = 60 \text{ MPa.}$$

Step 1: Design of the screw rods

$$\text{Strength of rods, } F = \left(\frac{\pi}{4} d_c^2\right) \cdot \sigma_t$$

$$60 \times 10^3 = \left(\frac{\pi}{4} d_c^2\right) \cdot 120$$

$$\therefore \quad \text{core area, } A_c = \frac{\pi}{4} dc^2 = \frac{60 \times 10^3}{120} = 500 \text{ mm}^2$$

Refer Table 9.2. Metric *V* threads,

Select standard screw = *M* 30 $\quad \therefore$ diameter, $d = 30$ mm.

Step 2: Design of the nut

Strength of the nut portion of couple against shearing at each end, $F = \pi l d \tau$

$$60 \times 10^3 = \pi \times l \times 30 \times 60$$

$$\therefore \quad \text{length of nut, } l = 10.61$$

Take length of nut = at least $d = 30$ mm

$l = 30$ mm at each end.

Step 3: Strength of the coupler at the nut portion in tension

$$F = \frac{\pi}{4} (D^2 - d^2)\sigma_t$$

$$60 \times 10^3 = \frac{\pi}{4} (D^2 - 30^2) \times 120$$

$$\therefore \quad \text{Outside dia of nut, } D = 39.199 \simeq 40 \text{ mm}$$

Step 4: Assume inside of coupler, $D_1 = 1.2d = 36$ mm

Step 5: Strength of the coupler at the middle portion

$$F = \frac{\pi}{4} (D_2^2 - D_1^2)\sigma_t$$

$$60 \times 10^3 = \frac{\pi}{4} (D_2^2 - 36^2) \times 120$$

$$\therefore \quad \text{Outside diameter of coupler, } D_2 = 43.96 \simeq 44 \text{ mm.}$$

Step 6: Length of coupler, $L = 4d = 4 \times 30 = 120$ mm

Total length couple $= 30 + 120 + 30 = 180$ mm.

Problem 9: A turn buckle is used to tighten ends of two wire ropes. The thread diameter at each end is *M*24. The allowable tensile and shear stresses for the rods and coupler are 90 MPa and 50 MPa respectively. Design the coupler.

Given data: Dia of the screw $= M24 \Rightarrow d = 24$ mm

Refer Table 9.2, core area, $A_c = 353$ mm^2

$\sigma_t = 90$ MPa, $\tau = 50$ MPa.

Step 1: Find load capacity of rods

$$F = A_c \cdot \sigma_t$$
$$= 353 \times 90 = 31{,}770 \text{ N}$$

(Use this F for further calculations)

Step 2: Strength of nut, $F = \pi l d \tau$

$$31{,}770 = \pi \times l \times 24 \times 50$$

$\therefore$ length of nut, $l = 8.43$

Take $l = d = 24$ mm.

Step 3: Strength of the nut based on the outside dia,

$$F = \frac{\pi}{4}(D^2 - d^2)\sigma_t$$

$$31{,}770 = \frac{\pi}{4}(D^2 - 24^2)90$$

$\therefore$ Outside dia of the nut, $D = 32.02 \simeq 32$ mm

Step 4: Assume inside diameter of the coupler

$$D_1 = 1.2d = 1.2 \times 24 = 28.8 \simeq 29 \text{ mm}$$

Step 5: Strength of the coupler based on the middle portion

$$F = \frac{\pi}{4}(D_2^2 - D_1^2)\sigma_t$$

$$31{,}770 = \frac{\pi}{4}(D_2^2 - 29^2) \times 90$$

$\therefore$ Outside diameter of the coupler, $D_2 = 35.9 \simeq 36$ mm

Step 6: Length of middle portion, $L = 4d$

$$L = 4 \times 24 = 96 \text{ mm}$$

Total length $= l + L + l = 24 + 96 + 24$

$= 144$ mm.

EXERCISE

1. Two pipes of outside diameter 30 mm are connected by means of a sleeve and taper pin. The pipes are subjected to a tensile load of 9 kN. Find the diameter of the pin taking allowable shear stress as 60 MPa.
2. Design a knuckle joint to connect two round rods subjected to a tensile load of 100 kN. The design stresses may be taken as 75 MPa in tension, 50 MPa in shear and 135 MPa in crushing.
3. Design a knuckle joint to connect two rods of 50 mm diameter. The following stresses may be used. σ_t = 81 MPa, τ = 54 MPa and σ_c = 135 MPa.
4. Design a cotter joint to support a load varying from 30 kN tension to 30 kN compression. Load is applied statically. The allowable stresses are 50 MPa in shear and 100 MPa in tension and compression.
5. Design a coupler to connect two round threaded rods and to support a tensile load of 90 kN.
6. Two mild steel rods of circular cross section are required to be conneted permitting angular movement. The allowable stresses for all the components in the joint are:

   70 MPa in tension and compression, 45 MPa in shear and 40 MPa in bearing.

   (Note: Since angular movement is required use Knukle Joint)

CHAPTER

11 Riveted Joints

Two or more plates are joined by means of rivets and such joints are called riveted joints. These joints are permanent in nature unlike bolted joints. These are used in bridges, ships, boilers, tanks, etc.

11.1 TYPES OF RIVETED JOINTS

Riveted joints are classified into two types viz, lap joint and butt joint. In lap joint, the two plates that are to be joined are placed on each other and are then riveted. Depending on the number of rows of rivets, the lap joints are called single riveted lap joint, double riveted lap joint, triple riveted lap joint, etc. In single riveted lap joint, there will be one row of rivets, as shown in Fig. 11.1(a). Fig. 11.1(b) shows a double riveted lap joint, Fig. 11.1(c) is a triple riveted lap joint.

In Fig. 11.1(b) and 11.1(c), the rivets are along same horizontal line. This type of rivets arrangement is called chain riveting.

Fig. 11.2(a) and 11.2(b) are examples of zig-zag riveting, wherein the rivet in side by side rows are arranged in the middle of the first row rivets.

In butt joint, the two main plates are put side by side touching each other (butting) and one or two covers are used that overlap these main plates. Accordingly, the butt joints are classified as single cover or double cover butt joints.

Depending on number of rows of rivets, the butt joints are classified as single riveted, double riveted, triple riveted etc. Fig. 11.3 shows single cover butt joints.

Fig. 11.4 shows the single cover butt joints with zigzag arrangement of rivets.

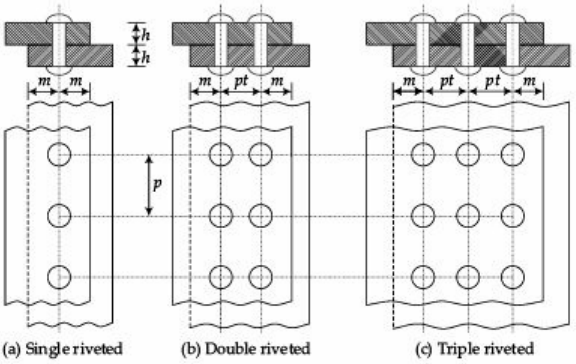


Fig. 11.1 Lap joint with chain riveting

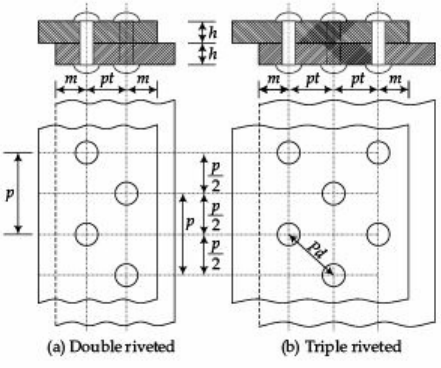


Fig. 11.2 Lap joint with zigzag riveting

In a double cover butt joint two cover plates are used one on each side of main plates (top and bottom). The cover plates may be of equal width or unequal width, according they are called double cover butt joints with equal covers and double cover butt joints with unequal covers. These may be chain riveting or zigzag riveting as shown in Figs. 11.5 and 11.6 respectively.

11.2 TERMINOLOGY

Pitch, p: It is the distance between the centres of two consecutive rivets in a row.

Margin, m: It is the distance from end of plate to first low of rivets.

Transverse pitch, p_t: The distance between two consecutive rows of rivets is called transverse pitch.

Diagonal pitch, p_d: In zigzag riveting, the distance between centres of ad jacent rivets in side by side rows is diagonal pitch.

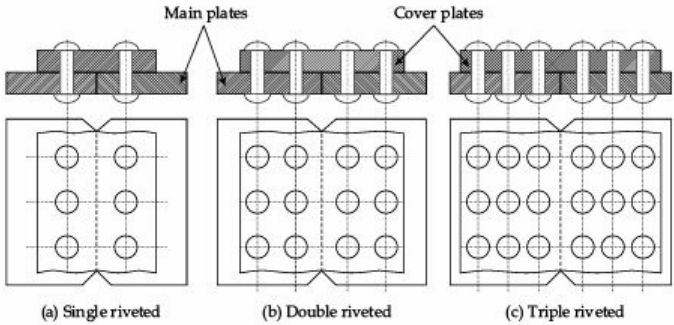


Fig. 11.3 Single cover butt joints with chain riveting

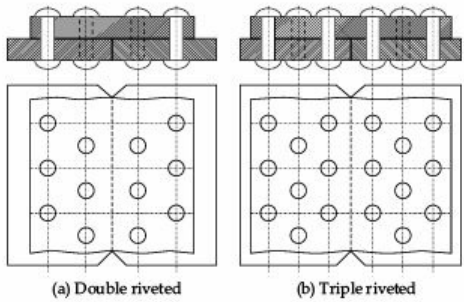


Fig. 11.4 Single cover butt joints with zigzag riveting

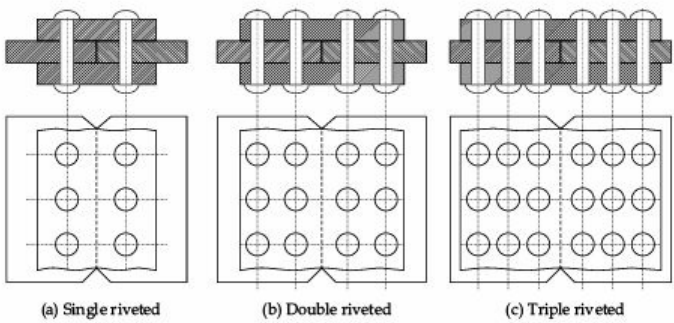


Fig. 11.5 Double cover butt joints (equal covers)

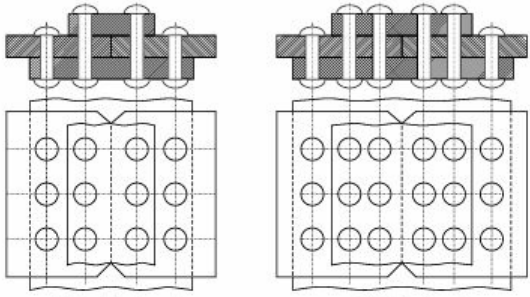


Fig. 11.6 Double cover butt joints (unequal covers)

11.3 FAILURE OF RIVETED JOINTS

A riveted joints can fail in one or more ways as mentioned below:

(i) Tearing of plate
(ii) Shearing of rivets and
(iii) Crushing of rivets.

(i) **Tearing of plate:** Joint may fail due to tearing of plate at margin or tearing of plate along line of rivet holes as shown in Fig. 11.7(a) and (b) respectively.

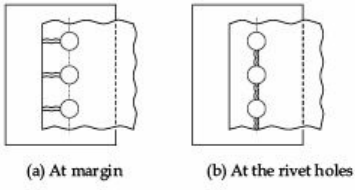


Fig. 11.7 Failure of plate in tearing

(ii) **Shearing of rivets:** The joint may fail due to shearing of rivets as shown in Fig. 11.8.

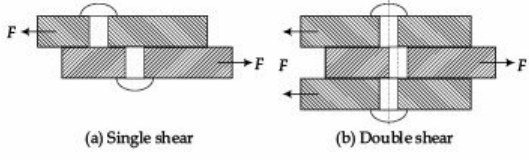


Fig. 11.8 Shearing of rivets

(iii) **Crushing of rivets:** A joint may fail due to crushing of rivets as shown in Fig. 11.9.

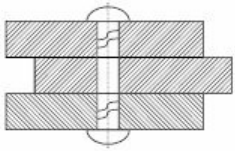


Fig. 11.9 Crushing of rivets

Symbols

F = tensile or compressive load on plates, N
d = diameter of the rivet, mm
d_h = diameter of rivet hole, mm
h = thickness of main plate, mm
h_1, h_2 = thickness of cover plates, mm
i_1 = no. of rivets in single shear in one pitch length or repetitive length of joint.
i_2 = no. of rivets in double shear in one pitch length
m = margin, mm
p = longitudinal pitch or pitch, mm
p_t = transverse pitch, mm
p_d = diagonal pitch, mm
C = corrosion allowance 1 to 3 mm
K = factor as per boiler code (Table 11.2)
η = efficiency
b = width of plate, mm
$\sigma_\theta = \sigma_t$ = allowable tensile stress for plates, MPa
τ = allowable shear stress for rivets, MPa
σ_c = allowable crushing stress for rivets, MPa
For structural joints, allowable stresses are:
σ_θ = 115 MPa
τ = 86 MPa for shop assembly
= 70 MPa for field assembly
σ_c = 210 MPa for shop assembly
= 140 MPa for field assembly

11.4 THEORETICAL STRENGTH OF RIVETED JOINTS

The theoretical strengths of a riveted joint are calculated for one pitch length or repetitive length.

(a) Strength of solid plate, $F = ph\sigma_\theta$ (13-20/P 13.2) ...(1)

(b) Strength of perforated plate

$$F_\tau = (p - d_h)\, h\sigma_\theta \quad (13\text{–}21) \qquad ...(2)$$

(c) Shear strength of rivets

$$F_\tau = (i_1 + 2i_2)\frac{\pi d^2}{4}\tau \qquad ...(13\text{-}22)$$

$2i_2$ is due to shear strength of rivet in double shear is twice that of single shear. However, Indian Boiler Regulation (IBR) recommends that strength of rivet in double shear as 1.875 times that of single shear, then

$$F_\tau = (i_1 + 1.875i_2)\frac{\pi d^2}{4}\tau \qquad ...(3)$$

(d) Crushing strength of rivets

$$F_c = (i_2 h + i_1 h_2) d\sigma_c \quad (13\text{-}23) \qquad ...(4)$$

(For lap joint $h_2 = h$).

Strength of riveted joint = Least of the above strengths [(2), (3), or (4)].

The joint fails wherever the strength is minimum. If tearing strength is minimum, then the joint fails due to tearing of plates. If shear strength of rivets is minimum, then the joint fails due to shearing of rivets and if the crushing strength is minimum, then the joint fails due to crushing of rivets.

$$\text{Efficiency of the joint} = \frac{\text{minimum strength}}{\text{strength of solid plate}}$$

Accordingly, there are three efficiencies of a riveted joints:

(i) Efficiency of the plate = $\eta_p = \dfrac{p - d_h}{p}$ ...(13-27/P 13.3)

(ii) Efficiency of the rivet in shear = $\eta_\tau = \dfrac{(i_1 + 1.875i_2)\pi d^2\tau}{4ph\sigma_\theta}$ ...(13-28)

(iii) Efficiency of the rivet in crushing

$$\eta_c = \frac{\left(i_2 + i_1\dfrac{h_2}{h}\right)\sigma_c}{\left(i_2 + i_1\dfrac{h_2}{h}\right)\sigma_c + \sigma_\theta} \qquad ...(13\text{-}28\ a)$$

Table. 11.1 Normal diameters of rivets and rivet holes

Diameter of rivet, (d) mm	*diameter of rivet hole (d_h) mm*
10 12 14 16 18 20 22 24	$d + 1$
27 30 33 36	$d + 1.5$
39 42 45 48	$d + 2$

Table 11.2 Values of K

No of rivets per pitch length	*Lap joint*	*Butt joint*	
		Single cover	*Double cover*
1	1.31	1.53	1.75
2	2.62	3.06	3.5
3	3.47	4.05	4.63
4	4.17	–	5.52
5	–	–	6.0

Design Procedure for Riveted Joints to Connect Two Plates of Thickness 'h'

Step 1: Diameter of rivet, $d = 6\sqrt{h}$ to $6.3\sqrt{h}$ ...(13-5 c/P 13-1)

Find d and select standard diameter, d of the rivet from Table 11.1 and diameter of the rivet hole, d_h. ...(T 13-2/P 13.4)

Step 2: Pitch

(a) Pitch in general case, considering the strength of perforated plate = shear strength of rivets,

$$p = \frac{(i_1 + 1.875 i_2)\pi d^2 \tau}{4h\sigma_\theta} + d_h \qquad \text{...(13-30/P 13.3)}$$

(b) As per IBR, $p = Kh + 40$

where K is a factor from Boiler Code (Refer Table 11.2 for K values). ...(T 13-14/P 13.11)

Take the smaller value of pitch as standard pitch.

Step 3: Transverse pitch $p_t = 2d$ to $2.5d$ ...(13-6 a/P 13.1)

Take $p_t = 2.25d$

Step 4: Margin $m = 1.5d$ ...(13-11 a)

Step 5: Thickness of the cover plates

(a) For lap joint, no cover plates.

(b) For single cover butt joint, thickness of the cover plate $h_1 = 1.125h$ ...(13-13)

(c) For double cover butt joint, with equal covers $h_1 = h_2 = 0.625h$. ...(13-15)

With unequal covers,

Thickness of the narrow plate, $h_1 = 0.625h$ ...(13-17 a)

Thickness of the wider plate, $h_2 = 0.75h$ ...(13-17 b)

Step 6: Efficiency of joints,

(a) Efficiency plate, $\eta_p = \dfrac{p - dn}{p}$ ...(13-27)

(b) Efficiency of rivet in shear

$$\eta_\tau = \frac{(i_1 + 1.875 i_2)\pi d^2 \tau}{4ph\sigma_\theta} \quad \text{...(13-28)}$$

(c) Efficiency of the rivet in crushing

$$\eta_c = \frac{\left(i_2 + i_1 \dfrac{h_2}{h}\right)\sigma_c}{\left(i_2 + i_1 \dfrac{h_2}{h}\right)\sigma_c + \sigma_\theta} \quad \text{...(13-28 a)}$$

(For lap joint and single cover butt joint take $h_2 = h$)

Efficiency of riveted joint is the least of the above three values. The joint fails at wherever the efficiency is minimum.

Problem 1: Design a double riveted lap joint to connect two plates each 20 mm thick. The allowable stress for rivets and plates are 90 MPa in tension, 60 MPa in shear and 150 MPa in crushing.

Given data: Thickness of the plates, $h = 20$

Allowable tensile stress, $\sigma_\theta = 90$ MPa

Allowable shear stress, $\tau = 60$ MPa

Allowable crushing stress, $\sigma_c = 150$ MPa

Fig. 11.10 shows a double lap joint. Since not mentioned, a chain type riveting is assumed.

Consider one pitch length marked as p

Theoretically the no. of rivets per pitch length are:

- in line (1) $\frac{1}{2} + \frac{1}{2} = 1$ rivet in single shear
- in line (2) $\frac{1}{2} + \frac{1}{2} = 1$ rivet in single shear

(Shaded portion of rivets)

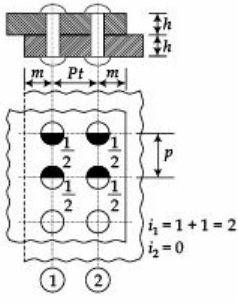


Fig. 11.10

$\therefore$ Total no. of rivets/pitch length

- in single shear $i_1 = 1 + 1 = 2$

  in double shear $i_2 = 0$ (no double shear for lap joint)

Step 1: Diameter of the rivet $d = 6\sqrt{h}$ to $6.3\sqrt{h}$

$= 6\sqrt{20}$ to $6.3\sqrt{20} = 26.83$ to 28.17

From Table 11.1, select standard rivet of diameter, $d = 27$ mm

Diameter of rivet hole, $d_h = 28.5$ mm

$(d + 1.5 = 27 + 1.5 = 28.5$ mm) ...(T 13-2)

Step 2: Pitch p

(a) Pitch in general case, $p = \dfrac{(i_1 + 1.875 i_2)\pi d^2 \tau}{4h\sigma_\theta} + d_h$

$$p = \frac{(2+0)\pi \times 27^2 \times 60}{4 \times 20 \times 90} + 28.5 = 66.67 \text{ mm}$$

(b) Pitch $P = Kh + 40$

From Table 11.2, for 2 rivets per pitch and lap joint $K = 2.62$ ...(Fig. b, T 13-14/P 13.13)

$\therefore$ $p = 2.62 \times 20 + 40 = 92.4$ mm

Recommended pitch, $p = 66.67 \simeq 67$ mm

(Smaller of these two values)

Step 3: Transverse pitch $p_t = 2.25d = 2.25 \times 27 = 60.75 \simeq 61$ mm

Step 4: Margin $m = 1.5d = 1.5 \times 27 = 40.5 \simeq 41$ mm

Step 5: Efficiency

(a) Efficiency of the plate, $\eta_p = \dfrac{p - d}{P}$

$$\eta_p = \frac{67 - 27}{67} = 0.5970 = 59.7\%$$

Efficiency rivet in shear

$$\eta_\tau = \frac{(i_1 + 1.875 i_2)\pi d^2 \tau}{4ph\sigma_\theta} = \frac{(2+0)\pi \times 27^2 \times 60}{4 \times 67 \times 20 \times 90}$$

$$= 0.5697 = 56.97\%$$

(c) Efficiency of the rivet in crushing

$$\eta_c = \frac{\left(i_2 + i_1 \frac{h_2}{h}\right)\sigma_c}{\left(i_2 + i_1 \frac{h_2}{h}\right)\sigma_c + \sigma_\theta} = \frac{\left(0 + 2 \times \frac{20}{20}\right)150}{\left(0 + 2 \times \frac{20}{20}\right)150 + 90}$$

($h_2 = h$ for lap joint)

$$\eta_n = 0.7692 = 76.92\%$$

$\therefore$ Efficiency of joint = 56.97%

(Least of the above three values)

Problem 2: Design a triple riveted zigzag lap joint to connect two plates each 12 mm thick. Draw neat sketch of the joint.

Given data: Thickness of plates, $h = 12$ mm

Assume allowable tensile stress, $\sigma_\theta = 115$ MPa

allowable shear stress, $\tau = 70$ MPa

and allowable crushing stress, $\sigma_c = 140$ MPa

(Considering structural joint)

Figure 11.11 shows a triple riveted zigzag lap joint.

From figure it is seen that

- no. of rivets in single shear per pitch $i_1 = 3$
- no. of rivets in double shear per pitch $i_2 = 0$

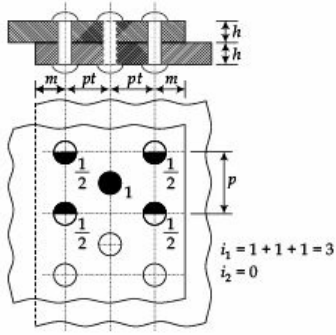


Fig. 11.11

Step 1: Diameter of rivet

$$d = 6\sqrt{h} \text{ to } 6.3\sqrt{h}$$

$$= 6\sqrt{12} \text{ to } 6.3\sqrt{12}$$

$$= 20.78 \text{ to } 21.82$$

From Table 11.1, select standard rivet diameter, $d = 22$ mm and rivet hole diameter

$$d_h = 22 + 1 = 23 \text{ mm}$$

Step 2: Pitch

(a) General case, $p = \frac{(i_1 + 1.875 i_2)\pi d^2 \tau}{4 h \sigma_\theta} + d_h = \frac{(3+0)\pi \times 22^2 \times 70}{4 \times 12 \times 115} + 23 = 80.85$ mm

(b) But $p = Kh + 40$ ...(Fig. f, T 13-14/P 13.22)

From Table 11.2, for lap joint with 3 rivets/pitch, $K = 3.47$, $p = 3.47 \times 12 + 40 = 81.64$ mm

$\therefore$ Recommended pitch $p = 80.85 \simeq 81$ mm

(Smaller value)

Step 3: Transverse pitch $p_t = 2.25 \quad d = 2.25 \times 22 = 49.5 = 50$ mm

Step 4: Margin $m = 1.5d = 1.5 \times 22 = 33$ mm

Step 5: Efficiency

(a) Efficiency of plate, $$\eta = \frac{p - d_h}{p} = \frac{81 - 23}{81} = 0.716$$

$$\eta_p = 71.6\%$$

(b) Efficiency of rivets in shear, $$\eta_\tau = \frac{(i_1 + 1.875 i_2)\pi d^2 \tau}{4 p h \sigma_\theta} = \frac{(3+0)\pi \times 22^2 \times 70}{4 \times 81 \times 12 \times 115}$$

$$\eta_\tau = 71.42\%$$

(c) Efficiency of rivet in crushing, $$\eta_c = \frac{\left(i_2 + i_1 \frac{h_2}{h}\right)\sigma_c}{\left(i_2 + i_1 \frac{h_2}{h}\right)\sigma_c + \sigma_\theta} = \frac{\left(0 + 3 \times \frac{12}{12}\right)140}{\left(0 + 3 \times \frac{12}{12}\right)140 + 115} = 0.785$$

$$= 78.5\%$$

($h_2 = h$ for lap joint)

$\therefore$ Efficiency of riveted joint = 71.42%

Problem 3: Two plates of 18 mm thick are joined by means of a triple riveted lap joint. The diameter of rivets is 25 mm. The allowable stresses are 90 MPa in tension, 60 MPa in shear and 145 MPa in crushing. The pitch is 120 mm. Find the strength of the joint in all the modes of failure and the efficiency.

Given data: Triple riveted lap joint: Ref. Fig. 11.12

Thickness of plate, $h = 18$ mm

Diameter of rivet, $d = 25$ mm

$\therefore$ Diameter of rivet hole, $d_h = 25 + 1.5 = 26.5$ mm

Allowable stresses, $\sigma_\theta = 90$ MPa

$\tau = 60$ MPa

$\sigma_c = 145$ MPa

Pitch, $p = 120$ mm

From figure, $i_1 = 3, \quad i_2 = 0$

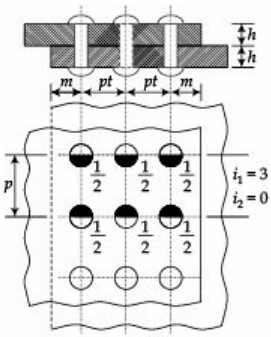


Fig. 11.12

Step 1: Strength of solid plate, $F = p h \sigma_\theta = 120 \times 18 \times 90$

$F = 194400$ N

Step 2: Strength of perforated plate, $F_\theta = (p - d_h) h \sigma_\theta$

$= (120 - 26.5) 18 \times 90$

$F_\theta = 151470$ N

Step 3: Shear strength of the rivets

$$F_\tau = (i_1 + 1.875 i_2)\frac{\pi d^2 \tau}{4}$$

$$F_\tau = (3 + 0)\frac{\pi}{4} \times 25^2 \times 60$$

$$= 88357.3 \text{ N}$$

Step 4: Crushing strength of rivets

$$F_c = (i_2 h + i_1 h_2) d \sigma_c$$

($h_c = h$ for lap joint)

$$F_c = (0 + 3 \times 18)\, 25 \times 145 = 195750 \text{ N}$$

Strength of joint = least of the above values

$$F_{min} = 88357.3 \text{ N}$$

Efficiency

$$\eta = \frac{F_{min}}{F} = \frac{88357.3}{194400} = 0.4545$$

$$\eta = 45.45\%$$

The joint fails due to shearing of rivets since the shear strength is the least.

Problem 4: Design a double riveted single cover butt joint with zigzag riveting to connect two plates of 16 mm thick. The allowable stresses for plates and rivets are 100 MPa in tension, 72 MPa in shear and 160 MPa in crushing.

Given data: Double riveted single cover butt joint with zigzag riveting (Fig. 11.13)

Thickness of plate h = 16 mm

σ_θ = 100 MPa

τ = 72 MPa

σ_c = 160 MPa

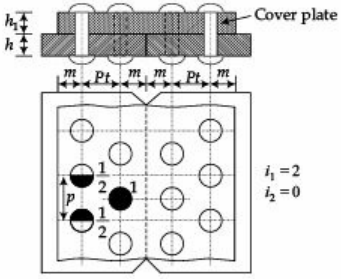


Fig. 11.13

From Fig. 11.13, No. of rivets/pitch

In single shear $i_1 = 1 + 1 = 2$

In double shear $i_2 = 0$ (No double shear)

(Consider only one side of the joint for selecting i_1 and i_2)

Step 1: Diameter of the rivet $d = 6\sqrt{h}$ to $6.3\sqrt{h}$

$= 6\sqrt{16}$ to $6.3\sqrt{16}$

$= 24$ to 25.2

Select standard diameter $d = 24$ mm

From Table 11.1

Diameter of the rivet hole $d_h = 24 + 1 = 25$ mm

Step 2: Pitch

(a) Pitch $p = \dfrac{(i_1 + 1.875 i_2)\pi^2 \tau}{4h\sigma_\theta} + d_h = \dfrac{(2+0)\pi \times 24^2 \times 72}{4 \times 16 \times 100} + 25 = 65.72$ mm

(b) As per IBR, $p = Kh + 40$ ...(Fig. c/P 13-12)

For two rivets/pitch and single cover butt joint, $K = 3.06$ (Table 11.2)

$p = 3.06 \times 16 + 40 = 88.96$ mm

$\therefore$ Recommended pitch $p = 65.72 \simeq 66$ mm

Step 3: Transverse pitch $p_t = 2.25d$

$p_t = 2.25 \times 24 = 54$ mm

Step 4: Margin $m = 1.5d = 1.5 \times 24 = 36$ mm

Step 5: For single cover plate, thickness of cover plate $h_1 = 1.125h = 1.125 \times 16 = 18$ mm

Step 6: Efficiency

(a) For plate $\eta_p = \dfrac{p - d_h}{p} = \dfrac{66 - 25}{66} = \eta_p = = 0.6212 = 62.12\%$

(b) Efficiency of rivet in shear $\eta_\tau = \dfrac{(i_1 + 1.875 i_2)\pi d^2 \tau}{4ph\sigma_\theta} = \dfrac{(2+0)\pi \times 24^2 \times 72}{4 \times 66 \times 16 \times 100}$

$\eta_\tau = 0.6169 \simeq 61.69\%$

(c) Efficiency of rivet in crushing, $\eta_c = \dfrac{\left(i_2 + i_1 \dfrac{h_2}{h}\right)\sigma_c}{\left(i_2 + i_1 \dfrac{h_2}{h}\right)\sigma_c + \sigma_\theta}$

($h_2 = h$ for single cover butt joint)

$$= \frac{\left(0 + 2 \times \frac{16}{16}\right)160}{\left(0 + 2 \times \frac{16}{16}\right)160 + 100} = 0.7619 = 76.19\%$$

$\therefore$ Efficiency of joint = 61.69%

(Least of the above 3 values).

Problem 5: Design a single riveted single cover butt joint to connect two plates 30 mm thick. The yield stress for plate and rivet materials is 324 MPa. Use factor of safety of 2.5 for tension, 4 for shear and 2 for crushing.

Given data: Single riveted single cover butt joint Ref. Fig. 11.14

From Fig. 11.14, $i_1 = 1$ and $i_2 = 0$ (no double shear)

Thickness of plate, $h = 30$ mm

Yield stress, $\sigma_y = 324$ MPa

FOS for tensile stress = 2.5

$\therefore$ Allowable tensile stress $\sigma_\theta = \dfrac{\sigma_y}{\text{FOS}}$

$$\sigma_\theta = \frac{324}{2.5} = 129.6 \text{ MPa.}$$

FOS for shear stress = 4

$\therefore$ Allowable shear stress, $\tau = \dfrac{\sigma_y}{\text{FOS}} = \dfrac{324}{4} = 81$ MPa

FOS for crushing stress = 2

$\therefore$ Allowable crushing stress, $\sigma_c = \dfrac{\sigma_y}{\text{FOS}} = \dfrac{324}{2} = 162$ MPa

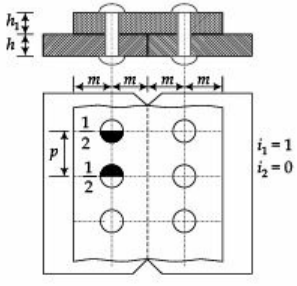


Fig. 11.14

Step 1: Diameter of rivet

$$d = 6\sqrt{h} \text{ to } 6.3\sqrt{h}$$
$$= 6\sqrt{30} \text{ to } 6.3\sqrt{30}$$
$$= 32.86 \text{ to } 34.5$$

Select standard diameter of rivet, $d = 33$ mm from Table 11.1.

Diameter of rivet hole, $d_h = d + 1.5$

$$d_h = 34.5 \text{ mm}$$

Step 2: Pitch

(a) $$p = \frac{(i_1 + 1.875 i_2)\pi d^2 \tau}{4 h \sigma_\theta} + d_h$$

$$= \frac{(1+0)\pi \times 33^2 \times 81}{4 \times 30 \times 129.6} + 34.5 = 52.32 \simeq 53 \text{ mm}$$

(b) Pitch $p = Kh + 40$ ...(Fig. a/P 13.12)

From Table 11.2, for single cover butt joint with one rivet/pitch, $K = 1.53$

$$p = 1.53 \times 30 + 40 = 85.9 \simeq 86 \text{ mm}$$

$\therefore$ Recommended pitch $p = 53$ mm (smaller one)

Step 3: Transverse pitch $p_t = 2.25d = 2.25 \times 33 = 74.25 \simeq 75$ mm

Step 4: Margin $m = 1.5d = 1.5 \times 33 = 49.5 \simeq 50$ mm

Step 5: Thickness of cover plate, $h_1 = 1.125h$

$$h_1 = 1.125 \times 30 = 33.75 \simeq 34 \text{ mm}$$

Step 6: Efficiency

(a) Efficiency of plate, $$\eta_p = \frac{p - d_h}{p} = \frac{53 - 34.5}{53}$$

$$\eta_p = 0.3491 = 34.91\%$$

(b) Efficiency of rivets in shear, $$\eta_\tau = \frac{(i_1 + 1.875 i_2)\pi d^2 \tau}{4 p h \sigma_\theta}$$

$$\eta_\tau = \frac{(1+0)\pi \times 33^2 \times 81}{4 \times 53 \times 30 \times 129.6} = 0.3362 = 33.62\%$$

(c) Efficiency of rivet in crushing $$\eta_c = \frac{\left(i_2 + i_1 \frac{h_2}{h}\right)\sigma_c}{\left(i_2 + i_1 \frac{h_2}{h}\right)\sigma_c + \sigma_\theta}$$

($h_2 = h$ for lap and single cover butt joints)

$$\eta_c = \frac{\left(0 + 1 \times \frac{30}{30}\right)162}{\left(0 + 1 \times \frac{30}{30}\right)162 + 129.6} = 0.555 = 55.56\%$$

$\therefore$ Efficiency of the joint $\eta = 33.62\%$.

Problem 6: A triple riveted single cover butt joint has the following details:

Thickness of plates = 12 mm

Thickness of cover plate = 10 mm

Diameter of rivet = 20 mm

Pitch of outer row = 210 mm

Pitch of the outer row is twice the pitch of the inner rows.

Allowable tensile stress for plates = 105 MPa

Allowable shear stress for rivets = 70 MPa

Allowable crushing stress for rivets = 165 MPa

Find the strength of joint by all modes of failure and its efficiency.

Given data: Triple riveted single cover butt joint with pitch of outer row = twice the pitch of inner rows.

Ref. Fig. 11.15, no. of rivets/pitch

$$i_1 = 1 + 2 + 2 = 5, \quad i_2 = 0 \quad \text{(no double shear)}$$

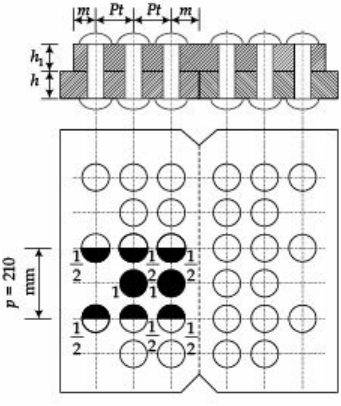


Fig. 11.15

Thickness of plates, h = 12 mm

Thickness of cover plate, h_1 = 10 mm

Pitch, p = 210 mm

Diameter of rivet, d = 20 mm

∴ Diameter of the rivet hole

$$d_h = d + 1 = 21 \text{ mm} \quad \text{(From Table 11.1)}$$

$$\sigma_\theta = 105 \text{ MPa}$$

$$\tau = 70 \text{ MPa}$$

$$\sigma_c = 165 \text{ MPa}$$

Step 1: Strength of the solid plate $F = ph\sigma_\theta = 210 \times 12 \times 105$

$$F = 264.6 \times 10^3 \text{ N}$$

Step 2: Strength of perforated plate, $F_\theta = (p - 2d_h)h\sigma_\theta$

(The plate is weak along inner rows where there are two holes in each inner row)

$$F_\theta = (210 - 2 \times 21)12 \times 105 = 211.6 \times 10^3 \text{ N}$$

Step 3: Shear strength of rivets $F_\tau = (i_1 + 1.875i_2)\dfrac{\pi d^2 \tau}{4}$

$$F_\tau = (5 + 0)\frac{\pi \times 20^2}{4} \times 70 = 109.96 \times 10^3 \text{ N}$$

Step 4: Crushing strength of rivets, $F_c = (i_2 h + i_1 h_2)d\sigma_c$

$(h_2 = h_1 = 10 \text{ mm}$ smaller of h and $h_2)$

$$F_\tau = (0 + 5 \times 10)20 \times 165 = 165 \times 10^3 \text{ N}$$

Step 5: Strength of joint

$$F_{min} = 109.96 \times 10^3 \text{ N}$$

The joint fails due to crushing of rivets.

Step 6: Efficiency $\eta = \dfrac{\text{Min. strength}}{\text{Strength of solid plate}} = \dfrac{109.96 \times 10^3}{264.6 \times 10^3}$

$$\eta = 0.4156 = 41.56\%.$$

Problem 7: Design a single riveted double cover butt joint to connect two plates 10 mm thick.

Given data: Single riveted double cover butt joint: Fig. 11.16

From Fig. 11.6, No. of rivets/pitch length

$$i_1 = 0, \quad i_2 = 1$$

Thickness of plates, $h = 10$ mm

Select $\sigma_\theta = 115$ MPa

$\tau = 70$ MPa

$\sigma_c = 140$ MPa for structural joint

Step 1: Diameter of the rivet

$$d = 6\sqrt{h} \text{ to } 6.3\sqrt{h}$$

$$= 6\sqrt{10} \text{ to } 6.3\sqrt{10}$$

$$= 18.97 \text{ to } 19.9$$

Take $d = 20$ mm and $d_h = 21$ mm from Table 11.1.

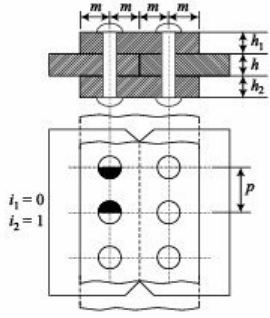


Fig. 11.16

Step 2: Pitch

(a) $$p = \frac{(i_1 + 1.875 i_2)\,\pi d^2 \tau}{4 h \tau_\theta} + d_h$$

$$= \frac{(0 + 1.875 \times 1)\,\pi \times 20^2 \times 70}{4 \times 10 \times 115} + 21 = 56.86 \text{ mm} = 57 \text{ mm}$$

(b) $p = Kh + 40$

For single riveted double cover butt joint,

(No. of rivets per pitch = 1) from Table 11.2 ...(Fig. f/P 13.13)

$$K = 1.75$$

$$\therefore \quad p = 1.75 \times 10 + 40 = 57.5 \simeq 58 \text{ mm}$$

Selecting smaller value, $p = 57$ mm

Step 3: For single riveted joint, there is no transverse pitch.

Step 4: Margin $m = 1.5d = 1.5 \times 20 = 30$ mm

Step 5: For equal corners, thickness of the cover plate

$$h_1 = h_2 = 0.625h = 0.625 \times 10 = 6.25 \simeq 7 \text{ mm}$$

Step 6: Efficiency

(a) For plate, $$\eta_p = \frac{p - d_h}{p} = \frac{57 - 21}{57} = 0.6316 = 63.16\%$$

(b) For rivet in shear, $$\eta_\tau = \frac{(i_1 + 1.875 i_2)}{4 p h \sigma_\theta} = \frac{(0 + 1.875 \times 1)\pi \times 20^2 \times 70}{4 \times 57 \times 10 \times 115}$$

$$\eta_\tau = 0.6290 = 62.9\%$$

(c) For rivet in crushing, $$\eta_c = \frac{\left(i_2 + i_1 \frac{h_2}{h}\right)\sigma_c}{\left(i_2 + i_1 \frac{h_2}{h}\right)\sigma_c + \sigma_\theta}$$

$$\eta_c = \frac{(1 + 0)140}{(1 + 0)140 + 115} = 0.5490 = 54.9\%$$

$\therefore$ Efficiency of joint = 54.9% (least of the above 3 values).

Problem 8: Design a double riveted double cover butt joint with equal covers to connect two plates each of 22 mm thickness. The following stresses may be used σ_θ = 90 MPa, τ = 60 MPa and σ_c = 150 MPa.

Given data: Double riveted double cover butt joint. Refer. Fig. 11.16.

$i_1 = 0$ (no single shear)
$i_2 = 1 + 1 = 2$ (both in double shear)
$h = 22$ mm
$\sigma_\theta = 90$ MPa
$\tau = 60$ MPa
$\sigma_c = 150$ MPa

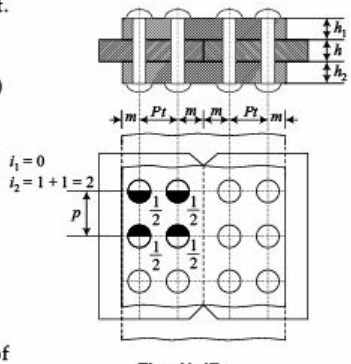


Fig. 11.17

Step 1: Diameter of the rivet

$$d = 6\sqrt{h} \text{ to } 6.3\sqrt{h}$$
$$= 6\sqrt{22} \text{ to } 6.3\sqrt{22}$$
$$= 28.14 \text{ to } 29.55$$

Take standard diameter, d = 30 mm and diameter of hole d_h = 31.5 mm from Table 11.1.

Step 2: Pitch

(c) $$p = \frac{(i_1 + 1.875 i_2)\pi d^2 \tau}{4 h \sigma_\theta} + d_h = \frac{(0 + 1.875 \times 2)\pi \times 30^2 \times 60}{4 \times 22 \times 90} + 31.5 = 111.82 \text{ mm} \simeq 112 \text{ mm}$$

(d) $p = Kh + 40$ ...(Fig g/P 13.13)

For double cover butt joint, with no. of rivets/pitch = 2

From Table 11.2,

$K = 3.5$

$\therefore \quad p = 3.5 \times 22 + 40 = 117$ mm

$\therefore$ Recommended, p = 112 mm (smaller value)

Step 3: Transverse pitch $p_t = 2.25\, d = 2.25 \times 30 = 67.5 \simeq 68$ mm

Step 4: Margin $m = 1.5d = 1.5 \times 30 = 45$ mm

Step 5: For equal covers, thickness of the cover plates

$$h_1 = h_2 = 0.625h = 0.625 \times 22 = 13.75 \simeq 14 \text{ mm}$$

Step 6: Efficiency

(a) For plate $$\eta_p = \frac{p - d_h}{p} = \frac{112 - 31.5}{112} = 0.7188$$

$$\eta_p \simeq 71.88\%$$

(b) For the rivet in shear $$\eta_\tau = \frac{(i_1 + 1.875 i_2)\pi d^2 \tau}{4 p h \sigma_\theta}$$

$$\eta_\tau = \frac{(0 + 1.875 \times 2)\pi \times 30^2 \times 60}{4 \times 112 \times 22 \times 90} = 0.7172 = 71.72\%$$

(c) For the rivets in crushing, $$\eta_c = \frac{\left(i_2 + i_1 \frac{h_2}{h}\right)\sigma_c}{\left(i_2 + i_1 \frac{h_2}{h}\right)\sigma_c + \sigma_\theta}$$

$$\eta_c = \frac{(2 + 0)\,150}{(2 + 0)\,150 + 90} = 0.7692 \simeq 76.92\%$$

$\therefore$ Efficiency of the joint = 71.72% (least).

Problem 9: Find the strength of the joint and efficiency for a triple riveted double cover butt joint with equal covers for the following data:

Thickness of plates = 20 mm
Thickness of cover plates = 16 mm
Diameter of rivet = 24 mm
Pitch = 150 mm

Strength of the rivet in double shear is 1.875 times that of rivet in single shear.

Allowable tensile stress, σ_θ = 100 MPa
Allowable shear stress, τ = 69 MPa
Allowable crushing stress, σ_c = 171 MPa

Given data: Triple riveted double cover butt joint with equal covers, Fig. 11.18.

From figure, $i_1 = 0$, $i_2 = 3$

h = 20 mm

$h_1 = h_2$ = 16 mm

d = 24 mm

p = 150 mm

σ_θ = 100 MPa

τ = 69 MPa

σ_c = 171 MPa

From Table 11.1, d_h = 25 mm for d = 24

Strength of the rivet in double shear = 1.875 times the strength of the rivet in single shear. Hence, multiply i_2 with 1.875 for strength calculations.

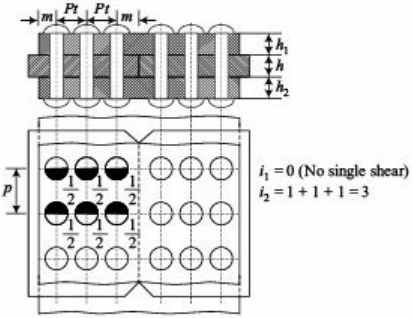


Fig. 11.18

Step 1: Strength of the solid plate $F = ph\sigma_\theta$

$$F = 150 \times 20 \times 100 = 300 \times 10^3 \text{ N}$$

Step 2: Strength of the perforated plate

$$F_\theta = (p - d_h)h\sigma_\theta$$

$$F_\theta = (150 - 25) \times 20 \times 100 = 250 \times 10^3 \text{ N}$$

Step 3: Shear strength of the rivets $F_\tau = (i_1 + 1.875i_2)\dfrac{\pi d^2 \tau}{4}$

$$= (0 + 1.875 \times 2)\frac{\pi \times 24^2}{4} \times 69 = 117.06 \times 10^3 \text{ N}$$

Step 4: Crushing strength of rivets

$$F_\tau = (i_1 h + i_1 h_2)d\sigma_c$$

$$F_\tau = (2 \times 20 + 0 \times 16)\, 24 \times 171 = 164.16 \times 10^3 \text{ N}$$

$\therefore$ Strength of the joint: $F_{min} = 117.06 \times 10^3$ N

The joint fails due to shearing of rivets, since the shear strength is minimum.

$$\text{Efficiency } \eta = \frac{F_{min}}{F} = \frac{117.06 \times 10^3}{300 \times 10^3} = 0.3902$$

$$\eta = 39.02\%.$$

Problem 10: Design a double riveted double cover butt joint with unequal covers and zigzag riveting to connect two plates of 25 mm thick.

Given data: Double riveted double cover zigzag but joint with unequal covers. Ref. Fig. 11.19. From figure, No. of rivets/pitch, $i_1 = 1$ and $i_2 = 1$

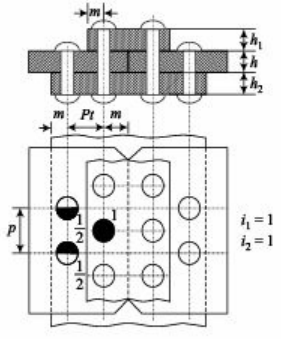


Fig. 11.19

Thickness of the plate, $h = 25$ mm

Assume, $\sigma_\theta = 115$ MPa, $\tau = 70$ MPa, $\sigma_c = 140$ MPa

(Assuming structural joint.)

Step 1: Diameter of the rivet $d = 6\sqrt{h}$ to $6.3\sqrt{h}$

$= 6\sqrt{25}$ to $6.3\sqrt{25}$

$= 30$ to 31.5

Take $d = 30$ mm standard diameter from Table 11.1 and diameter of the hole, $d_h = 31.5$ mm

Step 2: Pitch

(a) $p = \dfrac{(i_1 + 1.875 i_2)\pi d^2 \tau}{4h\sigma_\theta} + d_h = \dfrac{(1 + 1.875 \times 1)\pi \times 30^2 \times 70}{4 \times 25 \times 115} + 31.5 = 80.98 \simeq 81$ mm

(b) But pitch $p = Kh + 40$ ...(Fig. p/P 13.15)

From Table 11.2, for double cover butt joint with two rivets/pitch

$K = 3.5$

$\therefore \quad p = 3.5 \times 25 + 40 = 127.5$ mm

$\therefore$ Recommended pitch $p = 81$ mm (smaller one)

Step 3: Transverse pitch, $P_t = 2.25d = 2.25 \times 30 = 67.5 \simeq 68$ mm

Step 4: Margin $m = 1.5d = 1.5 \times 30 = 45$ mm

Step 5: Thickness of the cover plates

Narrow cover $h_1 = 0.625h = 0.625 \times 25 = 15.675 \simeq 16$ mm

Wider cover $h_2 = 0.75h = 0.75 \times 25 = 18.75 \simeq 19$ mm

Step 6: Efficiency:

(a) Efficiency of the plate $\eta_p = \dfrac{p - d_h}{p} = \dfrac{81 - 31.5}{81} = 0.6111 \simeq 61.11\%$

(b) Efficiency of the rivet based on shear $\eta_\tau = \dfrac{(i_1 + 1.875 i_2)\pi d^2 \tau}{4ph\sigma_\theta}$

$$\eta_\tau = \frac{(1 + 1.875 \times 1)\pi \times 30^2 \times 70}{4 \times 81 \times 25 \times 115} = 0.6109 \simeq 61.09\%$$

(c) Efficiency of the rivet in crushing, $\sigma_c = \dfrac{\left(i_2 + i_1 \dfrac{h_2}{h}\right)\sigma_c}{\left(i_2 + i_1 \dfrac{h_2}{h}\right)\sigma_c + \sigma_\theta}$

$$\sigma_c = \frac{\left(1 + 1 \times \frac{19}{25}\right) \times 140}{\left(1 + 1 \times \frac{19}{25}\right) 140 + 115} = 0.6377 = 63.77\%$$

$\therefore$ Efficiency of the joint = 61.09%

Problem 11: Design a triple riveted double cover butt joint with unequal cover plates to connect two plates of 20 mm thickness. Take σ_θ = 90 MPa, τ = 60 MPa, σ_c = 150 MPa.

Given data: Thickness of the plate h = 20 mm

σ_θ = 90 MPa

τ = 60 MPa

σ_c = 150 MPa

Triple riveted double cover butt joint with unequal cover, Fig. 11.20

No. of rivets in single shear i_1 = 1

No. of rivets in double shear i_2 = 1 + 1 = 2 (in one pitch length).

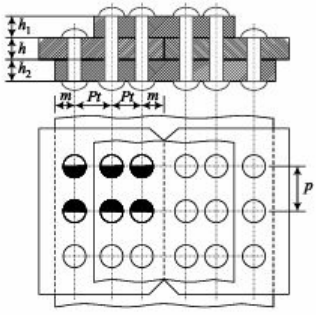


Fig. 11.20

Step 1: Diameter of the rivet $d = 6\sqrt{h}$ to $6.3\sqrt{h}$

$= 6\sqrt{20}$ to $6.3\sqrt{20}$

d = 26.83 to 28.17 mm

From Table 11.1, select standard

d = 27 mm and

d_h = 28.5 mm

Step 2: (a) Pitch in general case

(a) $p = \dfrac{(i_1 + 1.875 i_2)\pi d^2 \tau}{4h\sigma_\theta} + d_h = \dfrac{(1 + 1.875 \times 2)\,\pi \times 27^2 \times 60}{4 \times 20 \times 90} + 28.5 = 119.15 \text{ mm} \simeq 120 \text{ mm}$

(b) Pitch $p = Kh + 40$ ...(Fig. r/P 13.15)

From Table 11.2, for double cover butt joint, with no. of rivets/pitch = 3

$K = 4.63$

$\therefore$ $p = 4.63 \times 20 + 40 = 132.6 \text{ mm} \simeq 133 \text{ mm}$

Take p = 120 mm

Step 3: Transverse pitch $p_t = 2.25d = 2.25 \times 27 = 60.75 = 61$ mm

Step 4: Margin $m = 1.5d = 1.5 \times 27 = 40.5 \simeq 41$ mm

Step 5: Cover plates

Narrow cover $h_1 = 0.625h = 0.625 \times 20 = 12.5 \simeq 13$ mm

Wider cover $h_2 = 0.75h = 0.75 \times 20 = 15$ mm

Step 6: Efficiency

(a) Efficiency of plate, $$\eta_p = \frac{p - d_h}{p} = \frac{120 - 28.5}{120} = 0.7625$$

$$\eta_p = 76.25\%$$

(b) Efficiency of rivet in shear, $$\eta_\tau = \frac{(i_1 + 1.875 i_2)\pi d^2 \tau}{4 p h \sigma_\theta}$$

$$\eta_p = \frac{(1 + 1.875 \times 2)\pi \times 27^2 \times 60}{4 \times 120 \times 20 \times 90} = 0.7555 \simeq 75.55\%$$

(c) Efficiency of rivet in crushing, $$\eta_\tau = \frac{\left(i_2 + i_1 \frac{h_2}{h}\right)\sigma_c}{\left(i_2 + i_1 \frac{h_2}{h}\right)\sigma_c + \sigma_\theta}$$

$$\eta_c = \frac{\left(2 + 1 \times \frac{15}{20}\right)150}{\left(2 + 1 \times \frac{15}{20}\right)150 + 90} = 0.8209 = 82.09\%$$

$\therefore$ Efficiency of joint $\eta = 75.55\%$.

Problem 12: The following data refers to a triple riveted double cover butt joint with unequal cover plates.

Thickness of plates = 15 mm
Thickness of cover plates = 12 mm
Diameter of rivet = 18 mm
Pitch of outer row = 300 mm
Pitch of outer row = Twice the pitch of inner rows.
Allowable tensile stress = 90 MPa
Allowable shear stress = 60 MPa
Allowable crushing stress = 150 MPa

Investigate how the joint fails and find its efficiency.

Given data: $h = 15$ mm

$h_1 = h_2 = 12$ mm

$d = 18$ mm

$\therefore$ $d_h = 19$ mm (from Table 11.2)

$p = 300$ mm

$\sigma_\theta = 90$ MPa

$\tau = 60$ MPa

$\sigma_c = 150$ MPa

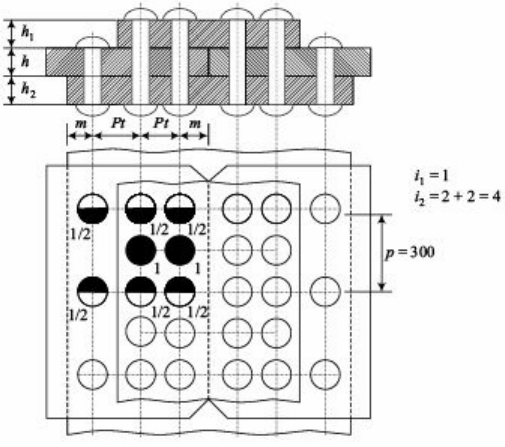


Fig. 11.21

Triple riveted double cover butt joint with unequal covers and pitch of outer row as double the pitch of the inner rows.

Ref. Fig. 11.21,

From figure, no. of rivets/pitch in single shear $i_1 = 1$

No. of rivets/pitch in double shear $i_1 = 2 + 2 = 4$

Step 1: Strength of solid plate $F = ph\sigma_\theta$

$= 300 \times 15 \times 90 = 405 \times 10^3$ N

Step 2: Strength of perforated plate, $F_\theta = (p - 2d_h)h\sigma_\theta$

(The plate is weakened by 2 holes in inner rows).

$F_\theta = (300 - 2 \times 19)15 \times 90 = 353.7 \times 10^3$ N

Step 3: Shear strength of the rivets $F_\tau = (i_1 + 1.875i_2)\frac{\pi}{4}d^2\tau$

$$F_\tau = (1 + 1.875 \times 4)\frac{\pi}{4} \times 18^2 \times 60 = 129.78 \times 10^3 \text{ N}$$

Step 4: Crushing strength of the rivets, $F_c = (i_2h + i_1h_2)d\sigma_c$

$$F_c = (4 \times 15 + 1 \times 12)18 \times 150 = 194.4 \times 10^3 \text{ N}$$

$\therefore$ Minimum strength, $F_{min} = F_\tau = 129.78 \times 10^3$ N

Hence, the joint fails due to shearing of the rivets

$$\text{Efficiency } \eta = \frac{F_{min}}{F} = \frac{129.78 \times 10^3}{405 \times 10^3}$$

$$\eta = 0.3204 = 32.04\%.$$

11.5 RIVETED JOINTS FOR BOILERS AND PRESSURE VESSELS

A pressure vessel stores pressurised fluid in the cylinder. If the fluid is steam, then it is known as boiler. A pressure vessel or boiler has two joints—longitudinal joint and circumferential joint. Figure 11.22 shows the riveted joints used for boiler.

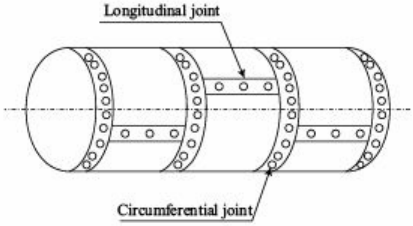


Fig. 11.22 Boiler joints

Symbols: D_i = Inside diameter of the boiler or pressure vessel, mm
p_f = Fluid pressure or steam pressure, MPa

Design Procedure of Boiler Joints

PART-I Longitudinal joint

Step 1: Thickness of plate $h = \frac{P_f D_i}{2\eta\sigma_\theta} + C$ ...(13-1/P 13.1)

η = Efficiency of joint $\simeq$ 0.7 to 0.8 ($\simeq$ 0.75)

C = Corrosion allowance = 1 to 3 mm ($\simeq$ 2 mm)

Step 2: Diameter of the rivet, $d = 6\sqrt{h}$ to $6.3\sqrt{h}$ ...(13-5 c)

Select standard diameter of the rivet and hole diameter.

Step 3: Pitch of the rivets:

(a) $p = \dfrac{(i_1 + 1.875 i_2)\pi d^2 \tau}{4ph\sigma_\theta} + d_h$ and ...(13-30)

(b) $p = Kh + 40$

Select K from Table 11.2 depending on type of joint.

Select smaller value of p from above two values. ...(T 13-14)

Step 4: Transverse pitch $p_t = 2.25d$ ...(13-6 a)

Step 5: Margin $m = 1.5d$ ...(13-11 a)

Step 6: Thickness of cover plates:

- For equal covers $h_1 = h_2 = 0.625h$ ...(13-15)
- For unequal covers $h_1 = 0.625h$ and $h_2 = 0.75h$ ...(13-17 a & b)

Step 7: Efficiency

(a) Efficiency of plate, $\eta_p = \dfrac{p - d_h}{p}$ ...(13-27)

(b) Efficiency of rivets in shear, $\eta_\tau = \dfrac{(i_1 + 1.875 i_2)\pi d^2 \tau}{4ph\sigma_\theta}$ ...(13-28)

(c) Efficiency of rivets in crushing, $\eta_c = \dfrac{\left(i_2 + i_1 \dfrac{h_2}{h}\right)\sigma_c}{\left(i_2 + i_1 \dfrac{h_2}{h}\right)\sigma_c + \sigma_\theta}$ ...(13-28)

PART-II Circumferential joints

The plates to be riveted being same in both longitudinal and circumferential joints (h being same), the diameter of rivets, transverse pitch and margin are same.

h, d, p_t and m are same as above.

Step 1: Total steam load $F = \left(\dfrac{\pi}{4} Di^2\right) P_f$

Step 2: Strength of each rivet

(a) In shear $F_\tau = \dfrac{\pi}{4} d^2 . \tau$

(b) In crushing $F_c = dh\sigma_c$

$\therefore$ Minimum strength of the rivet = F_1 = least of these two

Step 3: Number of rivets required = $\dfrac{\text{Steam load}}{\text{Minimum strength of rivet}}$

$$i = \frac{F}{F_1}$$

Step 4: Rivets/Row = i/row = $\dfrac{\text{Total no. of rivets}}{\text{No. of rows of rivets}}$

No. of rows = 1 for single riveted joint

= 2 for double riveted joint, etc

Step 5: Pitch for circumferential joint

$$p_c = \frac{\text{Circumference}}{\text{Rivets/Row}} = \frac{\pi D_i}{i/\text{row}}$$

Step 6: Efficiency

(a) For plate $\eta_p = \dfrac{p_c - d_h}{p_c}$

(b) For rivets in shear $\eta_\tau = \dfrac{(i_1 + 1.875 i_2)\pi d^2 \tau}{4 p_c h \sigma_\theta}$

and

(c) For rivets in crushing $\eta_c = \dfrac{\left(i_2 + i_1 \dfrac{h_2}{h}\right)\sigma_c}{\left(i_2 + i_1 \dfrac{h_2}{h}\right)\sigma_c + \sigma_\theta}$

$\therefore$ Efficiency of boiler joint = least of the above 6 values. (3 from part *I* and 3 from part II).

Problem 13: A penstock of 1 m diameter and made of steel plates is operated under 30 m of head of water. It is a single riveted longitudinal joint. Taking efficiency as 75% and 2 mm corrosion allowance for plate thickness, design the joint. The following allowable stresses may be used.

$$\sigma_t = 120 \text{ N/mm}^2, \quad \tau = 96 \text{ N/mm}^2, \quad \sigma_c = 162 \text{ N/mm}^2$$

Given data: Diameter of penstock $D_i = 1$ m = 1000 mm

Head of water = 30 m = 0.3 MPa

(10 m water head = 1 bar = 0.1 MPa)

$\sigma_t = \sigma_\theta = 120$ MPa $\quad \tau = 96$ MPa, $\quad \sigma_c = 162$ MPa

Longitudinal joint: Single riveted joint assume double cover butt joint, Ref. Fig. 11.22

$i_1 = 1, \quad i_2 = 0$ from figure.

$\eta = 0.75$

Step 1: Thickness of plate

$$h = \frac{P_f D_i}{2\eta\sigma_\theta} + C$$

$$h = \frac{0.3 \times 1000}{2 \times 0.75 \times 120} + 2$$

$$= 3.67 \simeq 4 \text{ mm}$$

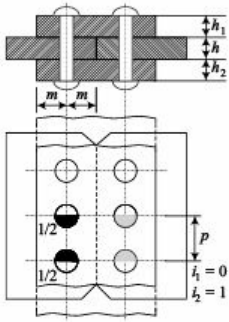


Fig. 11.23

Step 2: Diameter of rivet

$$d = 6\sqrt{h} \text{ to } 6.3\sqrt{h}$$

$$= 6\sqrt{4} \text{ to } 6.3\sqrt{4}$$

$$= 12 \text{ to } 12.6$$

Select standard diameter of the rivets

$d = 12$ mm and diameter of the rivets hole $d_h = 13$ mm

Step 3: Pitch

(a) $$p = \frac{(i_1 + 1.875 i_2)\pi d^2 \tau}{4h\sigma_\theta} + d_h$$

$$p = \frac{(0 + 1.875 \times 1)\pi \times 12^2 \times 96}{4 \times 4 \times 120} + 13 = 55.41 \text{ mm}$$

(b) But $p = Kh + 40$ From Table 11.2, for double cover butt joint with one rivet/pitch $K = 1.75$

$p = 1.75 \times 4 + 40 = 47$ ...(Fig. f/P 13.13)

$\therefore p = 47$ mm (smaller value)

Step 4: Margin $m = 1.5d = 1.5 \times 12 = 18$ mm

Step 5: Thickness of cover plates, $h_1 = h_2 = 0.625h$

$$h_1 = h_2 = 0.625 \times 4 = 2.5 \simeq 3 \text{ mm}$$

Step 6: Efficiency

(a) For plate $$\eta_p = \frac{p - d_h}{p} = \frac{47 - 12}{47} = 0.7449$$

$$\eta_p = 74.49\%$$

(b) Efficiency of rivets in shear, $$\eta_\tau = \frac{(i_1 + 1.875 i_2)\pi d^2 \tau}{4ph\sigma_\theta}$$

$$\eta_\tau = \frac{(0 + 1.875 \times 1)\pi \times 12^2 \times 96}{4 \times 47 \times 4 \times 120} = 0.9024 = 90.24\%$$

(c) Efficiency of the rivets in crushing

$$\eta_c = \frac{\left(i_2 + i_1 \frac{h_2}{h}\right)\sigma_c}{\left(i_2 + i_1 \frac{h_2}{h}\right)\sigma_c + \sigma_\theta}$$

$$\eta_c = \frac{(1+0)162}{(1+0)162+120} = 0.5745 = 57.45\%$$

$\therefore$ Efficiency of the joint = 57.45% (least of the above)

Problem 14: Design a double riveted double strap butt joint for the longitudinal seam of a boiler of diameter 1.3 m with a steam pressure of 2.4 MPa. The working stresses to be used are 77 MPa in tension, 54 MPa in shear and 120 MPa in crushing. Assume joint efficiency as 81%.

Given data: Double riveted double strap butt joint, refer Fig. 11.5(b). From figure, no. of rivets/ pitch in single shear, $i_1 = 0$

In double shear, $i_2 = 1 + 1 = 2$

Diameter of rivets, $D_i = 1.3$ m = 1300 mm

Steam pressure, $P_f = 2.4$ MPa

$\sigma_\theta = 77$ MPa, $\tau = 54$ MPa

$\sigma_c = 120$ MPa, $\eta = 0.81$

Longitudinal joint

Step 1: Thickness of the plate $h = \frac{P_f D_i}{2\eta\sigma_\theta} + C$

c = corrosional allowance = 2 mm

$$h = \frac{2.4 \times 1300}{2 \times 0.81 \times 77} + 2 = 27 \text{ mm}$$

Step 2: Diameter of the rivet $d = 6\sqrt{h}$ to $6.3\sqrt{h} = 6\sqrt{27}$ to $6.3\sqrt{27}$

= 31.18 to 32.74 mm

Take standard diameter $d = 33$ mm

Diameter of the rivet hole $d_h = 33 + 1.5 = 34.5$ mm

Step 3: Pitch

(a) $p = \frac{(i_1 + 1.875 i_2)\pi d^2 \tau}{4h\sigma_\theta} + d_h = \frac{(0 + 1.875 \times 2)\pi \times 33^2 \times 54}{4 \times 27 \times 77} + 34.5 = 117.81 \simeq 118$ mm

(b) $p = Kh + 40$. For double strap butt joint with 2 rivets/ pitch, $K = 3.5$ (from Table 11.2)

$p = 3.5 \times 27 + 40 = 134.5$ mm ...(Fig. g/P 13.13)

$\therefore$ Recommended pitch $p = 118$ mm (smaller)

Step 4: Transverse pitch $p_t = 2.25d = 2.25 \times 33 = 74.25 \simeq 75$ mm

Step 5: Margin $m = 1.5d = 1.5 \times 33 = 49.5 \simeq 50$ mm

Step 6: Thickness of the cover plates, for equal covers

$$h_1 = h_2 = 0.625h = 0.625 \times 27 = 16.875 \simeq 17 \text{ mm}$$

Step 7: Efficiency

(a) For plates $$\eta_p = \frac{p - d_h}{p} = \frac{118 - 34.5}{118} = 0.7076 = 70.76\%$$

(b) For the rivet in shear, $$\eta_\tau = \frac{(i_1 + 1.875 i_2)\pi d^2 \tau}{4ph\sigma_\theta}$$

$$\eta_\tau = \frac{(0 + 1.875 \times 2)\pi \times 33^2 \times 54}{4 \times 118 \times 27 \times 77} = 0.7060 = 70.60\%$$

(c) For the rivets in crushing, $$\eta_c = \frac{\left(i_2 + i_1 \frac{h_2}{h}\right)\sigma_c}{\left(i_2 + i_1 \frac{h_2}{h}\right)\sigma_c + \sigma_\theta}$$

$$\eta_c = \frac{(2 + 0)120}{(2 + 0)120 + 77} = 0.7571 = 75.71\%$$

$\therefore$ Efficiency of the joint, $\eta = 70.6\%$ (least one)

Problem 15: Design the main dimensions for longitudinal and circumferential joints for a boiler whose inner diameter is 1.7 m and steam pressure of 2.05 MPa. The rivet in double shear will have an effective resistance not greater than 87.5% over that in single shear.

Given data: Diameter of the boiler, $D_i = 1.7$ m $= 1700$ mm

Steam pressure, $p_f = 2.05$ MPa

Assume $\sigma_\theta = 90$ MPa, $\tau = 60$ MPa, $\sigma_c = 150$ MPa.

Resistance in double shear = 1.875 times that in single shear.

Use triple riveted double cover butt joint with unequal cover for longitudinal joint and double riveted lap joint for circumferential joint.

Part-I Longitudinal joint

Ref. Figure 11.20

From Figure, $i_1 = 1$ and $i_2 = 2$ rivets/row.

Step 1: Thickness of the plate, $$h = \frac{P_f D_i}{2\eta\sigma_\theta} + C$$

Assume $\eta = 0.75$ and $C = 2$ mm

$$h = \frac{2.05 \times 1700}{2 \times 0.75 \times 90} + 2 = 27.82 \simeq 28 \text{ mm}$$

Step 2: Diameter of the rivet $d = 6\sqrt{h} + 6.3\sqrt{h}$

$d = 6\sqrt{28}$ to $6.3\sqrt{28} = 31.74$ to 33.34 mm

From Table 11.1, select standard rivet diameter, $d = 33$ mm and

Diameter of the rivet hole $d_h = d + 1.5 = 34.5$ mm

Step 3: Pitch

(a) $p = \dfrac{(i_1 + 1.875 i_2)\pi d^2 \tau}{4 h \sigma_\theta} + d_h = \dfrac{(1 + 1.875 \times 2)\pi \times 33^2 \times 60}{4 \times 28 \times 90} + 34.5 = 131.23 \simeq 132$ mm

(b) $p = Kh + 40$. From Table 11.2, for double cover butt joint with 3 rivets/pitch, $K = 4.63$

$\therefore\ p = 4.63 \times 28 + 40 = 169.64$ mm ...(Fig. r/P 13.15)

$\therefore$ Recommended pitch, $p = 132$ mm (smaller one)

Step 4: Transverse pitch $p_t = 2.25d = 2.25 \times 33 = 74.25 \simeq 75$ mm

Step 5: Margin $m = 1.5d = 1.5 \times 33 = 49.5 \simeq 50$ mm

Step 6: Thickness of the cover plates,

$h_1 = 0.625h = 0.625 \times 28 = 17.5 \simeq 18$ mm

$h_2 = 0.75h = 0.75 \times 28 = 21$ mm

Step 7: Efficiency

(a) For plates $\eta_p = \dfrac{p - d_h}{p} = \dfrac{132 - 34.5}{132} = 0.7386 = 73.86\%$

(b) For rivets in shear, $\eta_\tau = \dfrac{(i_1 + 1.875 i_2)\pi d^2 \tau}{4 p h \sigma_\theta}$

$$\eta_\tau = \frac{(1 + 1.875 \times 2)\pi \times 33^2 \times 60}{4 \times 132 \times 28 \times 90} = 0.7328 \simeq 73.28\%$$

(c) For rivets in crushing, $\eta_c = \dfrac{\left(i_2 + i_1 \dfrac{h_2}{h}\right)\sigma_c}{\left(i_2 + i_1 \dfrac{h_2}{h}\right)\sigma_c + \sigma_\theta}$

$$\eta_c = \frac{\left(2 + 1 \times \frac{21}{28}\right)150}{\left(2 + 1 \times \frac{21}{28}\right)150 + 90} = 0.8209 = 82.09\%$$

Part-II Circumferential joint

Use double rivet lap joint for circumferential joint.

Ref. Figure 11.24,

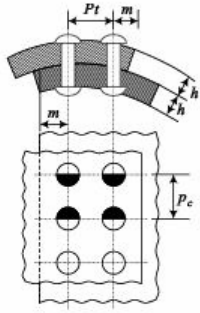


Fig. 11.24

From Figure

$i_1 = 1 + 1 = 2$

$i_2 = 0$ (for lap joint)

$h = 28$ mm

$d = 33$ mm

$p_t = 75$ mm

$m = 50$ mm

(Same as Part I)

Step 1: Total steam load F = Area of boiler × pressure

$$F = \frac{\pi}{4} Di^2 p_f = \frac{\pi}{4} 1700^2 \times 2.05 = 4.654 \times 10^3 \text{ N}$$

Step 2: Shear strength of each rivet, $F_\tau = \frac{\pi}{4} d^2 \tau$

$$F_\tau = \frac{\pi}{4} 33^2 \times 60 = 51.32 \times 10^3 \text{ N}$$

Step 3: Crushing strength of the rivet, $F_c = dh\sigma_c$

$$F_c = 33 \times 28 \times 150 = 138.6 \times 10^3 \text{ N}$$

$\therefore$ Strength of the rivet, $F_1 = 51.32 \times 10^3$ (minimum of these two)

Step 4: No of rivets reqd. $i = \dfrac{\text{Total steam load}}{\text{Strength of one rivet}} = \dfrac{F}{F_1}$

$$i = \frac{4.654 \times 10^6}{51.32 \times 10^3} = 90.86 \text{ rivets} \simeq 92 \text{ rivets (Even number)}$$

$\therefore$ $$\text{rivets/row} = \frac{i}{\text{no. of rows}} = \frac{92}{2} = 46 \text{ rivets/row}$$

(For double riveted lap joint, no. of rows = 2)

Step 5: Pitch for circumferential joint $p_c = \dfrac{\pi D_i}{i/\text{row}}$

$$p_c = \frac{\pi \times 1700}{46} = 116.1 \text{ mm}$$

Step 6: Efficiency

(a) For plate, $$\eta_p = \frac{p_c - d_h}{p_c} = \frac{116.1 - 34.5}{116.1} = 0.703 = 70.3\%$$

(b) For rivets in shear, $$\eta_\tau = \frac{(i_1 + 1.875 i_2)\pi d^2 \tau}{4 p_c h \sigma_\theta}$$

$$\eta_\tau = \frac{(2+0)\pi \times 33^2 \times 60}{4 \times 116.1 \times 28 \times 90} = 0.3508 = 35.08\%$$

(c) For rivet in crushing, $$\eta_c = \frac{\left(i_2 + i_1 \frac{h_2}{h}\right)\sigma_c}{\left(i_2 + i_1 \frac{h_2}{h}\right)\sigma_c + \sigma_\theta}$$

(for lap joint $h_2 = h$)

$$\eta_c = \frac{\left(0 + 2 \times \frac{28}{28}\right)150}{\left(0 + 2 \times \frac{28}{28}\right)150 + 90} = 0.7692 = 76.92\%$$

∴ Efficiency of the boiler joint, $\eta = 35.08\%$
(Least of the above six value)

Problem 16: Design the longitudinal and circumferential joints for a boiler of 2 m diameter subjected to a pressure of 1 MPa. The longitudinal joint is a triple riveted butt joint with equal covers and efficiency of 85% and the circumferential joint of double riveted lap joint. The pitch of the outer row is twice the pitch of inner rows. The yeild stress for plate and rivet material is 353 MPa. Take factor of safety of 3 for tension, 5 for shear and 2 for crushing.

Given data: Inside diameter of boiler, $D_i = 2$ m $= 2000$ mm
Internal pressure, $p_f = 1$ MPa
Efficiency, $\eta = 0.85$
Yield stress, $\sigma_y = 353$ MPa
FOS = 3 for tension
= 5 for shear and
= 2 for crushing

∴ $$\sigma_\theta = \frac{\sigma_y}{FOS} = \frac{353}{3} = 117.67 \text{ MPa}$$

$$\tau = \frac{\sigma_y}{FOS} = \frac{353}{5} = 70.6 \text{ MPa}$$

$$\sigma_c = \frac{\sigma_y}{FOS} = \frac{353}{2} = 176.5 \text{ MPa}.$$

Part-I Longitudinal joint

Triple riveted double cover butt joint with equal covers and pitch of the outer rwo is twice the pitch of inner rows.

The joint is shown in Figure 11.25.

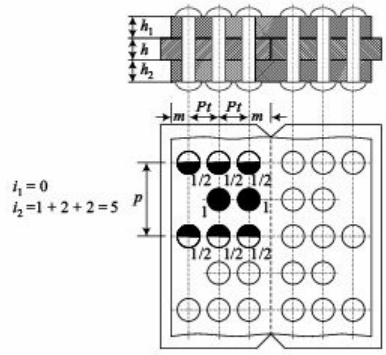


Fig. 11.25

From Fig. $i_1 = 0$ (no signal shear)

$i_2 = 1 + 2 + 2 = 5$ rivet/pitch

Step 1: Thickness of the plate $h = \dfrac{P_f D_i}{2\eta\sigma_\theta} + C$

Assume corrosion allowance, $C = 2$ mm

$$h = \frac{1 \times 2000}{2 \times 0.85 \times 117.67} + 2 = 11.998 \simeq 12 \text{ mm}$$

Step 2: Diameter of the rivet

$$d = 6\sqrt{h} \text{ to } 6.3\sqrt{h}$$

$$d = 6\sqrt{12} \text{ to } 6.3\sqrt{12}$$

$$= 20.75 \text{ to } 21.82.$$

From Table 11.1, select standard diameter of the rivet, $d = 22$ mm and rivet hole diameter, $d_h = 23$ mm.

Step 3: Pitch

(a) $$p = \frac{(i_1 + 1.875 i_2)\pi d^2 \tau}{4h\sigma_\theta} + d_h = \frac{(0 + 1.875 \times 5)\pi \times 22^2 \times 70.6}{4 \times 12 \times 117.67} + 23 = 201.18 \text{ mm}$$

(b) $p = Kh + 40$ for double cover butt joint with 5 rivets/pitch,

From Table 11.2, $K = 6$,

$$p = 6 \times 12 + 40 = 112 \text{ mm}$$

$\therefore$ Pitch, $p = 112$ mm

Step 4: Transverse pitch, $p_t = 2.25d = 2.25 \times 22 = 49.5 \simeq 50$ mm

Step 5: Margin $m = 1.5d = 1.5 \times 22 = 33$ mm

Step 6: Thickness of cover plates,

For equal covers, $h_1 = h_2 = 0.625h = 0.625 \times 12 = 7.5 \simeq 8$ mm

Step 7: Efficiency

(a) For the plate, $$\eta_p = \frac{p - d_h}{p} = \frac{112 - 23}{112} = 0.7946 = 79.46\%$$

(b) For the rivet in shear, $$\eta_\tau = \frac{(i_1 + 1.875 i_2)\pi d^2 \tau}{4ph\sigma_\theta}$$

$$= \frac{(0 + 1.875 \times 5)\pi \times 22^2 \times 70.6}{4 \times 112 \times 12 \times 117.67} = 1.5909 = 159.09\%$$

(c) For the rivet in crushing, $$\eta_c = \frac{\left(i_2 + i_1 \frac{h_2}{h}\right)\sigma_c}{\left(i_2 + i_1 \frac{h_2}{h}\right)\sigma_c + \sigma_\theta}$$

$$\eta_c = \frac{(5 + 0)176.5}{(5 + 0)176.5 + 117.67} = 0.8824 \simeq 88.24\%$$

Part-II Circumferential joint

Double riveted lap joint (Ref. Fig. 11.24)

$i_1 = 2$, $i_2 = 0$ $h = 12$ mm,

$d = 22$ mm, $d_h = 23$ mm, $p_t = 50$ mm, $m = 33$ mm

(Same as longitudinal joint)

Step 1: Total steam load $$F = \frac{\pi}{4} D i^2 p_f = \frac{\pi}{4} 2000^2 \times 1 = 3.14 \times 10^6 \text{ N}$$

Step 2: Shear strength of the rivet $F_\tau = \frac{\pi}{4} d^2 \tau$

$$F_\tau = \frac{\pi}{4} \times 22^2 \times 70.6 = 26.837 \times 10^3 \text{ N}$$

Step 3: Crushing strength of the rivet, $F_c = hd\sigma_c$

$$F_c = 12 \times 22 \times 176.5 = 46.596 \times 10^3 \text{ N}$$

$\therefore$ Strength of the rivet = $F_1 = 26.837 \times 10^3$ N

Step 4: No. of rivets required $\quad i = \dfrac{F}{F_1} = \dfrac{3.14 \times 10^6}{26.837 \times 10^3}$

$$i = 117.00 \simeq 118 \text{ rivets}$$

Step 5: Rivet/row, i/row $= \dfrac{118}{2} = 59$ rivets

Step 6: Pitch for circumferential joint

$$p_c = \frac{\text{circumference}}{\text{rivets/row}} = \frac{\pi D_i}{i/\text{row}}$$

$$p_c = \frac{\pi \times 2000}{59} = 106.49 \text{ mm}$$

Step 7: Efficiency

(a) For the plate $\quad \eta_p = \dfrac{p_c - d_h}{p_c} = \dfrac{106.49 - 23}{106.49}$

$$\eta_p = 0.7840 = 78.4\%$$

(b) For the rivets in shear $\quad \eta_\tau = \dfrac{(i_1 + 1.875 i_2)\pi d^2 \tau}{4 p_c h \sigma_\theta}$

$$\eta_\tau = \frac{(2+0)\pi \times 22^2 \times 70.6}{4 \times 106.49 \times 12 \times 117.68} = 0.3569 = 35.69\%$$

(c) For the rivets in crushing, $\quad \eta_c = \dfrac{\left(i_2 + i_1 \dfrac{h_2}{h}\right)\sigma_c}{\left(i_2 + i_1 \dfrac{h_2}{h}\right)\sigma_c + \sigma_c}$

($h_2 = h = 12$ mm for lap joint)

$$\eta_c = \frac{\left(0 + 2 \times \dfrac{12}{12}\right)176.5}{\left(0 + 2 \times \dfrac{12}{12}\right)176.5 + 117.68} = 0.7499 = 74.99\%$$

$\therefore$ Efficiency of the boiler joint, $\eta = 35.69\%$

(Least of the above all 6 efficiencies).

11.6 TIE BAR OR LOZENGE JOINT

This is a riveted joint used in structures. This is basically a double cover butt joint with equal covers. It has a special feature for the plate not being weakened by more than one rivet hole. It has one hole at the extreme rows, with a difference of not more than one hole in the side-by-side rows. This is also called diamond joint since the rivets are arranged in a diamond shape. Fig. 11.26 shows a typical arrangement of rivets in a lozenge joint.

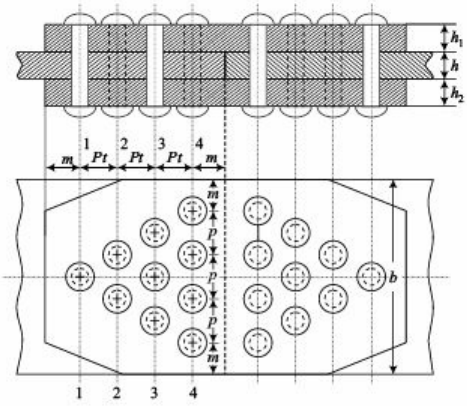


Fig. 11.26 Lozenge Joint

Symbols: F = force, N

b = width of plate, mm

h = thickness of plates, mm

σ_θ = allowable tensile stress in plate, MPa

τ = allowable shear stress in rivets, MPa

σ_c = allowable crushing stress in rivets, MPa

i = no. of rivets

d = diameter of rivet

d_h = diameter of rivet hole

F_τ = strength of rivet in shear

F_c = strength of rivet in crushing

F_1 = strength of rivet

Design Procedure of Lozenge Joint

Step 1: Diameter of the rivet $d = 6\sqrt{h}$ to $6.3\sqrt{h}$

Select standard diameter of the rivet and diameter of the rivet hole d_h from Table 11.1

Step 2: To find width of the plate

For the plate not to be weakened by more than one rivet hole, $F = (b - d_h)h\sigma_\theta$

Find width of the plate b.

If 'b' is given, find the load capacity of joint F.

Step 3: Strength of the rivet

(a) Shear strength of one rivet in double shear $F_\tau = 1.875\left(\frac{\pi d^2}{4}\right)\tau$

(b) Crushing strength of one rivet, $F_c = hd\sigma_c$

$\therefore$ Minimum strength of rivet, F_1 = minimum of these two

Step 4: No. of rivets required $i = \dfrac{\text{Total load } (F)}{\text{Minimum strength or rivet } (F_1)}$

Step 5: Transverse pitch $p_t = 2.25d$

Step 6: Margin $m = 1.5d$

Step 7: Thickness of the cover plates, $h_1 = h_2 = 0.625h$.

Step 8: Arrange the number of rivets in a diamond shape, keeping in mind that the first row should always have only one rivet and the difference in no. of rivets in subsequent consecutive rows should not be greater than one (i.e., either one or zero). Table 11.3 below shows some examples of arrangement of rivets for Lozenge joint.

Step 9: Strength and efficiency

(i) Strength of the solid plate, $F_s = bh\sigma_\theta$.

(ii) The joint may fail due to tearing of plate along 1 – 1

Strength of the plate along line 1 – 1, $F_\theta = (b - d_h)h\sigma_\theta$.

(iii) The joint may fail due to shearing of all rivets.

Shear strength of all the rivets, $F_\tau = i(1.875)\dfrac{\pi d^2\tau}{4}$.

(iv) The joint may fail due to crushing of all rivets.

Crushing strength of all the rivets $F_c = i(dh\sigma_c)$

(v) Failure along 2 – 2:

The joint may fail due to tearing of plate along 2 – 2 and failure of rivets in the previous rows.

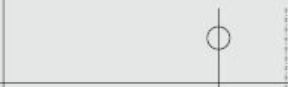
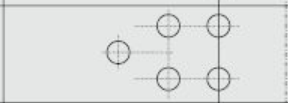
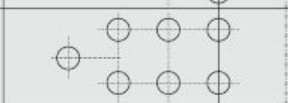
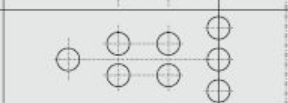
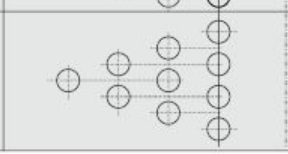
$F_{2-2} = (b - 2d_h)h\sigma_\theta$ + (no. of rivets in the previous rows) $\times F_1$.

(vi) Failure along 3 – 3:

The joint may fail due to tearing of the plate along 3 – 3 and failure of rivets in the previous rows.

$F_{3-3} = (b - 3d_h)h\sigma_\theta$ + (no. of rivets in previous row 1 – 1 and 2 – 2) $\times F_1$ etc.

Table 11.3 Arrangement of rivets in Lozenge joint

No. of rivets	Arrangement of rivets
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

$$\text{Strength of joint} = \text{Minimum of the above strengths} = F_{min}$$

$$\text{Efficiency, } \eta = \frac{\text{Minimum strength of joint } (F_{min})}{\text{Strength of solid plate } (F_s)}$$

Problem 17: Design a Lozenge riveted joint to connect two M.S. plates of 12 mm thick and to carry a load of 400 kN. The allowable stresses are: tensile stress = 100 MPa, shear stress 60 MPa and compressive stress 160 MPa.

Given data: Thickness of the plates, $h = 12$ mm

Load, $F = 400 \times 10^3$ N

Tensile stress, $\sigma_\theta = 100$ MPa

Shear stress, $\tau = 60$ MPa

Compressive or crushing stress, $\sigma_c = 160$ MPa

Step 1: Diameter of the rivet $d = 6\sqrt{h}$ to $6.3\sqrt{h}$

$d = 6\sqrt{12}$ to $6.3\sqrt{12} = 20.78$ to 21.82.

Select standard diameter, $d = 22$ mm and diameter of rivet hole, $d_h = 23$ mm (Table 11.1)

Step 2: Width of the plate: For the joint not to be weakened by more than one rivet hole,

$$F = (b - d_h)h\sigma_\theta$$

$$400 \times 10^3 = (b - 23)12 \times 100$$

$\therefore$ width of the plate $b = 356.33 \simeq 357$ mm

Step 3: Strength of a single rivet

(a) Shear strength of the rivet in double shear, $F_\tau = (1.875)\dfrac{\pi d^2 \tau}{4}$

$$F_\tau = (1.875)\frac{\pi \times 22^2}{4} \times 60 = 42.765 \times 10^3 \text{ N}$$

(b) Crushing strength of rivet, $F_c = dh\sigma_c$

$$F_c = 22 \times 12 \times 160 = 42.240 \times 10^3 \text{ N}$$

Minimum strength of single rivet $F_1 = 42.24 \times 10^3$ N

Step 4: No. of rivets required

$$i = \frac{\text{Total force}}{\text{Strength of single rivet}} = \frac{F}{F_1} = \frac{400 \times 10^3}{42.24 \times 10^3}$$

$i = 9.47 \simeq 10$ rivets

Step 5: Transverse pitch $p_t = 2.25d$

$p_t = 2.25 \times 22 = 49.5 \simeq 50$ mm

Step 6: Margin, $m = 1.5d = 1.5 \times 22 = 33$ mm

Step 7: Thickness of the cover plates $h_1 = h_2 = 0.625h = 0.625 \times 12 = 7.5 \simeq 8$ mm

Step 8: Arrange the total no. of rivets 10 in a diamond shape as shown in Fig. 11.27.

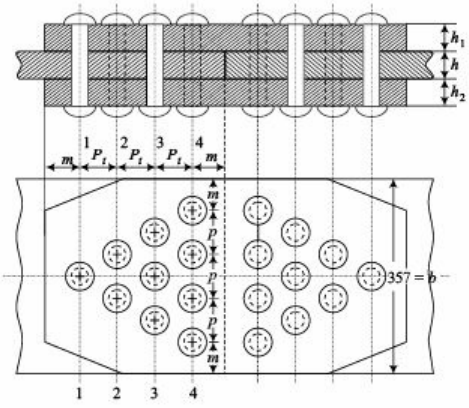


Fig. 11.27 Lozenge Joint

Step 9: Strength of the joint and efficiency

(i) Strength of the solid plate $F_s = bh\sigma_\theta = 357 \times 12 \times 100 = 428.4 \times 10^3$ N

(ii) Strength of the perforated plate $F_\theta = (b - d_h)h\sigma_\theta$

$$F_\theta = (357 - 23)12 \times 100 = 400.8 \times 10^3 \text{ N}$$

(iii) Shear strength of the total rivets in double shear $F_\tau = i(1.875)\dfrac{\pi d^2}{4}\tau$

$$F_\tau = 10(1.875)\frac{\pi \times 22^2}{4} \times 60 = 427.65 \times 10^3 \text{ N}$$

(iv) Crushing strength of the rivets, $F_c = i(dh\sigma)$

$$F_c = 10(22 \times 12 \times 160) = 422.4 \times 10^3 \text{ N}$$

(v) Strength of the joint along line 1 – 1

$$F_{1-1} = F_\theta = (b - d_h)h\sigma_\theta = 400.8 \times 10^3 \text{ N}$$

(vi) Strength of the joint along line 2 – 2

F_{2-2} = tearing of plate along 2 – 2 + failure of rivets in previous row 1 – 1 (one rivet)

$F_{2-2} = (b - 2d_h)h\sigma_\theta + 1(F_1) = (357 - 2 \times 23)12 \times 100 + 1 \times 42.24 \times 10^3 = 415.44 \times 10^3$ N

(vii) Strength of the joint along line 3 – 3

F_{3-3} = tearing of plate along 3 – 3 + failure of rivets in previous rows 1 – 1 and 2 – 2(1 + 2 = 3 rivets)

$= (b - 3d_h)h\sigma_\theta + (1 + 2)F_1 = (357 - 3 \times 23)12 \times 100 + (1 + 2)42.24 \times 10^3 = 472.32 \times 10^3$ N

(viii) Strength of joint along line 4 – 4

F_{4-4} = tearing of plate along 4 – 4 + failure of rivets in the previous rows 1 – 1, 2 – 2 and 3 – 3 (1 + 2 + 3 = 6 rivets) = $(b - 4d_h)h\sigma_\theta + (1 + 2 + 3)F_1$

$= (357 - 4 \times 23)12 \times 100 + 6 \times 42.24 \times 10^3 = 571.44 \times 10^3$ N

$\therefore$ Strength of joint = minimum of the above all values

$$F_{min} = 400.8 \times 10^3 \text{ N}$$

Efficiency $$\eta = \frac{\text{strength of joint } (F_{min})}{\text{strength of solid plate } (F_s)} = \frac{400.8 \times 10^3}{428.4 \times 10^3}$$

$$\eta = 0.9356 \simeq 93.56\%$$

Problem 18: Design a tie bar to connect two plates 20 mm thick and 250 mm wide. The allowable tensile stress for the plates is 90 MPa. Allowable shear and compressive stresses for the rivet material are 55 MPa and 110 MPa respectively. Sketch the designed joint showing main dimensions and calculate the joint efficiency.

Given data: Thickness of the plate $h = 20$ mm

Width of the plate $b = 250$ mm

$\sigma_\theta = 90$ MPa

$\tau = 55$ MPa

$\sigma_c = 110$ MPa

Step 1: Diameter of the rivet $d = 6\sqrt{h}$ to $6.3\sqrt{h}$

$d = 6\sqrt{20}$ to $6.3\sqrt{20} = 26.83$ to 28.17

Select standard diameter $d = 27$ mm and

diameter of the rivet hole $d_h = 28.5$ mm (Table 11.1)

Step 2: Force capacity based on the plate

$F = (b - d_h)h\sigma_\theta = (250 - 28.5)20 \times 90$

$F = 398.7 \times 10^3$ N

Step 3: Strength of the single rivet

(a) Shear strength of each rivet in double shear

$$F_\tau = (1.875)\frac{\pi}{4}d^2\tau = 1.875 \times \frac{\pi}{4} \times 27^2 \times 55 = 59.04 \times 10^3 \text{ N}$$

(b) Crushing strength of each rivet, $F_c = hd\sigma_c$

$F_c = 20 \times 27 \times 110 = 59.4 \times 10^3$ N

$\therefore$ Strength of each rivet = $F_1 = 59.04 \times 10^3$ N (minimum these two values)

Step 4: No. of rivets required $$i = \frac{\text{Force capacity } (F)}{\text{Strength of each rivet } (F_1)}$$

$$i = \frac{3.987 \times 10^3}{59.04 \times 10^3} = 6.75 \simeq 7 \text{ rivets}$$

Step 5: Transverse pitch $p_t = 2.25d = 2.25 \times 27 = 60.75 \simeq 61$ mm

Step 6: Margin, $m = 1.5d = 1.5 \times 27 = 40.5 \simeq 41$ mm

Step 7: Thickness of the cover plates $h_1 = h_2 = 0.625h = 0.625 \times 20 = 12.5 \simeq 13$ mm

Step 8: Arrange these 7 rivets in diamond shape as shown in Fig. 11.28.

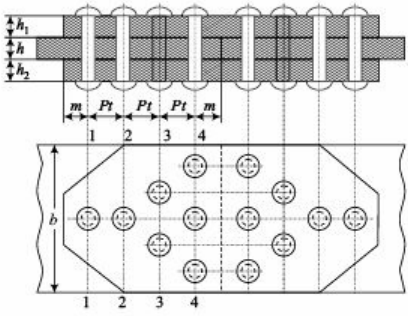


Fig. 11.28

Step 9: Strength of the joint and efficiency

(i) Strength of the solid plate $F_S = bh\sigma_\theta = 250 \times 20 \times 90 = 450 \times 10^3$ N

(ii) Strength of 7 rivets in double shear $F_\tau = i(1.875)\frac{\pi}{4}d^2\tau$

$$F_\tau = 7(1.875)\frac{\pi}{4} \times 27^2 \times 55 = 413.32 \times 10^3 \text{ N}$$

(iii) Crushing strength of 7 rivets, $F_c = 7(dh\sigma_c) = 7(27 \times 20 \times 110) = 415.8 \times 10^3$ N

(iv) Strength of the joint along line 1 – 1

F_{1-1} = Tearing of plate along 1 – 1 = $(b - d_h)h\sigma_\theta = (250 - 28.5)20 \times 90 = 398.7 \times 10^3$ N

(v) Strength of joint along 2 – 2

F_{2-2} = Tearing of plate along 2 – 2 + failure of rivets in previous row 1 – 1 (1 rivet)

$F_{2-2} = (b - dh)h\sigma_\theta + (1)F_1 = (250 - 28.5)20 \times 90 + (1) \times 59.04 \times 10^3 = 457.74 \times 10^3$ N

(vi) Strength of joint along 3 – 3

F_{3-3} = Tearing of plate along 3 – 3 + failure of rivets in previous rows 1 – 1 and 2 – 2 (1 + 1 = 2 rivets) = $(b - 2d_h)h\sigma_\theta + (1 + 1)\,F_1$

$F_1 = (250 - 2 \times 28.5)20 \times 90 + (2)59.04 \times 10^3 = 465.48 \times 10^3$ N

(vii) Strength of joint along 4 – 4

F_{4-4} = Tearing of plate along 4 – 4 + failure of rivets in previous rows 1 – 1, 2 – 2 and 3 – 3(1 + 1 + 2 = 4 rivets) = $(b - 3d_h)h\sigma_\theta + (4)F_1$

$F_{4-4} = (250 - 3 \times 28.5)20 \times 90 + 4 \times 59.04 \times 10^3 = 532.26 \times 10^3$ N

∴ Strength of the joint = least of the above values; $F_{min} = 398.7 \times 10^3$ N

∴ Efficiency $$\eta = \frac{\text{strength of joint } (F_{min})}{\text{strength of solid plate } (F_s)}$$

$$\eta = \frac{398.7 \times 10^3}{450 \times 10^3} = 0.886$$

$$\eta = 88.6\%.$$

11.7 ECCENTRIC LOADING OF RIVETED JOINTS

An eccentrically loaded joint is one in which the line of application of the load does not pass through c.g. of rivets. It passes away from c.g. axis. This has two effects, direct load and bending load due to couple. The analysis is similar to eccentric loading of bolted joints explained earlier.

11.8 TYPE -1

Symbols: F = Force

c.g. = centre of gravity of rivets.

e = eccentricity, the distance from c.g. to force.

$\bar{x}$, $\bar{y}$ = position of c.g. from left side lower rivet.

l_1, l_2, l_3, etc. = distance from c.g. to rivets 1, 2, 3, etc.

F_d = Direct load per rivet, acts parallel to given load.

F_1, F_2, F_3, etc. = Normal loads acting on rivets 1, 2, 3, etc. act perpendicular to l_1, l_2, l_3, etc respectively.

The direction of F_1, F_2, F_3, etc should give the same moment as that of external force about c.g. If $(F \times e)$ is clockwise then F_1l_1, F_2l_2, F_3l_3 etc. are clockwise about c.g. and vice versa.

F_{R1}, F_{R2}, F_{R3} etc. = Resultant forces acting on rivets 1, 2, 3, etc.

θ_1 = angle between F_d and F_1

θ_2 = angle between F_d and F_2, etc.

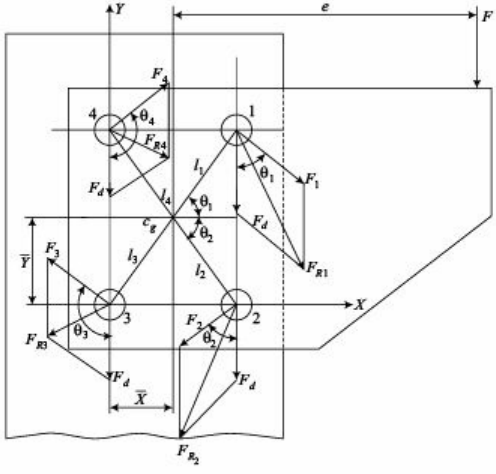


Fig. 11.29

τ = allowable shear stress for rivet
A = area of c/s of rivet
d = diameter of rivet

Procedure

Step 1: Find c.g. of the rivets

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \ldots}{\text{no. of rivets}} \quad \text{and}$$

$$\bar{y} = \frac{y_1 + y_2 + y_3 + \ldots}{\text{no. of rivets}}$$

where x_1, x_2, x_3, etc are horizontal distances from y-axis to rivets 1, 2, 3, etc. and y_1, y_2, y_3, etc. are vertical distances from x-axis to rivets 1, 2, 3 etc.

Step 2: Find eccentricity 'e' and distances l_1, l_2, l_3, etc.,

l_1, l_2, l_3 etc. are diagonals of some triangles.

For example, $l_1 = \sqrt{x^2 + y^2}$, etc.

Identify the rivets that have $\theta < 90°$.

Among these, identify the farthest rivet from c.g.

Farthest rivet from c.g. (having more l) and $\theta < 90°$ is heavily loaded.

Find cos θ for that rivet $\cos\theta = \frac{x}{l}$

Step 3: Direct load per rivet $F_d = \frac{F}{\text{no. of rivets}}$

Step 4: Normal load on heavily loaded rivet

$$F_n = \frac{F.e.l_n}{(l_1^2 + l_2^2 + l_3^2 + \ldots)}$$

i.e.

$$F_1 = \frac{F.e\,l_1}{(l_1^2 + l_2^2 + l_3^2 + \ldots)}$$

$$F_2 = \frac{F.e\,l_2}{(l_1^2 + l_2^2 + l_3^2 + \ldots)} = \frac{F_1}{l_1} l_2. \text{ etc.}$$

$$\left(\frac{F_1}{l_1} = \frac{F_2}{l_2} = \frac{F_3}{l_3} \ldots\right)$$

Step 5: Resultant load on the heavily loaded rivet

$$F_R = \sqrt{F_d^2 + F_n^2 + 2F_dF_n\cos\theta_n}$$

Step 6: Area of the rivets $A = \dfrac{F_R}{\text{stress}}$

But $A = \dfrac{\pi d^2}{4}$

Find diameter of the rivet, d.

Select standard rivet diameter, d from Table 11.1.

Problem 19: Determine the diameter of the rivet for a joint loaded as shown in Fig. 11.30. The allowable shear stress for the rivets may be taken as 90 MPa.

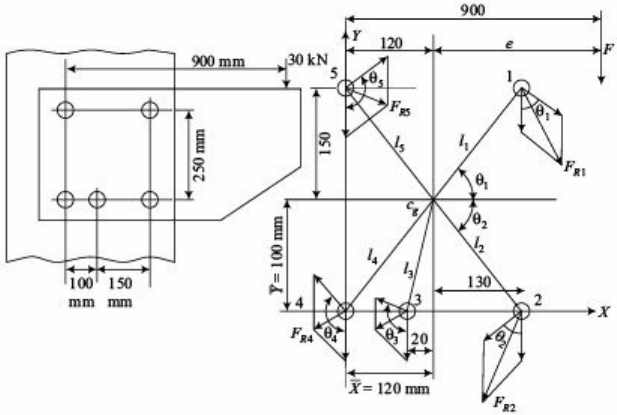


Fig. 11.30

Given data: Draw the rivets separately as shown above.

$$F = 30 \times 10^3 \text{ N}, \quad \tau = 90 \text{ MPa}, \quad \text{No. of rivets } i = 5$$

Step 1: To find centre of gravity $\bar{x} = \dfrac{x_1 + x_2 + x_3 + \ldots}{i} = \dfrac{(250 + 250 + 100 + 0 + 0)}{5} = 120 \text{ mm}$

$$\bar{y} = \frac{y_1 + y_2 + y_3 + \ldots}{i} = \frac{250 + 0 + 0 + 0 + 250}{5} = 100 \text{ mm}$$

Step 2: Eccentricity $e = 900 - \bar{x} = 900 - 120 = 780 \text{ mm}$

$$l_1 = \sqrt{x^2 + y^2} = \sqrt{130^2 + 150^2} = 198.49 \text{ mm}$$

$$l_2 = \sqrt{130^2 + 100^2} = 164.01 \text{ mm}$$

$$l_3 = \sqrt{20^2 + 100^2} = 101.98 \text{ mm}$$

$$l_4 = \sqrt{120^2 + 100^2} = 156.20 \text{ mm}$$

$$l_5 = \sqrt{120^2 + 150^2} = 192.09 \text{ mm}$$

It is seen from figure that for rivets 1 and 2, the angle between F_d and F_n is less than 90° and $l_1 > l_2$, hence rivet 1 is heavily loaded.

(Farthest rivet from c.g. having $\theta < 90°$ is heavily loaded)

Hence, find cos θ_1, F_1 and F_{R1} for that heavily loaded rivet.

$$\cos \theta_1 = \frac{x}{l_1} = \frac{130}{198.49} = 0.655$$

Step 3: Direct load/rivet, $F_d = \frac{F}{i} = \frac{30000}{5} = 6000$ N

Step 4: Normal load on heavily loaded rivet '1'

$$F_1 = \frac{F.e\, l_1}{\Sigma\, l^2} = \frac{30 \times 10^3 \times 780 \times 198.49}{198.49^2 + 164.01^2 + 101.98^2 + 156.20^2 + 192.09^2}$$

$$F_1 = 33.66 \times 10^3 \text{ N}$$

This acts perpendicular to l_1.

Step 5: Resultant load on heavily loaded rivet 1

$$F_{R\max} = F_{R1} = \sqrt{F_d^2 + F_1^2 + 2F_dF_1 \cos \theta_1}$$

$$F_{R1} = \sqrt{6000^2 + (33.66 \times 10^3)^2 + 2 \times 6000 \times (33.66 \times 10^3) \times 0.655}$$

$$F_{R1} = F_{R\max} = 37.86 \times 10^3 \text{ N}$$

Step 6: Area of the rivet $A = \frac{F_{R\max}}{\text{stress}} = \frac{37.86 \times 10^3}{90}$

$$A = \frac{\pi}{4} d^2 = 420.69$$

$\therefore$ diameter of rivet, $d = 23.14$ mm

Select standard diameter of rivet from Table 11.1

$$d = 24 \text{ mm}$$

Problem 20: Determine the diameter of the rivet for a joint loaded as shown in Fig. 11.31. The allowable shear stress for rivet material is 75 MPa.

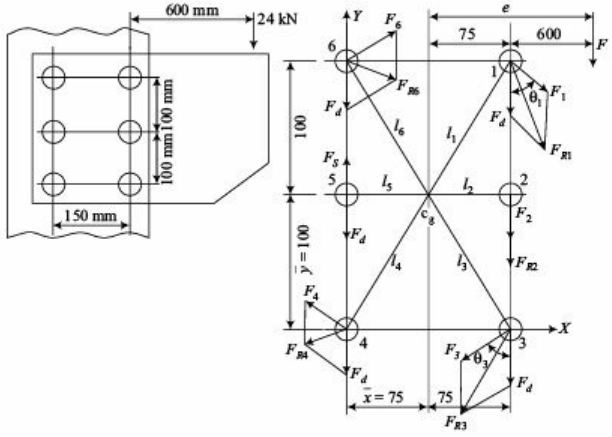


Fig. 11.31

Given data: Load, $F = 24 \times 10^3$ N

Stress, $\tau = 75$ MPa

Draw the given rivets separately as shown.

Step 1: To find centre of gravity

$$\bar{x} = \frac{x_1 + x_2 + x_3 + y_4 + \ldots}{i}$$

$$\bar{x} = \frac{150 + 150 + 150 + 0 + 0 + 0}{6} = 75 \text{ mm}$$

$$\bar{y} = \frac{y_1 + y_2 + y_3 + \ldots}{i} = \frac{200 + 100 + 0 + 0 + 100 + 200}{6} = 100 \text{ mm}$$

(Due to symmetry, c.g. lies in the middle)

Step 2: Eccentricity $e = 600 + 75 = 675$ mm

$$l_1 = l_3 = l_4 = l_6 = \sqrt{75^2 + 100^2} = 125 \text{ mm}$$

$l_2 = l_5 = 75$ mm (Symmetric)

It is observed that for rivets 1, 2 and 3, θ is less than 90° and rivet 1 and 3 are farthest from c.g., either rivet 1 or 3 is heavily loaded. Hence find cos θ, F_n and F_R for that rivet. (Say rivet 1)

$$\cos \theta_1 = \frac{x}{l_1} = \frac{75}{125} = 0.6$$

Step 3: Direct load per rivet, $F_d = \frac{F}{i} = \frac{24000}{6} = 4000\ \text{N}$

This acts parallel to 'F' (vertical downward at all rivets)

Step 4: Normal load on heavily loaded rivet '1'

$$F_1 = \frac{Fel_1}{\Sigma l^2} = \frac{24000 \times 675 \times 125}{4 \times 125^2 + 2 \times 75^2} = 27.46 \times 10^3\ \text{N}$$

This acts perpendicular to l_1.

Step 5: Resultant load on heavily loaded rivet '1'

$$F_{R1} = \sqrt{F_d^2 + F_1^2 + 2F_dF_1 \cos\theta}$$

$$= \sqrt{4000^2 + (27.46 \times 10^3)^2 + 2 \times 4000 \times 27.46 \times 10^3 \times 0.6}$$

$$= 30.03 \times 10^3\ \text{N}$$

Step 6: Area of the rivet $A = \frac{F_R}{\text{stress}} = \frac{30.03 \times 10^3}{75} = 400.384\ \text{mm}^2$

But $A = \frac{\pi d^2}{4} = 400.384$

$\therefore$ Diameter of the rivet $d = 22.58$ mm

From Table 11.1, select standard diameter of the rivet $d = 24$ mm.

Problem 21: Determine the size of rivets required for the bracket shown in Fig. 11.32, take the permissible shear stress for the rivet material as 100 MPa.

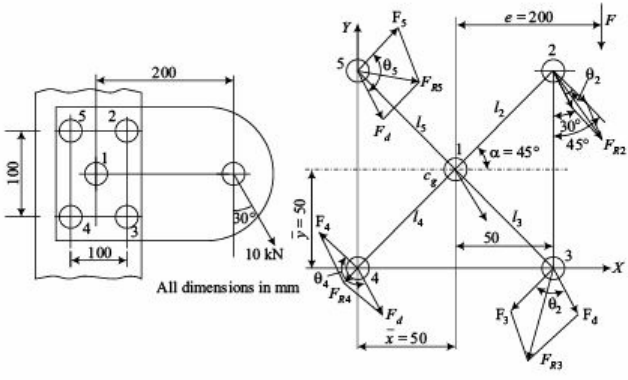


Fig. 11.32

Given data: $F = 10 \times 10^3$ N, $\tau = 100$ MPa.

Step 1: To find c.g. Draw the given rivets separately as shown. The rivets are arranged in symmetric manner. Hence, the c.g. lies at the centre at rivet '1'.

$$\bar{x} = \frac{100}{2} = 50 \text{ mm and } \bar{y} = \frac{100}{2} = 50 \text{ mm.}$$

Step 2:

$$l_1 = 0$$

$$l_2 = l_3 = l_4 = l_5 = \sqrt{50^2 + 50^2} = 70.71 \text{ mm (symmetric)}$$

It is observed that for rivet 1, there is no normal load that acts perpendicular to l_1 (since $l_1 = 0$)

For rivets 2 and 3, the $\theta < 90°$ and rivet 2 is heavily loaded since $l_2 = l_3$ and $\theta_2 < \theta_3$.

$$\cos \theta_2 = \cos 15 = 0.966$$

(The angle of F_d is 30°, which is parallel to F. The angle of F_2 is perpendicular to l_2, $\alpha = 45°$

$\therefore$

$$\theta_2 = \alpha - 30 = 45 - 30 = 15°$$

$$e = 200 \text{ mm (directly given)}$$

Step 3: Direct load on each rivet $F_d = \frac{F}{i} = \frac{10000}{5} = 2000$ N

(This acts parallel to F at 30° to vertical on all the rivets).

Step 4: Normal load on the heavily loaded rivet $F_2 = \frac{Fel_2}{\Sigma l^2}$

$$F_2 = \frac{10000 \times 200 \times 70.71}{4 \times 70.71^2 + 0^2} = 7071.14 \text{ N}$$

(This acts perpendicular to l_2)

Step 5: Resultant load on the heavily loaded rivet 2

$$F_{R2} = \sqrt{F_d^2 + F_2^2 + 2F_dF_2 \cos \theta_2}$$

$$= \sqrt{2000^2 + 7071.14^2 + 2 \times 2000 \times 7071.14 \times 0.966}$$

$$= 9017.97 \text{ N}$$

Step 6: Area of the rivet $A = \frac{F_R}{\text{stress}} = \frac{9017.97}{100} = 90.18 \text{ mm}^2$

But $A = \frac{\pi d^2}{4} = 90.18$

$\therefore$ Diameter of the rivet, $d = 10.72$ mm

Ref. Table 11.1 and select standard diameter, $d = 12$ mm

Problem 22: For the riveted joint shown in the Fig. 11.33, determine the size of the rivet taking permissible shear stress as 60 MPa.

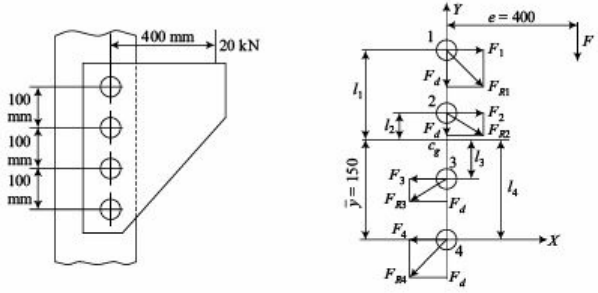


Fig. 11.33

Given data: $F = 20 \times 10^3$ N, $\tau = 60$ MPa, $i = 4$ rivets

Draw the given rivets separately as shown.

Step 1: To locate c.g.

Due to symmetry, cg lies at middle

$$\bar{x} = 0, \quad \bar{y} = 150 \text{ mm}$$

Step 2: e = 400 mm

$$l_1 = l_4 = 150 \text{ mm (symmetric)}$$

$$l_2 = l_3 = 50 \text{ mm}$$

$$\theta = 90° \text{ for all 4 rivets.}$$

Since l_1 and l_4 are equal, either rivet 1 or 4 is heavily loaded.

$$\cos\theta = \cos 90° = 0$$

Step 3: Direct load/rivet $$F_d = \frac{F}{i} = \frac{20000}{4} = 5000 \text{ N}$$

(This acts parallel to given load, vertical)

Step 4: Normal load on heavily loaded rivet

$$F_1 = \frac{Fel_1}{\Sigma l^2} = \frac{20 \times 10^3 \times 400 \times 150}{2(150^2 + 50^2)} = 24 \times 10^3 \text{ N}$$

(This acts perpendicular to l_1 i.e. horizontal)

Step 5: Resultant load $$F_{R1} = \sqrt{F_d^2 + F_1^2 + 2F_dF_1\cos\theta}$$

$$F_{R1} = \sqrt{F_d^2 + F_1^2} = \sqrt{5000^2 + 24000^2} = 24.515 \times 10^3 \text{ N}$$

Step 6: Area of the rivet $A = \dfrac{F_R}{\text{stress}} = \dfrac{24.515 \times 10^3}{60} = 408.59 \text{ mm}^2$

But $A = \dfrac{\pi d^2}{4} = 408.59$

$\therefore$ Diameter of the rivet, $d = 22.81$ mm

From Table 11.1, select standard rivet diameter $d = 24$ mm.

Problem 23: Determine the diameter of rivet for a joint loaded as shown in the Fig. 11.34. The allowable shear stress for the rivet material is 72 MPa.

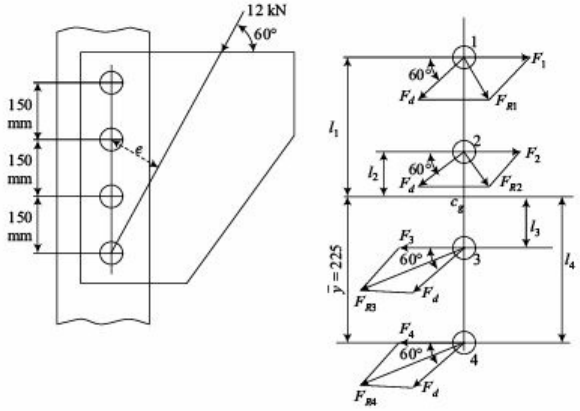


Fig. 11.34

Given data: $F = 12000$ N, $i = 4$ rivets, $\tau = 72$ MPa.

Draw the given rivets separately as shown.

Step 1: To locate c.g.

Due to symmetry, $\bar{x} = 0$

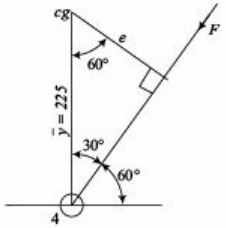
$$\bar{y} = 150 + \frac{150}{2} = 225 \text{ mm}$$

Step 2: e = eccentricity = perpendicular distance from c.g. to force.

From figure,

$$\cos 60° = \frac{e}{\bar{y}}$$

$\therefore$ $e = \bar{y} \cos 60° = 225 \cos 60° = 112.5$ mm



$l_1 = l_4 = 225$ mm

$l_2 = l_3 = 75$ mm (symmetric)

It is seen that for rivets 3 and 4, $\theta < 90°$ and rivet 4 is farthest from c.g., rivet 4 is heavily loaded.

$$\theta_4 = 60° \quad \therefore \cos\theta_4 = \cos 60° = 0.5$$

Step 3: Direct load/rivet $$F_d = \frac{F}{i} = \frac{12000}{4} = 3000 \text{ N}$$

(This acts parallel to F at 60° with horizontal)

Step 4: Normal load on heavily loaded rivet '4'

$$F_4 = \frac{Fel_4}{\Sigma l^2} = \frac{12000 \times 112.5 \times 225}{2(225^2 + 75^2)} = 2700 \text{ N}$$

Step 5: Resultant load on heavily loaded rivet '4'

$$F_{R4} = \sqrt{F_d^2 + F_4^2 + 2F_dF_4\cos\theta_4}$$

$$= \sqrt{3000^2 + 2700^2 + 2 \times 3000 \times 2700 \times 0.5} = 4938.62 \text{ N}$$

(This acts perpendicular to l_4)

Step 6: $\therefore$ Area of the rivet $$A = \frac{F_R}{\tau} = \frac{4938.62}{72} = 68.59 \text{ mm}$$

But $$A = \frac{\pi d^2}{4} = 68.59$$

$\therefore$ Diameter of rivet, $d = 9.35$ mm

From table select standard diameter of rivet

$d = 10$ mm.

Problem 24: Determine the diameter of the rivet for a bracket loaded as shown in Fig. 11.35. The allowable shear stress for rivets is 60 MPa.

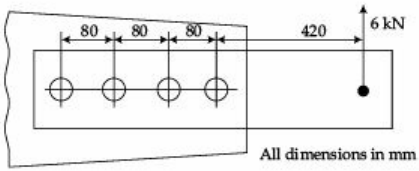
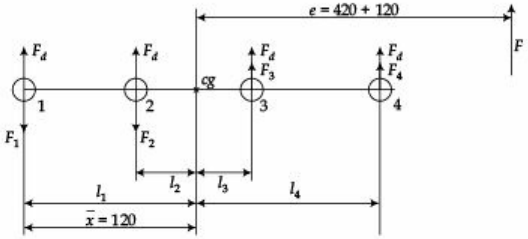


Fig. 11.35

Given data: $F = 6000$ N, $i = 4$ rivets, $\tau = 60$ MPa.

Draw the given rivets separately as shown below:



Step 1: To locate c.g.

Due to symmetry, $\bar{y} = 0$ and $\bar{x} = 80 + \frac{80}{2} = 120$ mm

Step 2:

$$l_1 = l_4 = 120 \text{ mm}$$
$$l_2 = l_3 = 40 \text{ mm}$$
$$e = 420 + 120 = 540 \text{ mm}$$

Direct load acts parallel to F, Vertical upward at all rivets.

Normal loads act perpendicular to l_1, l_2, l_3, etc. in the same moment as external moment $F.e$.

Here, ($F.e$.) is counter clockwise about c.g. Hence, F_1, F_2, F_3 and F_4 also act such way that they give ccw moment about c.g.

For rivets 3 and 4, $\theta = 0$ and rivet 4 is farthest from c.g., hence rivet 4 is heavily loaded.

$$\cos \theta_4 = \cos 0 = 1$$

Step 3: Direct load/rivet $$F_d = \frac{F}{i} = \frac{6000}{4} = 1500 \text{ N}$$

Step 4: Normal load on heavily loaded rivet 4

$$F_4 = \frac{Fel_4}{\Sigma l^2} = \frac{6000 \times 540 \times 120}{2(120^2 + 40^2)} = 12150 \text{ N}$$

Step 5: Resultant load on heavily loaded rivet 4

$$F_{R4} = \sqrt{F_d^2 + F_4^2 + 2F_dF_4 \cos \theta_4}$$
$$= \sqrt{1500^2 + 12150^2 + 2 \times 1500 \times 12150 \times 1}$$
$$= 13560 \text{ N}$$

($\because$ $\theta = 0$, $F_{R4} = F_d + F_4$)

Step 6: Area of rivet $$A = \frac{F_{R4}}{\tau} = \frac{13650}{60} = 227.5 \text{ mm}^2$$

But $$A = \frac{\pi d^2}{4} = 227.5$$

$\therefore$ Diameter of rivet, $d = 17.02$ mm

Select standard diameter, $d = 18$ mm from Table 11.1.

Problem 25: Determine the diameter of the rivet for a bracket loaded as shown in Fig. 11.36. Take allowable shear stress as 54 MPa for the rivets.

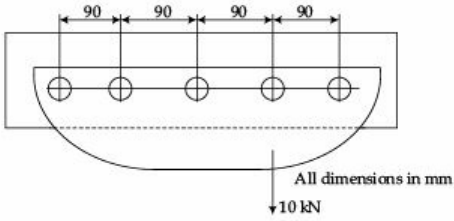
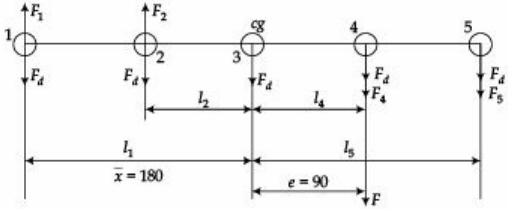


Fig. 11.36

Given data: $F = 10000$ N, $\tau = 54$ MPa, $i = 5$ rivets

Draw the given rivets separately as shown below.



Step 1: $\bar{x} = 180$ mm (symmetric)

Step 2: $\bar{y} = 0$

$e = 90$ mm (from figure)

$l_1 = l_5 = 180$ mm,

$l_2 = l_4 = 90$ mm

$l_3 = 0$ (c.g. lies at rivet 3)

For rivets 4 and 5, $\theta = 0°$, and since $l_5 > l_4$, rivet 5 is heavily loaded.

$$\cos\theta = \cos 0 = 1$$

Step 3: Direct load/rivet $$F_d = \frac{F}{i} = \frac{10000}{5} = 2000 \text{ N}$$

This acts parallel to F, vertical downward at all 5 rivets.

Step 4: Normal load

It is seen that rivet 5 is farthest from c.g. and has θ less than 90°, rivet 5 is heavily loaded rivet.

$$F_5 = \frac{Fel_5}{\Sigma l^2} = \frac{10000 \times 90 \times 180}{2(180^2 + 90^2) + 0^2} = 2000 \text{ N}$$

Since $F \times e$ is a cw moment about *c.g.*, $F_1 l_1$, $F_2 l_2$, etc., will also have cw direction as shown.

Step 5: Resultant load on having loaded rivet 5

$$F_{R5} = F_d + F_5 \quad (\therefore \theta = 0)$$
$$= 2000 + 2000 = 4000 \text{ N}$$

Step 4: Area, rivet $$A = \frac{F_{R_5}}{\text{stress}} = \frac{4000}{54} = 74.07 \text{ mm}^2$$

But $$A = \frac{\pi}{4} d^2 = 74.07$$

$\therefore$ diameter of the rivet, $d = 9.7115$ mm

Take standard diameter, $d = 10$ mm from Table 11.1

Problem 26: Determine the load carrying capacity of a riveted jointed loaded as shown in Fig. 11.37 the diameter of the rivets used is 24 mm and has an allowable shear stress 60 MPa.

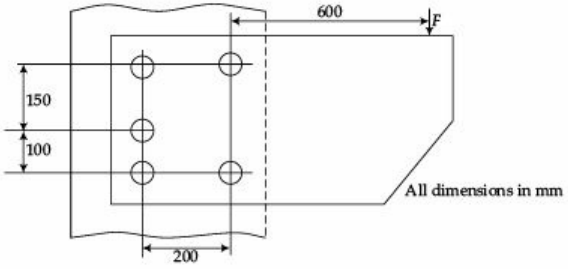


Fig. 11.37

Given data: Diameter of the rivet, $d = 24$ mm

Allowable shear stress, $\tau = 60$ MPa

No. of rivets, $i = 5$

Draw the given rivets separately as shown below.

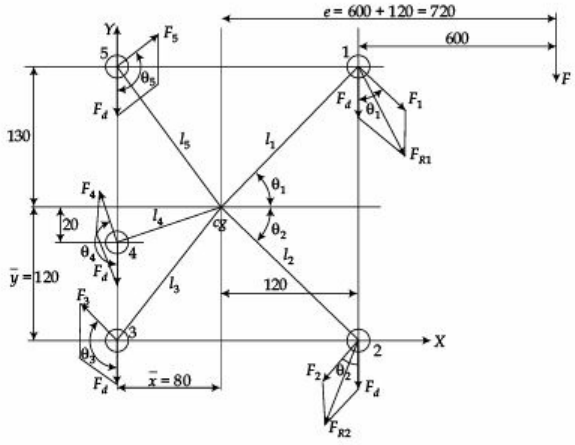
Step 1: To locate c.g. $$\bar{x} = \frac{x_1 + x_2 + x_3 + \ldots}{i} = \frac{200 + 200 + 0 + 0 + 0}{5} = 80 \text{ mm}$$

$$\bar{y} = \frac{y_1 + y_2 + y_3 + \ldots}{i} = \frac{200 + 0 + 0 + 100 + 250}{5} = 120 \text{ mm}$$

Step 2: $e = 600 + 120 = 720$ mm

$$l_1 = \sqrt{120^2 + 130^2} = 176.92 \text{ mm}$$

$$l_2 = \sqrt{120^2 + 120^2} = 169.71 \text{ mm}$$



$$l_3 = \sqrt{80^2 + 120^2} = 144.22 \text{ mm}$$

$$l_4 = \sqrt{80^2 + 20^2} = 82.46 \text{ mm}$$

$$l_5 = \sqrt{80^2 + 130^2} = 152.64 \text{ mm}$$

It is seen that for rivets 1 and 2, $\theta < 90°$ and $l_1 > l_2$, rivet 1 is heavily loaded

$$\cos \theta_1 = \frac{x}{l_1} = \frac{120}{176.92} = 0.678$$

Step 3: Direct load/rivet $\quad F_d = \dfrac{F}{i} = \dfrac{F}{5} = 0.2F$

(This acts parallel to F, vertical downward)

Step 4: Normal load heavily loaded rivet 1

$$F_1 = \frac{Fel_1}{\Sigma l^2} = \frac{F \times 720 \times 176.92}{(176.92^2 + 169.71^2 + 144.22^2 + 82.46^2 + 152.64^2)}$$

$$F_1 = 1.148\, F$$

Step 5: Resultant load on heavily loaded rivet 1

$$F_{R_1} = \sqrt{F_d^2 + F_1^2 + 2F_dF_1 \cos \theta_1}$$

$$= \sqrt{(0.2F)^2 + (1.148F)^2 + 2(0.2F)(1.148F)\,0.678}$$

$$F_{R_1} = 1.2919\, F.$$

Step 6: Area of the rivet $A = \dfrac{F_R}{\tau}$

But $$A = \frac{\pi d^2}{4} = \frac{\pi \times 24^2}{4}$$

$$\frac{\pi \times 24^2}{4} = \frac{1.2919F}{60}$$

$\therefore$ Load capacity, $F = 21 \times 10^3$ N

Problem 27: Determine the safe load that can be applied to an eccentrically loaded riveted bracket shown in Fig. 11.38. The allowable shear stress for 25 mm diameter rivets used is 90 MPa.

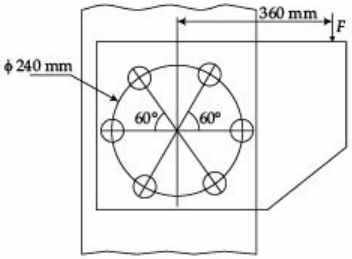
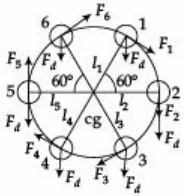


Fig. 11.38



Given data: $d = 25$ mm, $\tau = 90$ MPa, $i = 6$ rivets, $e = 360$ mm.

Draw the given rivets separately as shown in figure.

Step1: To locate c.g.

Due to symmetry, c.g. lies in the middle of circle of diameter = 240 mm.

Step 2: $e = 360$ mm

$l_1 = l_2 = l_3 = l_4 = l_5 = l_6 = 120$ mm (radius of circle)

It is seen that rivets 1, 2 and 3, $\theta < 90°$ and l is same for all of them. Hence, rivet 2 with $\theta = 0$ will be heavily loaded since $\cos\theta = 1$(other things being same, $\cos\theta$ is max)

Step 3: Direct load/rivet $F_d = \dfrac{F}{i} = \dfrac{F}{6} = 0.1667F$

(This is parallel to F at all rivets)

Step 4: Normal load on heavily loaded rivet 2

$$F_2 = \frac{Fel_2}{\Sigma l^2} = \frac{F \times 360 \times 120}{6 \times 120^2} = 0.5\ F$$

Step 5: Resultant load on the heavily loaded rivet 2

$$F_{R2} = \sqrt{F_d^2 + F_2^2 + 2F_dF_2\cos\theta_2} = F_d + F_2$$

(Since $\theta = 0$, $\cos\theta = 1$)

$$F_{R2} = 0.1667F + 0.5F = 0.6667F$$

Step 6: Area of the rivet $$A = \frac{F_R}{\text{stress}} = \frac{0.6667F}{90}$$

But $$A = \frac{\pi d^2}{4} = \frac{\pi \times 25^2}{4}$$

$$\frac{\pi \times 25^2}{4} = \frac{0.6667F}{90}$$

$\therefore$ Safe load that can be applied on bracket $F = 66.26 \times 10^3$ N.

11.9 ECCENTRIC LOADING-TYPE-II

(Load Acting Perpendicular to Rivet Axis)

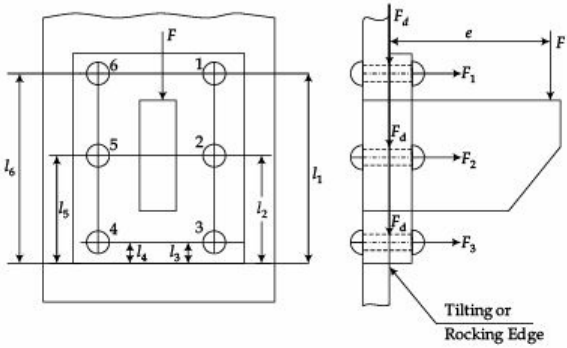


Fig. 11.39

F = load, N

e = eccentricity, distance from force to rocking edge, mm

l_1, l_2, l_3 etc = distances from rocking edge to rivets 1, 2, 3, etc. mm

F_d = direct load/rivet acts parallel to F, N

F_1, F_2, F_3 etc. L = normal loads on rivets 1, 2, 3 etc. acting along axis of rivets, N

τ_{max} = allowable or maximum shear stress for rivets, MPa

σ_{max} = allowable or maximum normal stress for rivets, MPa

A = area of rivet, mm^2

d = diameter of rivet, mm

i = no. of rivets

Direct load induces shear stress and normal load induces tensile stress in the rivets.

Design procedure

Step 1: Find l_1, l_2, l_3, etc. from given figure.

Step 2: Direct load/rivet $F_d = \dfrac{F}{i}$

Step 3: Shear stress in the rivet due to direct load

$$\tau = \frac{F_d}{A}$$

Step 4: Normal load on farthest rivet from rocking edge, that is heavily loaded,

$$F_1 = \frac{Fel_1}{\Sigma l^2}$$

Step 5: Tensile stress induced in the rivet due to normal load $\sigma = \dfrac{F_1}{A}$

Step 6: Combined or principal stresses

(a) Maximum normal stress

$$\sigma_{max} = \frac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right]$$

Equate this to allowable σ_{max} find A and d.. .

(b) Maximum shear stress $\tau_{max} = \dfrac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$

Equate this to allowable τ_{max}. Find A and d.

Recommend bigger diameter of these two. Refer Table 11.1, select standard diameter of rivet.

Note: If diameter of the rivet is given, then find F from both σ_{max} and τ_{max} equations.

Recommend the smaller one as maximum load capacity of the joint.

Problem 28: For the eccentrically loaded bracket shown in Fig. 11.40, determine the diameter of the rivet limiting the normal stress to 120 MPa in tension and 75 MPa in shear.

Given data: $F = 18000$ N, $e = 600$ mm

$\sigma_{max} = 120$ MPa, $\tau_{max} = 75$ MPa, $i = 6$ rivets

Step 1: $l_1 = l_6 = 50 + 100 + 100 = 250$ mm

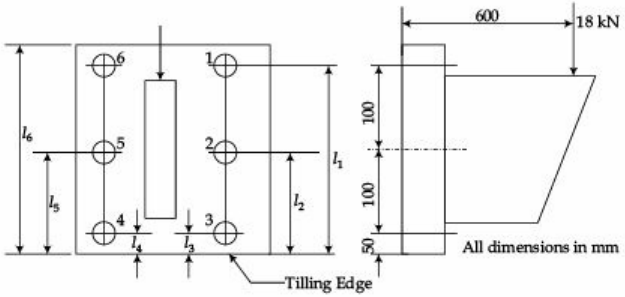


Fig. 11.40

$$l_2 = l_5 = 50 + 100 = 150 \text{ mm}$$
$$l_3 = l_4 = 50 \text{ mm}$$

Step 2: Direct load/rivet $$F_d = \frac{F}{i} = \frac{18000}{6} = 3000 \text{ N}$$

Step 3: $\therefore$ Shear stress due to direct load.

$$\tau = \frac{F_d}{A} = \frac{3000}{A}$$

Step 4: Normal load on the farthest rivet 1

which is heavily loaded, $$F_1 = \frac{Fel_1}{\Sigma l^2}$$

$$F_1 = \frac{18000 \times 600 \times 250}{2(250^2 + 150^2 + 50^2)} = 15.43 \times 10^3 \text{ N}$$

Step 5: Tensile stress due to normal load

$$\sigma = \frac{F_1}{A} = \frac{15.43 \times 10^3}{A}$$

Step 6: Combined stresses

(a) Maximum normal stress $$\sigma_{max} = \frac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right]$$

$$120 = \frac{1}{2}\left[\frac{15.43 \times 10^3}{A} + \sqrt{\left(\frac{15.43 \times 10^3}{A}\right)^2 + 4\left(\frac{3000}{A}\right)^2}\right]$$

$$A = 133.267 = \frac{\pi d^2}{4}$$

$\therefore$ diameter of the rivet, $d = 13.03$ mm

(b) Maximum shear stress $\tau_{max} = \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$

$$75 = \frac{1}{2}\sqrt{\left(\frac{15.43 \times 10^3}{A}\right)^2 + 4\left(\frac{3000}{A}\right)^2}$$

$\therefore$ $$A = 110.37 = \frac{\pi d^2}{4}$$

$\therefore$ dia of the rivet, $d = 11.86$ mm

Recommended diameter of the rivet, $d = 13.03$ mm

Standard diameter from Table 11.1, $d = 14$ mm

Problem 29: Determine the diameter of the rivet for a bracket riveted as shown in Fig. 11.41. The yield stress for the rivet material is 324 MPa. Take factor of safety as 2.5.

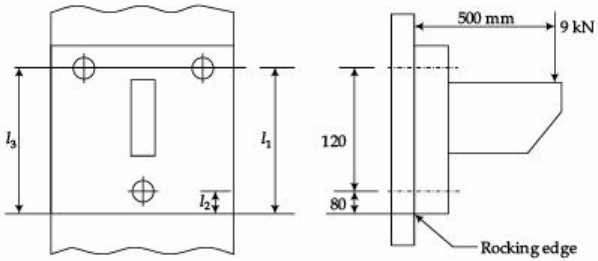


Fig. 11.41

Given data: $F = 9000$ N

$e = 500$ mm, $i = 3$ rivet

$\sigma_y = 324$ MPa and FOS = 2.5

$\therefore$ Allowable tensile stress $\sigma_{max} = \frac{\sigma_y}{FOS} = \frac{324}{2.5} = 129.6$ MPa

Allowable shear stress $\tau_{max} = 0.5\sigma_{max} = 64.84$ MPa

Step 1: $l_1 = l_3 = 120 + 80 = 200$ mm

$l_2 = 80$ mm. $e = 500$ mm

Step 2: Direct load/rivet $F_d = \frac{F}{i} = \frac{9000}{3} = 3000\,N$

Step 3: Shear stress induced $\tau = \frac{F_d}{A} = \frac{3000}{A}$

Step 4: Normal load on farthest rivet that is heavily loaded (rivet 1)

$$F_1 = \frac{Fel_1}{\Sigma l^2} = \frac{9000 \times 500 \times 200}{2 \times 200^2 + 80^2}$$

$$F_1 = 10416.67 \text{ N}$$

Step 5: Tensile stress induced $\sigma = \frac{F_1}{A} = \frac{10416.67}{A}$

Step 6: Combind stresses

(a) Maximum normal stress, $\sigma_{max} = \frac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right]$

$$129.6 = \frac{1}{2}\left[\frac{10416.67}{A} + \sqrt{\left(\frac{10416.67}{A}\right)^2 + 4\left(\frac{3000}{A}\right)^2}\right]$$

$$A = 86.57 \text{ mm}^2 = \frac{\pi d^2}{4}$$

$\therefore$ diameter of rivet, $d = 10.498$ mm

(b) Maximum shear stress, $\tau_{max} = \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$

$$64.8 = \frac{1}{2}\sqrt{\left(\frac{10416.67}{A}\right)^2 + 4\left(\frac{3000}{A}\right)^2}$$

$\therefore$ $$A = 92.76 \text{ mm}^2 = \frac{\pi d^2}{4}$$

$\therefore$ Diameter of rivet, $d = 10.867$ mm

Recommended diameter, $d = 10.867$

Select standard diameter, $d = 12$ mm (From Table 11.1)

Problem 30: Determine the diameter of the rivet for a wall bracket with circular flange, loaded as shown in Fig. 11.42. The allowable normal and shear stresses for the rivet material are 150 MPa and 90 MPa respectively.

Given data: $F = 30000$ N, $e = 600$ mm

$\sigma_{max} = 150$ MPa, $\tau_{max} = 90$ MPa, $2a = 500$ mm and $2b = 400$ mm

Step 1: $l_1 = a + b = 250 + 200 = 450$ mm

$l_2 = l_6 = a + b \cos 60 = 250 + 200 \cos 60 = 350$

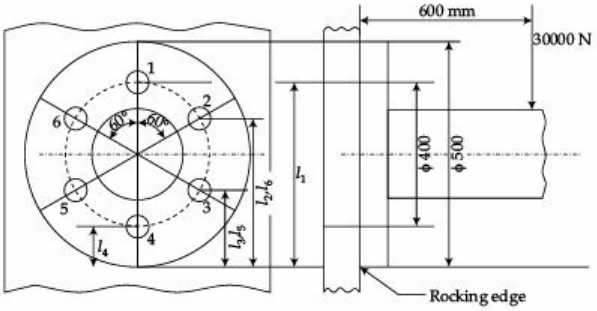


Fig. 11.42

$$l_3 = l_5 = a - b \cos 60 = 250 - 200 \cos 60 = 150$$

$$l_4 = a - b = 250 - 200 = 50$$

Step 2: Direct load/rivet $\quad F_d = \dfrac{F}{i} = \dfrac{30000}{6} = 5000 \text{ N}$

Step 3: Shear stress due to direct load

$$\tau = \frac{F_d}{A} = \frac{5000}{A} \text{ MPa}$$

Step 4: Normal load on the heavily loaded rivet '1' that is farthest from rocking edge

$$F_1 = \frac{Fel_1}{\Sigma l^2} = \frac{30000 \times 600 \times 450}{450^2 + 2(350^2) + 2(150^2) + 50^2}$$

$$F_1 = 16363.64 \text{ N}$$

Step 5: Tensile load on the heavily loaded rivet '1' due to normal load,

$$\sigma = \frac{F_1}{A} = \frac{16363.64}{A} \text{ MPa}$$

Step 6: Combined stresses.

(a) Maximum normal stress, $\quad \sigma_{max} = \dfrac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right]$

$$150 = \frac{1}{2}\left[\frac{16363.64}{A} + \sqrt{\left(\frac{16363.64}{A}\right)^2 + 4\left(\frac{5000}{A}\right)^2}\right]$$

$$\therefore \qquad A = 118.47 = \frac{\pi d^2}{4}$$

$$\therefore \qquad d = 12.28 \text{ mm}$$

(b) Maximum shear stress, $\tau_{max} = \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$

$$90 = \sqrt{\left(\frac{16363.64}{A}\right)^2 + 4\left(\frac{5000}{A}\right)^2}$$

$\therefore$ $$A = 106.54 = \frac{\pi d^2}{4}$$

$$d = 11.65$$

Recommended dia of rivet, $d = 12.28$ mm (bigger one)

standard diameter from Table 11.1, $d = 14$ mm.

Problem 31: Determine the load carrying capacity of a riveted joint loaded as shown in Fig. 11.43. The allowable tensile and shear stresses in the rivets are 100 MPa and 54 MPa respectively. The diameter of rivets is 30 mm.

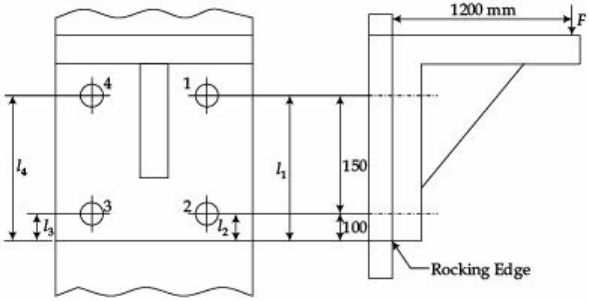


Fig. 11.43

Given data: $e = 1200$ mm, $i = 4$ rivets

$\sigma_{max} = 100$ MPa, $\tau_{max} = 54$ MPa, $d = 30$ mm.

Step 1: $l_1 = l_4 = 100 + 150 = 250$ mm

$l_2 = l_3 = 100$ mm

Step 2: Direct load/rivet $$F_d = \frac{F}{i} = \frac{F}{4} = 0.25F$$

Step 3: Shear stress induced $$\tau = \frac{F_d}{A} = \frac{0.25F}{A}$$

Step 4: Normal load on farthest rivet '1' which is heavily loaded,

$$F_1 = \frac{Fel_1}{\Sigma l^2}$$

$$F_1 = \frac{F \times 1200 \times 250}{2(250^2 + 100^2)} = 2.069F$$

Step 5: Tensile stress induced, $\sigma = \frac{F_1}{A} = \frac{2.069\,F}{A}$

Step 6: Combined stresses in the rivet

(a) Maximum normal stress, $\sigma_{max} = \frac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right]$

$$100 = \frac{1}{2}\left[\frac{2.069F}{A} + \sqrt{\left(\frac{2.069F}{A}\right)^2 + 4\left(\frac{0.25F}{A}\right)^2}\right]$$

$$100 = \frac{2.099F}{A}$$

But $A = \frac{\pi d^2}{4} = \frac{\pi \times 30^2}{4}$

$\therefore$ $F = \frac{100A}{2.099} = 33.68 \times 10^3$ N

(b) Maximum shear stress $\tau_{max} = \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$

$$54 = \frac{1}{2}\sqrt{\left(\frac{2.069F}{A}\right)^2 + 4\left(\frac{0.25F}{A}\right)^2}$$

$$54 = \frac{1.0643F}{A}$$

$\therefore$ $F = 35.86 \times 10^3$ N

Recommended capacity $F = 33.68 \times 10^3$ N (smaller value)

Problem 32: Determine the safe value 'P' for a joint loaded as shown in Fig. 11.44, limiting the normal and shear stresses to 90 MPa and 50 MPa respectively

Given data: $\sigma_{max} = 90$ MPa. $\tau_{max} = 50$ Mpa

Diameter of rivets in 1 and 6, $d_1 = 25$ mm

2 and 5, $d_2 = 20$ mm

3 and 4, $d_3 = 15$ mm

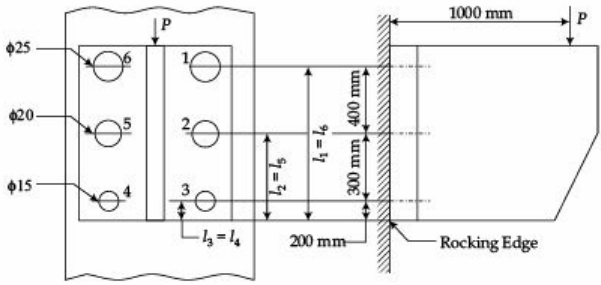


Fig. 11.44

Step 1: $l_1 = l_6 = 200 + 300 + 400 = 900$ mm

$l_2 = l_5 = 200 + 300 = 500$ mm

$l_3 = l_4 = 200$ mm

$e = 1000$ mm.

Step 2: Direct load/rivet $\quad F_d = \dfrac{F}{i} = \dfrac{P}{6} = 0.1667P$

Step 3: Shear stress $\quad \tau = \dfrac{F_d}{A}$

For rivets 1 and 6, $\quad \tau_1 = \dfrac{F_d}{A_1} = \dfrac{0.1667P}{A_1}$

where A_1 = area of the rivet at 1 and 6 $= \dfrac{\pi}{4} \times 25^2$

For rivets 2 and 5, $\quad \tau_2 = \dfrac{F_d}{A_2} = \dfrac{0.1667P}{A_2}$

$$A_2 = \frac{\pi}{4} d_2^2 = \frac{\pi}{4} \times 20^2$$

and for rivets 3 and 4, $\quad \tau_3 = \dfrac{F_d}{A_3} = \dfrac{0.1667P}{A_3}$

$$A_3 = \frac{\pi}{4} d_3^2 = \frac{\pi}{4} \times 15^2$$

Step 4: Normal load

For rivets 1 and 6, $\quad F_1 = \dfrac{Fel_1}{\Sigma l^2}$

$$F_1 = \frac{P \times 1000 \times 900}{2(900^2 + 500^2 + 200^2)} = 0.4091P$$

For rivets 2 and 5, $\frac{F_2}{l_2} = \frac{F_1}{l_1}$

$$F_2 = \frac{F_1}{l_1} \times l_2 = \frac{0.4091P}{900} \times 500 = 0.2273\,P$$

For rivets 3 and 4, $\frac{F_3}{l_3} = \frac{F_1}{l_1}$

$\therefore$ $$F_3 = \frac{F_1}{l_1} \times l_3 = \frac{0.4091P}{900} \times 200 = 0.091\,P$$

Step 5: Tensile stresses

For rivets 1 and 6 $$\sigma_1 = \frac{F_1}{A_1} = \frac{0.4091P}{A_1}$$

For rivets 2 and 5, $$\sigma_2 = \frac{F_2}{A_2} = \frac{0.2273P}{A_2}$$

For rivets 3 and 4, $$\sigma_3 = \frac{F_3}{A_3} = \frac{0.091P}{A_3}$$

Step 6: Combined stresses

(a) For rivets 1 and 6

(i) $\sigma_{max} = \frac{1}{2}\left[\sigma_1 + \sqrt{\sigma_1^2 + 4\tau_1^2}\right]$

$$90 = \frac{1}{2}\left[\frac{0.4091P}{A_1} + \sqrt{\left(\frac{0.4091P}{A_1}\right)^2 + 4\left(\frac{0.1667\,P}{A_1}\right)^2}\right]$$

$$A_1 = \frac{\pi}{4} \times 25^2$$

$$90 = \frac{P}{A_1}(0.4684) \Rightarrow P = \frac{90 \times \frac{\pi}{4} \times 25^2}{0.4684} = 94.31 \times 10^3\ \text{N}$$

(ii) $\tau_{max} = \frac{1}{2}\sqrt{\sigma_1^2 + 4\tau_1^2}$

$$50 = \frac{1}{2}\sqrt{\left(\frac{0.4091P}{A_1}\right)^2 + 4\left(\frac{0.1667}{A_1}\right)^2}$$

$\therefore$ $$P = 93.003 \times 10^3\ \text{N}$$

(b) For rivets 2 and 5,

(i) $\sigma_{max} = \frac{1}{2}\left[\sigma_2 + \sqrt{\sigma_2^2 + 4\tau_2^2}\right]$

$$90 = \frac{1}{2}\left[\frac{0.2273P}{A_2} + \sqrt{\left(\frac{0.2273P}{A_2}\right)^2 + 4\left(\frac{0.1667P}{A_2}\right)^2}\right]$$

$$A_2 = \frac{\pi}{4} \times 20^2$$

$\therefore$ $P = 89.64 \times 10^3$ N

(ii) $\tau_{max} = \frac{1}{2}\sqrt{\sigma_2^2 + 4\tau_2^2}$

$$50 = \frac{1}{2}\sqrt{\left(\frac{0.2273P}{A_2}\right)^2 + 4\left(\frac{0.1667P}{A_2}\right)^2}$$

$$P = 77.86 \times 10^3 \text{ N}$$

(c) For rivets 3 and 4

(i) $\sigma_{max} = \frac{1}{2}\left[\sigma_3 + \sqrt{\sigma_3^2 + 4\tau_3^2}\right]$

$$90 = \frac{1}{2}\left[\left(\frac{0.091P}{A_3}\right) + \sqrt{\left(\frac{0.091P}{A_3}\right)^2 + 4\left(\frac{0.1667P}{A_3}\right)^2}\right]$$

$$A_3 = \frac{\pi}{4} \times 15^2$$

$\therefore$ $P = 72.86 \times 10^3$ N

(ii) $\tau_{max} = \frac{1}{2}\sqrt{\sigma_3^2 + 4\tau_3^2}$

$$50 = \frac{1}{2}\sqrt{\left(\frac{0.091P}{A_3}\right)^2 + 4\left(\frac{0.1667P}{A_3}\right)^2}$$

$$P = 51.13 \times 10^3 \text{ N}$$

$\therefore$ Load capacity $P = 51.13 \times 10^3$ N

(Smallest of all the above 6 values).

Problem 33: Determine the maximum stresses induced in the worst stressed rivets in a riveted joint loaded as shown in the Fig. 11.45.

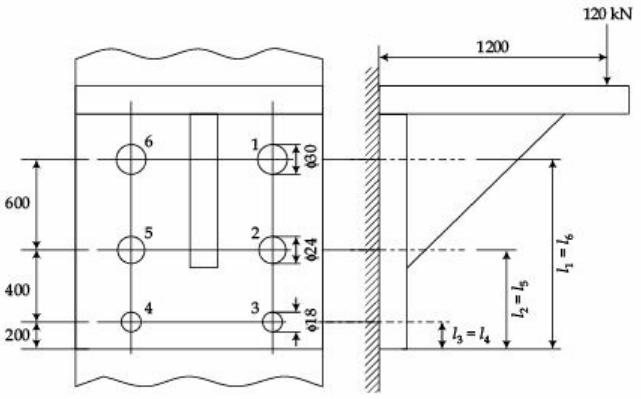


Fig. 11.45

Given data: $F = 120 \times 10^3$ N, $e = 1200$ mm

dia of rivets $d_1 = 30$ mm, $d_2 = 24$ mm, $d_3 = 18$ mm

No. of rivets $i = 6$

Step 1:

$$l_1 = l_6 = 200 + 400 + 600 = 1200 \text{ mm}$$

$$l_2 = l_5 = 200 + 400 = 600 \text{ mm}$$

$$l_3 = l_4 = 200 \text{ mm}$$

$$A_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} \times 30^2$$

$$A_2 = \frac{\pi}{4} d_2^2 = \frac{\pi}{4} \times 24^2$$

$$A_3 = \frac{\pi}{4} d_3^2 = \frac{\pi}{4} \times 18^2$$

Step 2: Direct load/rivet $\quad F_d = \dfrac{F}{i} = \dfrac{120 \times 10^3}{6} = 20000$ N

Step 3: Shear stress

(i) For rivets 1 and 6, $\tau_1 = \dfrac{F_d}{A_1} = \dfrac{20000}{A_1} = 28.29$ MPa

(ii) For rivets 2 and 5, $\tau_2 = \dfrac{F_d}{A_2} = \dfrac{20000}{A_2} = 44.21$ MPa

(iii) For rivets 3 and 4, $\tau_3 = \dfrac{F_d}{A_3} = \dfrac{20000}{A_3} = 78.60$ MPa.

Step 4: Normal loads

(i) For rivets 1 and 6,
$$F_1 = \frac{Fel_1}{\Sigma l^2}$$
$$F_1 = \frac{120 \times 10^3 \times 1200 \times 1200}{2(1200^2 + 600^2 + 200^2)} = 46.957 \times 10^3 \text{ N}$$

(ii) For rivets 2 and 5,
$$F_2 = \frac{F_1}{l_1} \cdot l_2 = \frac{46.957 \times 10^3}{1200} \times 600 = 23.478 \times 10^3 \text{ N}$$

(iii) For rivets 3 and 4,
$$F_3 = \frac{F_1}{l_1} \times l_3 = \frac{46.957 \times 10^3}{1200} \times 200 = 7.826 \times 10^3 \text{ N}$$

Step 5: Tensile stresses

(i) For rivets 1 and 6,
$$\sigma_1 = \frac{F_1}{A_1} = \frac{46.957 \times 10^3}{\frac{\pi}{4} \times 30^2} = 66.43 \text{ MPa}$$

(ii) For rivets 2 and 5,
$$\sigma_2 = \frac{F_2}{A_2} = \frac{23.478 \times 10^3}{\frac{\pi}{4} \times 24^2} = 51.90 \text{ MPa}$$

(iii) For rivets 3 and 4,
$$\sigma_3 = \frac{F_3}{A_3} = \frac{7.826 \times 10^3}{\frac{\pi}{4} \times 18^2} = 30.75 \text{ MPa}$$

Step 6: Resultant or combined stresses

(a) For rivets 1 and 6

(i) $\sigma_{1max} = \frac{1}{2}\left[\sigma_1 + \sqrt{\sigma_1^2 + 4\tau_1^2}\right]$

$$\sigma_{1max} = \frac{1}{2}\left[66.43 + \sqrt{66.43^2 + 4 \times 28.29^2}\right]$$

$$\sigma_{1max} = 76.85 \text{ MPa}$$

(ii) $\tau_{1max} = \frac{1}{2}\sqrt{\sigma_1^2 + 4\tau_1^2} = \frac{1}{2}\sqrt{66.43^2 + 4 \times 28.29^2} = 43.63 \text{ MPa}$

(b) For rivets 2 and 5

(i) $\sigma_{2max} = \frac{1}{2}\left[51.90 + \sqrt{51.90^2 + 4 \times 44.21^2}\right] = 77.21 \text{ MPa}$

(ii) $\tau_{2max} = \frac{1}{2}\sqrt{51.90^2 + 4 \times 44.21^2} = 51.26 \text{ MPa}$

(c) For rivets 3 and 4

(i) $\sigma_{3max} = \frac{1}{2}\left[30.75 + \sqrt{30.75^2 + 4 \times 78.6^2}\right] = 95.46$ MPa

(ii) $\tau_{2max} = \frac{1}{2}\sqrt{30.75^2 + 4 \times 78.6^2} = 80.09$ MPa

$\therefore$ Maximum normal stress $\sigma_{max} = 95.46$ MPa
Experienced by rivets 3 and 4
Maximum shear and stress $\tau_{max} = 80.09$ MPa
Experienced by rivets 3 and 4.

EXERCISE

1. Design a triple riveted lap joint with staggered riveting (zigzag) to connect two plates of 20 mm thick. The allowable stresses are $\sigma_\theta = 90$ MPa, $\tau = 60$ MPa and $\sigma_c = 150$ MPa.
2. Design a double riveted single cover butt joint to connect two plates 30 mm thick. The allowable tensile, shear and crushing stresses for the plates and rivets are 90 MPa, 60 MPa and 150 MPa respectively.
3. Design a double riveted double cover butt joint with unequal cover plates to connect two plates 20 mm thick. Take $\sigma_\theta = 90$ MPa, $\tau = 60$ MPa, $\sigma_c = 150$ MPa.
4. Design longitudinal and circumferential joints for a boiler of 1.2 m diameter with steam pressure of 1.5 MPa. Use triple riveted double cover butt joint with equal covers for longitudinal joint and double riveted lap joint with zigzag riveting for circumferential joint. Sketch the joints.
5. Design the longitudinal seam of a boiler of 2 m diameter to sustain a steam pressure of 0.9 MPa. The material of plates and rivets has a yield stress of 328.6 MPa. Take factor of safety of 3 for tension, 5 for shear and 2 for crushing.
6. Design a Lozenge joint to connect two plates of 18 mm thickness. The joint carriers a tensile load of 300 N.
7. Determine the diameter of the rivet for a bracket loaded as shown in the Fig. 11.46. The allowable shear stress for the rivets is 60 MPa.

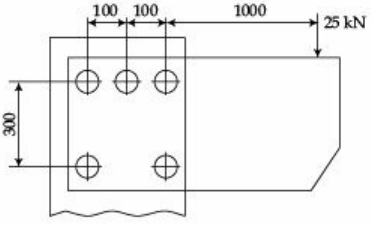


Fig. 11.46

8. Determine the dia of the rivet for a bracket loaded as shown in Figure 11.47. The allowable shear stress for the rivets is 70 MPa.

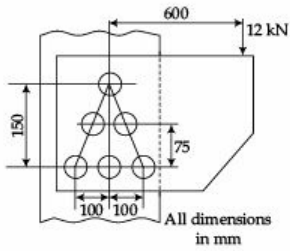


Fig. 11.47

9. Determine the load carrying capacity of a bracket riveted as shown in Figure 11.48. The allowable shear stress for the rivets is 75 MPa and diameter of the rivets is 20 mm.

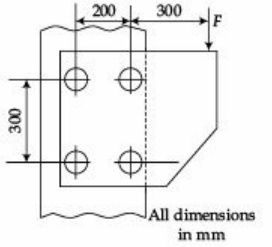


Fig. 11.48

10. Determine the load carrying capacity of a joint riveted as shown as Fig. 11.49. The allowable shear stress for 30 mm rivets used is 60 MPa. If the load to be 50% more than the value calculated above, what should be the diameter of the rivet.
11. Determine the stresses induced in each rivet of diameter 25 mm for a bracket loaded as shown in Fig. 11.50.
12. Determine the size of rivets for a joint loaded as shown in Fig. 11.51. The allowable tensile and shear stresses are 120 MPa and 69 MPa.
13. Determine the diameter of the rivet for a bracket loaded as shown in Fig. 11.52. The allowable shear and normal stresses for the rivets are 72 MPa and 144 MPa respectively.

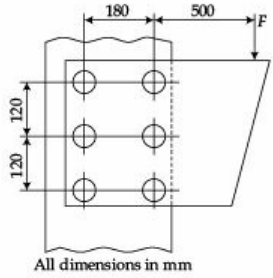


Fig. 11.49

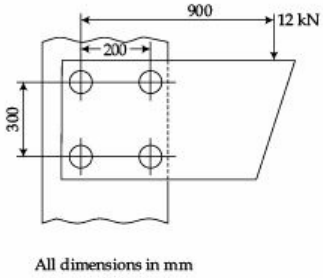


Fig. 11.50

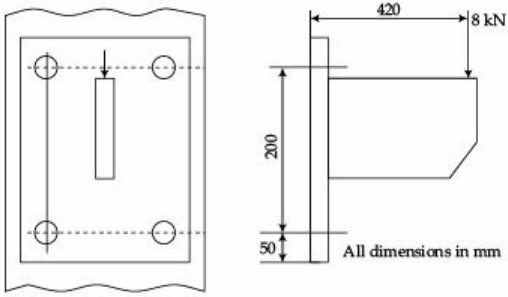


Fig. 11.51

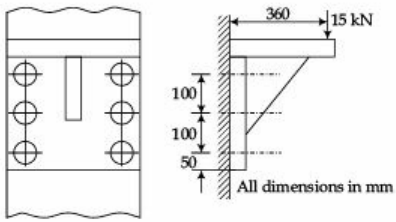


Fig. 11.52

14. Determine the diameter of the rivets for a bracket shown in Fig. 11.53. The allowable tensile and shear stresses for the rivet material are MPa are 100 MPa and 60 MPa respectively.

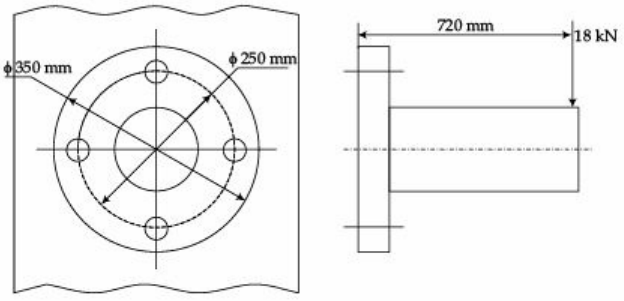


Fig. 11.53

15. Determine the load carrying capacity of a riveted joint loaded as shown in Fig. 11.54. The yield stress for rivet material is 353 MPa. Use factor of safety of 2.5.

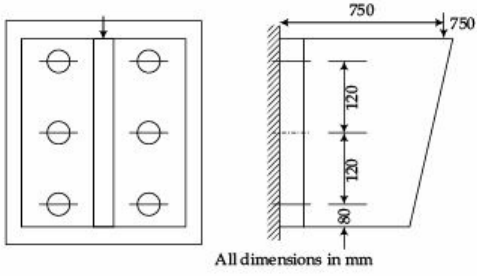


Fig. 11.54

16. Determine the load carrying capacity of a bracket loaded as shown in Fig. 11.55. The tensile and shear stresses in the rivets may be limited to 105 MPa and 54 MPa respectively.

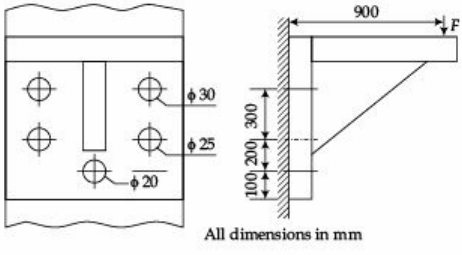


Fig. 11.55

CHAPTER

12 Welded Joints

INTRODUCTION

Unlike bolted joints, welted joints are permanent in nature and form very strong joints. Welded joints find their application from house to workshops, to factories to ship building and aeroplanes.

The welded joints are classified as lap joints, butt joints, corner or edge joints and *T*-joints.

Lap Joint

Similar to riveted joint, in lap joint two plates are kept one above other and welded. There may be transverse weld or parallel weld. Typical lap joints are shown in Fig. 12.1.

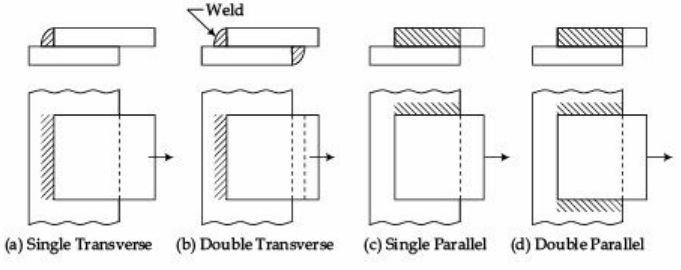


Fig. 12.1 Lap joints

Butt Joint

In this type of joint, the plates are kept one by the side of other and welded. Sometimes the edges are chamfered to fill weld. Fig. 12.2 shows examples of single butt and double butt welds.

(a) Single butt (b) Double butt

Fig. 12.2 Butt welds

Corner or Edge Joints

Figure 12.3 shows corner and edge joints.

(a) Corner joint (b) Edge joint

Fig. 12.3

T-Joint

One plate is kept perpendicular to other and weld is performed either on one side or both sides. Fig. 12.4 shows a *T*-joint.

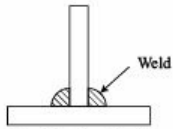


Fig. 12.4 T-Joint

Symbols: $F = P$ = load, N
w = size of weld, mm
h = thickness of plates, mm
l = length of weld, mm
σ = allowable tensile or normal stress for weld, MPa
τ = allowable shear stress for weld, MPa
M_b = bending moment, N-mm
M_t = twisting moment or torque, N-mm
t = throat thickness of weld = $w \cos 45 = 0.707\ W$

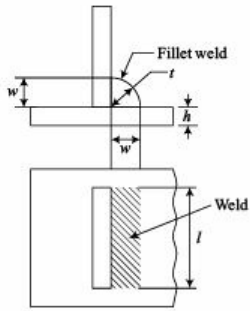


Fig. 12.5 Fillet weld.

12.1 MEMBERS SUBJECTED TO SIMPLE-DIRECT LOADS

Table 12.1 shows some commonly adopted welded joints with simple loads and relevant equations for stresses. For a given problem, select relevant sketch and corresponding equation given there and solve for the unknown.

Table 12.1 Weld-stress Formulae

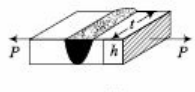
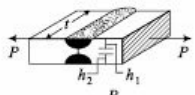
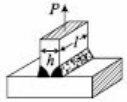
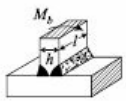
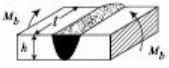
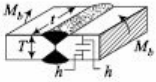
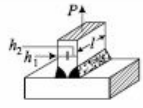
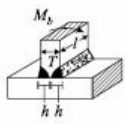
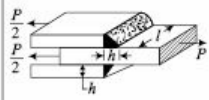
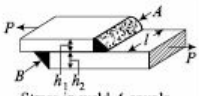
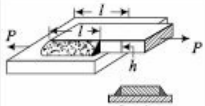
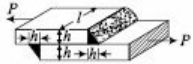
 $\sigma = \frac{P}{hl}$	 $\sigma = \frac{P}{(h_1 + h_2)l}$	 $\sigma = \frac{P}{hl}$	 $\sigma = \frac{6M_b}{lh^2}$
 $\sigma = \frac{M_b}{lh^2}$	 $\sigma = \frac{3TM_b}{lh(3T^2 - 6Th + 4h^2)}$	 $\sigma = \frac{P}{(h_1 + h_2)l}$	 $\sigma = \frac{3TM_b}{lh(3T^2 - 6Th + 4h^2)}$
 $\sigma = \frac{0.707P}{hl}$	 Stress in weld A equals stress in weld B $\sigma = \frac{1.414P}{(h_1 + h_2)l}$	 $\sigma = \frac{0.707P}{hl}$	 Both plates same thickness $\sigma = \frac{0.707P}{hl}$

Table 12.1 (Contd.)

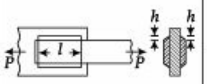
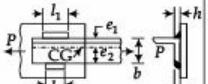
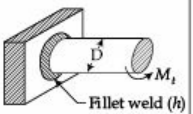
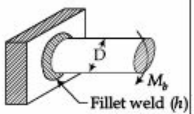
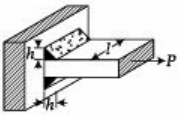
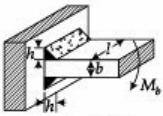
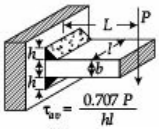
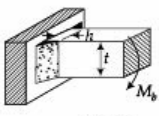
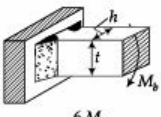
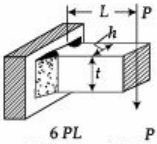
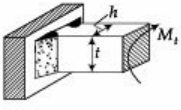
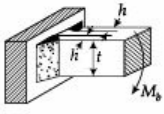
 $\sigma = \frac{0354\ P}{hl}$	 $\sigma = \frac{1414\ P}{h(l_1 + l_2)}$ or $l_1 = \frac{1414\ Pe_2}{\sigma hb}$ $l_1 = \frac{1414\ Pe_1}{\sigma hb}$	 $\tau = \frac{2.83\ M_t}{hD^2\pi}$	 $\sigma = \frac{5.66\ M_b}{hD^2\pi}$
 $\sigma = \frac{0.707\ P}{hl}$	 $\sigma = \frac{1.414\ M_b}{hl(b+h)}$	 $\tau_{av} = \frac{0.707\ P}{hl}$ $\sigma_{max} = \frac{P}{hl(b+h)}\sqrt{2L^2 + \frac{(b+h)^2}{2}}$	 $\sigma = \frac{4.24\ M_b}{hl^2}$
 $\sigma = \frac{6\ M_b}{hl^2}$	 $\sigma = \frac{6\ PL}{hl^2}$ $\tau = \frac{P}{hl}$	 $\tau = \frac{M_t(3l + 1.8h)P}{h^2l^2}$	 $\sigma = \frac{3\ M_b}{hl^2}$

Table 12.2 gives recommended allowable stresses for welds. ...(T 12-8/P 12.12)

Table 12.2 Strength of shielded-arc Flush Steel Welds

Kind of Stress	*Recommended Design Stress*		
	Static Load	*Load Varies from O to F*	*Load Varies from +F to –F*
Tension MPa	110.30	100.00	55.20
Compression MPa	124.20	110.30	55.20
Bending MPa	124.20	110.30	62.10
Shear MPa	75.80	68.90	34.50
Shear and Tension MPa	75.80	68.90	34.50

PROBLEMS

Problem 1: Determine the size of weld for the joint shown in Fig. 12.6 taking the allowable stress in weld as 70 MPa.

Given data: Stress, $\tau = 70$ MPa

Torque, $M_t = 300$ Nm $= 300 \times 10^3$ N-mm

diameter, $D = 60$ mm

From Table 12.1, 4th line, 3rd figure,

$$\text{stress, } \tau = \frac{2.83 M_t}{hD^2\pi}$$

$$70 = \frac{2.83 \times 300 \times 10^3}{h \cdot 60^2 \times \pi}$$

$\therefore$ Size of weld, $h = w = 1.07$ mm

$w \simeq 2$ mm.

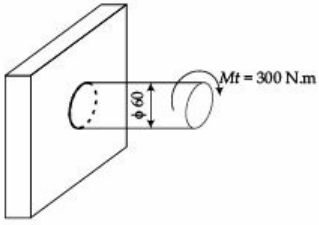


Fig. 12.6

Problem 2: Find bending moment that can be safely carried by a welded joint shown in Fig. 12.7. The allowable stress in weld is 100 MPa and size of weld is 12 mm.

Given data: Refer Table 12.1, Fig. 5 in 4th line,

$$\sigma = \frac{4.24 M_b}{h[b^2 + 3l(b+h)]}$$

where σ = allowable stress = 100 MPa

M_b = bending moment

$b = 25$ mm

$l = 20$ mm from fiure

h = size of weld = 12 mm

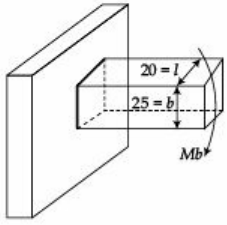


Fig. 12.7

$$100 = \frac{4.24 M_b}{12[25^2 + 3 \times 20(25 + 12)]}$$

$\therefore$ Bending moment $M_b = 805.19 \times 10^3$ N-mm.

Problem 3: Find the size of weld for a bracket loaded eccentrically as shown in the Fig. 12.8. Take the allowable shear stress in the weld as 25 MPa and allowable bending stress as 54 MPa.

Given data: $P = 2.5$ kN $= 2500$ N

$\tau = 25$ MPa, $L = 120$ mm, $\sigma = 54$ MPa

Refer Table 12.1, 5th line, 5th figure,

(a) Shear stress $\tau = \dfrac{0.707P}{hl}$

where $l = 40$ mm

$$25 = \frac{0.707 \times 2500}{h \times 40}$$

$\therefore$ size of weld $h = w = 1.77$ mm

(b) Bending stress $\sigma = \dfrac{4.24PL}{hl^2}$

$$54 = \frac{4.24 \times 2500 \times 120}{h \times 40^2}$$

$\therefore$ Size of weld $h = w = 14.72$

Recommended size of weld $= 14.72 \simeq 15$ mm.

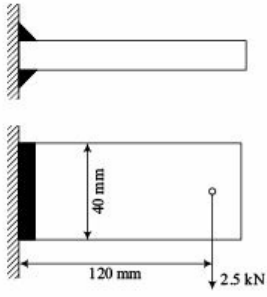


Fig. 12.8

Problem 4: Find the safe load that can be applied to a bracket loaded as shown in Fig. 12.9, taking the allowable tensile stress in 10 mm weld used as 90 MPa.

Given data: Size of weld $w = h = 10$ mm

Allowable stress $\sigma = 90$ MPa

Refer Fig. 1, 5th line, Table 12.1,

$$\text{Stress, } \sigma = \frac{0.707P}{hl}$$

where $l = 60$ mm

$$90 = \frac{0.707P}{10 \times 60}$$

$\therefore$ Load capacity, $P = 76.38 \times 10^3$ N.

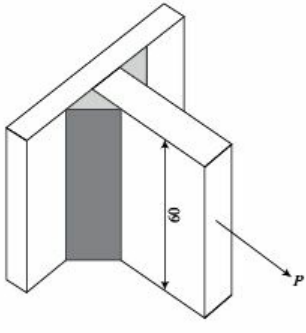


Fig. 12.9

12.2 TO FIND LENGTHS OF WELD FOR ANGLES OR CHANNELS WELDED TO A GUSSET (SUPPORT)

Figure 12.10 shows two angles welded to a gusset. ...(Fig. 12-5/P 12.6)

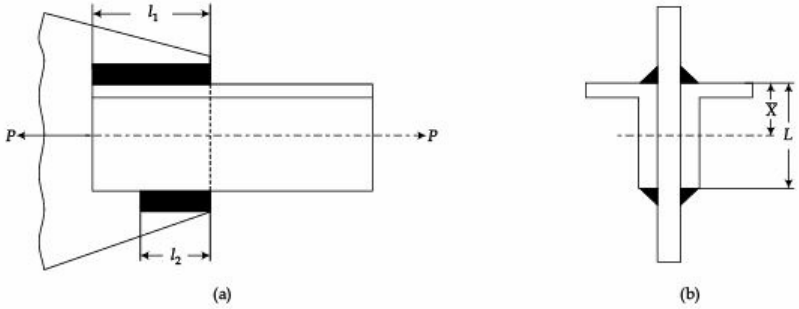


Fig. 12.10 Parallel fillet weld

P = tensile load, N

L = distance between welds, mm

l_1, l_2 = length of welds

$\bar{x}$ = distance to c.g. axis from top of angle

σ = allowable tensile stress in weld

l_t = total length of weld

= $2(l_1 + l_2)$ for two angles welded one of each side

= $(l_1 + l_2)$ for one angle welded on any one side only.

Procedure

Step 1: Find distance to c.g. = $\bar{x}$ from top side

$$\bar{x} = \frac{A_1X_1 + A_2X_2}{A_1 + A_2}$$

Divide the angle (any one) into two rectangles.

A_1 = area of rectangle (1),

A_2 = area of rectangle (2),

X_1 and X_2 are the distances from top to c.g. of rectangles (1) and (2).

Step 2: Total length of weld $l_t = \dfrac{P}{0.707 w \sigma}$ ...(12-7/P 12.2)

(Refer Table 12.2 for allowable tensile stress if not given in problem).

Step 3: Find $l_1 + l_2 = ?$

For 2 angles as shown in Fig. 12.10,

$$l_1 + l_2 = \frac{l_t}{2}$$

For one angle, $l_1 + l_2 = l_t$

Step 4: Taking moments about c.g. axis for welds, and rearranging,

$$\frac{l_1}{L - \overline{X}} = \frac{l_2}{\overline{X}} = \frac{l_1 + l_2}{L} \qquad ...(12\text{-}8)$$

Find weld lengths l_1 and l_2.

Add a length of 5 to 8 mm for start and stop of weld for each length.

Problem 5: Two angles 150 mm × 100 mm × 12 mm are welded to a gusset by means of parallel welds along the broader side of the angle. Determine the lengths of welds required. The angles carry a tensile load of 300 kN.

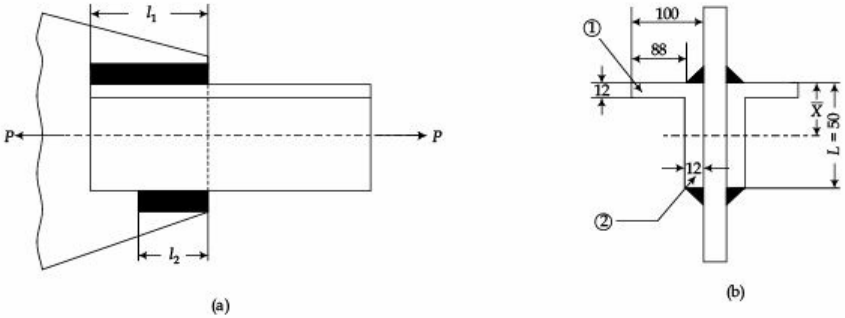


Fig. 12.11

Given data: $P = 300 \text{ kN} = 300 \times 10^3 \text{ N}$

$L = 150$ mm (broader side of angle)

Divide one angle into two rectangles, (1) and (2) as shown.

Assume size of weld = thickness of plate $h = w = 12$ mm.

From Table 12.2, select allowable tensile stress $\sigma = 110.3$ MPa (static stress) ...(T 12-8)

Step 1: To find $\overline{X} = \dfrac{A_1 X_1 + A_2 X_2}{A_1 + A_2}$

$A_1 = 88 \times 12, \qquad A_2 = 150 \times 12$

$$X_1 = \frac{12}{2}, \qquad X_2 = \frac{150}{2}$$

$$\therefore \qquad \bar{X} = \frac{\left(88 \times 12 \times \frac{12}{2}\right) + 150 \times 12 \times \frac{150}{2}}{88 \times 12 + 150 \times 12} = 49.49 \text{ mm}$$

Step 2: Total length of weld, $l_t = \dfrac{P}{0.707 w \sigma}$

$$l_t = \frac{300 \times 10^3}{0.707 \times 12 \times 110.3} = 320.59 \text{ mm}$$

Step 3: For 2 angles, $l_1 + l_2 = \dfrac{l_t}{2} = \dfrac{320.59}{2} = 160.3$ mm

(Total length $l_t = l_1 + l_2 + l_1 + l_2$)

Step 4: $\dfrac{l_1}{l - \bar{X}} = \dfrac{l_2}{\bar{X}} = \dfrac{l_1 + l_2}{L}$

consider $\dfrac{l_2}{\bar{X}} = \dfrac{l_1 + l_2}{L}$

$$l_2 = \frac{l_1 + l_2}{L} \times \bar{X} = \frac{160.3}{150} \times 49.49$$

$$l_2 = 52.89$$

$$\therefore \qquad l_1 = (l_1 + l_2) - l_1 = 160.3 - 52.89 = 107.41 \text{ mm}$$

Adding 5 mm for start and stop of welds

$$l_1 = 107.41 + 5 = 112.41 \simeq 113 \text{ mm}$$

$$l_2 = 52.89 + 5 = 57.89 \simeq 58 \text{ mm}$$

Problem 6: An angle 100 mm × 90 mm × 10 mm is welded to a steel plate along the broader side of the angle. The size of weld is 8 mm and allowable stress in weld is 90 MPa. The angle is subjected to a tensile load of 180 kN.

Given data: Angle 100 × 90 × 10

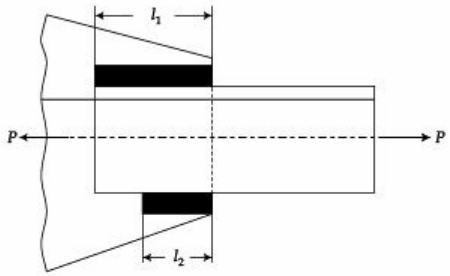
Longer side of angle = 100 mm

Welds are made along the broader side hence $L = 100$ mm

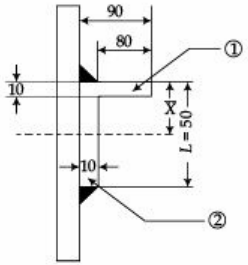
Size of weld, $w = 8$ mm.

Allowable stress, $\sigma = 90$ MPa

Load, $P = 180 \text{ kN} = 180 \times 10^3 \text{ N}$



(a)



(b)

Fig. 12.12

Step 1: $$\bar{X} = \frac{A_1X_1 + A_2X_2}{A_1 + A_2} = \frac{\left(80 \times 10 \times \frac{10}{2}\right) + \left(100 \times 10 \times \frac{100}{2}\right)}{80 \times 10 + 100 \times 10} = 30 \text{ mm}$$

$$\bar{X} = 30 \text{ mm}$$

Step 2: Length of weld required $$l_t = \frac{P}{0.707 w \sigma} = \frac{180 \times 10^3}{0.707 \times 8 \times 90}$$

$$l_t = 353.6 \simeq 354 \text{ mm}$$

Step 3: For one angle $l_t = (l_1 + l_2) = 354$ mm.

Step 4: $$\frac{l_1}{L - \bar{X}} = \frac{l_2}{\bar{X}} = \frac{l_1 + l_2}{L}$$

$$l_2 = \frac{l_1 + l_2}{L} \times \bar{X} = \frac{354}{100} \times 30 = 106.2 \text{ mm}$$

$$l_1 = (l_1 + l_2) - l_2 = 247.8 \text{ mm}.$$

Add 5 mm for start and stop of weld.

$$l_1 = 247.8 + 5 = 252.8 \simeq 253 \text{ mm}$$

$$l_2 = 106.2 + 5 = 111.2 \simeq 112 \text{ mm}.$$

Problem 7: A channel welded to a steel plate as shown in the Fig. 12.13, carries a tensile load of 150 kN along the c.g. axis of the channel. Find suitable lengths of welds taking allowable tensile stress in the weld as 120 MPa.

Given data: $P = 150 \text{ kN} = 150 \times 10^3 \text{ N}$

$\sigma = 120$ MPa

Assume size of the weld, $w = h = 10$ mm

$L = 200$ mm (distance between welds)

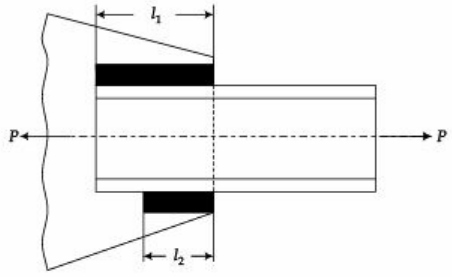


Fig. 12.13

Step 1:
$$\bar{X} = \frac{A_1X_1 + A_2X_2 + A_3X_3}{A_1 + A_2 + A_3}$$

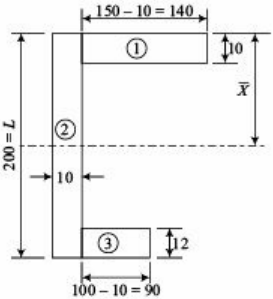
$$\bar{X} = \frac{(140 \times 10)\frac{10}{2} + (200 \times 10) \times \frac{200}{2} + (90 \times 12)\left(200 - \frac{12}{2}\right)}{(140 \times 10 + 200 \times 10 + 90 \times 12)}$$

$$= 92.97 \text{ mm}$$

Step 2: Total length of the weld required

$$l_t = \frac{P}{0.707 w \sigma}$$

$$l_t = \frac{150 \times 10^3}{0.707 \times 10 \times 120} = 176.8 \text{ mm}$$



Step 3: For one channel (only on one side)

Total length of weld $l_t = l_1 + l_2 = 176.8$ mm

Step 4:
$$\frac{l_1}{L - \bar{X}} = \frac{l_2}{\bar{X}} = \frac{l_1 + l_2}{L}$$

$$\therefore \quad l_2 = \left(\frac{l_1 + l_2}{L}\right) \times \bar{X} = \frac{176.8}{200} \times 92.97 = 82.19 \text{ mm}$$

$$\therefore \quad l_1 = 176.8 - l_2 = 94.61 \text{ mm}$$

Add 5 mm for start and stop of weld

$$l_1 = 94.61 + 5 = 99.61 \simeq 100 \text{ mm}$$

$$l_2 = 82.19 + 5 = 87.19 \simeq 88 \text{ mm}.$$

12.3 PARALLEL AND TRANSVERSE WELDS

12.3.1 Transverse welds

Figure 12.14 shows a bracket with transverse welds subjected to tensile load P.

P = load, N

l_t = length of each transverse weld; mm

σ = allowable tensile stress in weld, MPa

l = total length of weld, mm

= l_t for one transverse weld, mm

= $2l_t$ for 2 welds, mm

w = size of weld, mm

Maximum normal stress in weld

$$\sigma = \frac{P}{0.707wl} \qquad \text{...(12-9/P 12.1)}$$

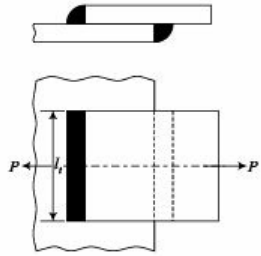


Fig. 12.14

12.3.2 Parallel Welds

Figure 12.15 shows a bracket with two parallel fillet welds subjected to a tensile load P.

l_p = length of each parallel weld, N

τ = allowable shear stress in weld, MPa

l = total length of weld, mm

= l_p for one weld, mm

= $2l_p$ for 2 welds, mm

w = size of weld, mm

Maximum shear stress in weld,

$$\tau = \frac{P}{0.707wl} \qquad \text{...(12-5 a/P 12.1)}$$

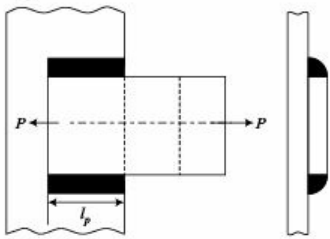


Fig. 12.15

12.3.3 Combined Transverse and Parallel Fillet Welds

Figure 12.16 shows a bracket with combined transverse and parallel fillet welds carrying a tensile load. For such a case, the total strength of joint is the addition of strength of transverse weld and strength of parallel welds.

$P = P_\tau + P_p$

P = total load

P_τ = strength of transverse weld

= $0.707wl_t\sigma$

P_p = strength of 2 parallel welds = $2(0.707w\tau l_p)$

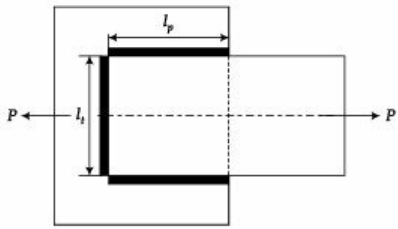


Fig. 12.16

$\therefore \quad P = 0.707 w\tau l_t + 2(0.707 w\tau l_p)$

Problem 8: A rectangular plate 60 mm wide, 12 mm thick is welded to a support as shown in Fig. 12.17. Find the length of welds required if the allowable tensile stress is 90 MPa for the weld.

Given data: $P = 30 \text{ kN} = 30 \times 10^3 \text{ N}$

$l = 2l_t$ (Two welds)

$\sigma = 90 \text{ MPa}, \quad h = 12 \text{ mm}$

Assume size of weld

$w = h = 12 \text{ mm}$

For transverse fillet weld

$$\sigma = \frac{P}{0.707 wl}$$

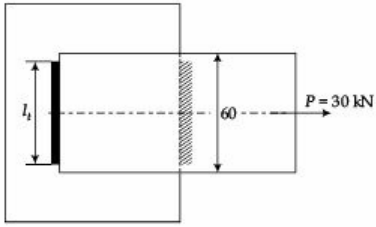


Fig. 12.17

$$90 = \frac{30 \times 10^3}{0.707 \times 12 \times l} \Rightarrow \text{length of weld}$$

$$l = 39.28 \simeq 40 \text{ mm}$$

$$\therefore \quad l_t = \frac{l}{2} = 20 \text{ mm}$$

Add 5 mm each for start and stop of weld

$l_t = 20 + 5 = 25$ mm each

Problem 9: For a bracket loaded as shown in Fig. 12.18, find the load carrying capacity taking size of weld as 10 mm and allowable stresses 120 MPa for weld.

Given data: $l_t = 45 \text{ mm}$

$l = 2l_t = 90 \text{ mm}$

$w = 10 \text{ mm}$

$\sigma = 120 \text{ MPa}$

For transverse weld $\sigma = \dfrac{P}{0.707 wl}$

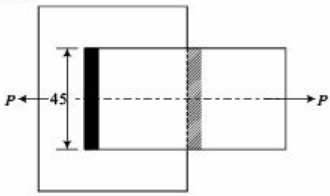


Fig. 12.18

$P = \sigma(0.707) wl = 120 \times 0.707 \times 10 \times 90$

$P = 76.356 \times 10^3 \text{ N}.$

Problem 10: Two plates are weld by means of parallel fillet welds as shown in Fig. 12.19. The thickness of plates is 10 mm. Find the length of welds required to sustain a tensile load of 90 kN.

Given data: $P = 90 \text{ kN} = 90000 \text{ W}$

Thickness of plate $h = 10$ mm

Assume size of weld $w = h = 10$ mm

Select allowable shear stress for weld $\tau = 75.8$ MPa (From Table 12.2)

Total length of weld, $l = 2l_p$

For parallel fillet welds, $\tau = \dfrac{P}{0.707wl}$

$$75.8 = \frac{90 \times 10^3}{0.707 \times 10 \times l}$$

$\Rightarrow$ Length of weld $= l = 167.94$

$l = 168$ mm $= 2l_p$

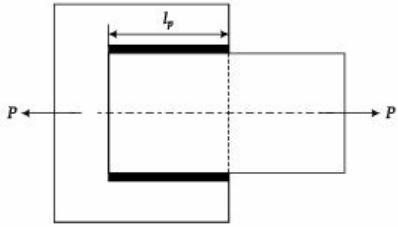


Fig. 12.19

$\therefore$ Length of each parallel weld $l_p = \dfrac{l}{2} = \dfrac{168}{2} = 84$ mm

Add 5 mm for start and stop of weld, $l_p = 89$ mm

Problem 11: Determine the safe load that can be applied to a bracket welded as shown in Fig. 12.20. The allowable stress in 6 mm weld is 60 MPa.

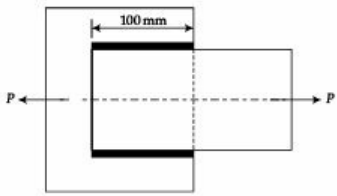


Fig. 12.20

Given data: Size of weld, $w = 6$ mm

Allowable shear stress, $\tau = 60$ MPa

Length of each parallel weld, $l_p = 100$ mm

$\therefore$ $l = 2 \times l_p = 200$ mm

For parallel fillet welds, $\tau = \dfrac{P}{0.707wl} = \dfrac{P}{0.707 \times 6 \times 200}$

$\therefore$ Load capacity, $P = 50.904 \times 10^3$ N.

Problem 12: A 80 mm wide, 12 mm thick plate subjected to axial tensile load is welded to a vertical support by single transverse fillet weld and a double parallel fillet weld as shown in Fig. 12.21, the maximum tensile and shear stress in weld are 100 MPa and 70 MPa respectively. Find the length of each parallel weld.

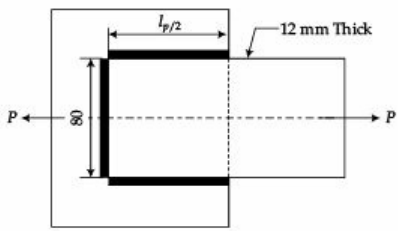


Fig. 12.21

Given data: $b = 80$ mm $= l_t$

$h = 12$ mm

assume $w = h = 12$ mm

$\sigma = 100$ MPa

$\tau = 70$ MPa

Since the load is not given, find the load capacity of the plate and design the weld to carry the same load capacity of plate.

Strength of plate $P = bh\sigma = 80 \times 12 \times 100 = 96 \times 10^3$ N

For combined transverse fillet weld and parallel fillet welds, total load capacity of the weld

$$P = P_\tau + P_P$$

$$= 0.707 w\sigma l_t + 2\left(0.707 w\tau \frac{l_p}{2}\right)$$

$$96 \times 10^3 = 0.707 \times 12 \times 100 \times 80 + 2\left(0.707 \times 12 \times 70 \times \frac{l_p}{2}\right)$$

$\therefore$ Length of each parallel weld $\frac{l_p}{2} = 23.68$ mm

Add 5 mm for start and stop of weld,

$$\text{Length of each weld } \frac{l_p}{2} = 23.68 + 5 = 28.68 \simeq 29 \text{ mm.}$$

Problem 13: For a welded bracket shown in Fig. 12.22, find the load carrying capacity if the weld size is 9 mm and allowable stresses in welds are 96 MPa in tension and 72 MPa in shear.

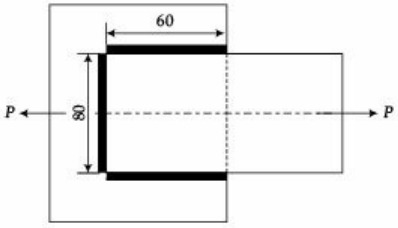


Fig. 12.22

Given data: Size of weld, $w = 9$ mm

Allowable tensile stress, $\sigma = 96$ MPa

Allowable shear stress, $\tau = 72$ MPa

Length of each parallel weld = 60 mm = l_p

Length of transverse weld = 80 mm = l_t

Load capacity of combined transverse and parallel fillet welded joint

$$P = P_\tau + P_P$$

$$= (0.707 w\sigma l_t) + 2(0.707 w\tau l_p)$$

$$= (0.707 \times 9 \times 96 \times 80) + 2(0.707 \times 9 \times 72 \times 60)$$

$$P = 103.844 \times 10^3 \text{ N}.$$

12.4 ECCENTRIC LOADING OF WELDED BRACKETS TYPE-I

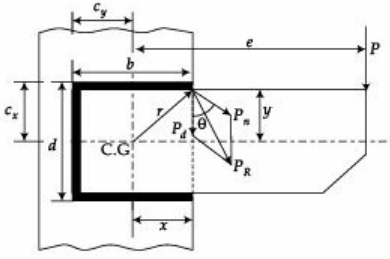


Fig. 12.23

(Fig. 12-7/P 12-6)

Figure 12.23 shows an eccentrically loaded welded bracket. The load P is acting away from centre of gravity (c.g.) of weld.

Symbols: P = load, N
b = horizontal length of weld, mm
d = vertical length of weld, mm
c.g. = centre of gravity of weld.
c_x = distance of c.g. of weld from top, mm
c_y = distance of c.g. of weld from left, mm
r = distance from c.g. of weld to right side top corner, mm
e = eccentricity, distance from c.g. to load, mm
P_d = direct load/unit length of weld, N/mm
P_n = normal load/unit length of weld, N/mm
θ = angle between P_d and P_n, deg.
P_R = resultant load/unit length of weld
t = throat thickness of weld = $w \cos 45 = 0.707\ w$ mm
w = size of weld, mm
τ = allowable shear stress in weld, MPa.
l = total length of weld, mm

Design procedure

Step 1: Locate the c.g. of weld. Refer Table 12.3, and select appropriate equation for C_x and C_y depending on the shape of the weld. (For symmetric figures, the c.g. lies in the middle i.e, $C_x = d/2$, $C_y = b/2$). ...(T 12-3/P 12.10)

Step 2: Find r, e and $\cos\theta$ $\quad r = \sqrt{x^2 + y^2}$ (where $x = b - c_y$ and $y = c_x$)

$$\cos\theta = \frac{x}{r}.$$

Total length of weld = l.

Step 3: Direct load/unit length of weld $P_d = \frac{P}{l}$ N/mm. ...(12-17/P 12.2)

Step 4: Normal load/unit length of weld

$$P_n = \frac{Per}{J} \text{ N/mm} \qquad \text{...(12-20)}$$

J = polar MI of weld,

select appropriate equation of J depending on the shape of the weld from Table 12.3.

Step 5: Resultant load/unit length of weld

$$P_R = \sqrt{P_d^2 + P_n^2 + 2P_dP_n \cos\theta} \text{ N/mm} \qquad \text{...(12-21)}$$

Step 6. Size of weld

$$\text{Thickness of weld, } t = \frac{P_R}{\text{stress}}$$

$$\therefore \qquad w = \frac{t}{0.707} \qquad \text{...(12-1 a/P 12.1)}$$

Note: If size of weld w is given, then find load capacity P.

Problem 14: Determine the size of the weld for a plate welded as shown in Fig. 12.24. The allowable stress for the weld is 75 MPa.

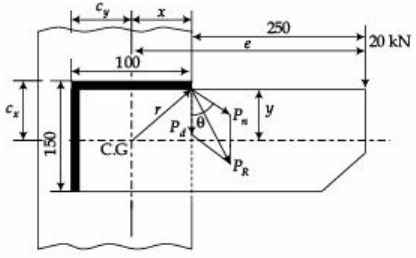


Fig. 12.24

Given data: Load $P = 20$ kN $= 20 \times 10^3$ N

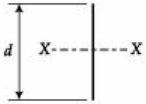
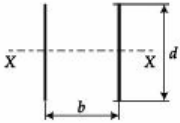
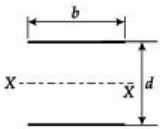
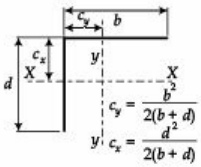
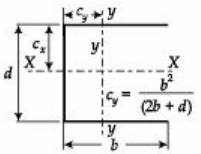
Allowable shear stress, $\tau = 75$ MPa

Total length of weld, $l = b + d = 150 + 100 = 250$ mm

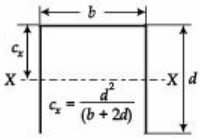
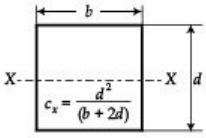
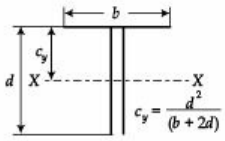
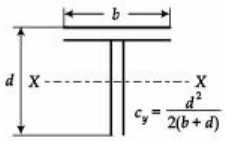
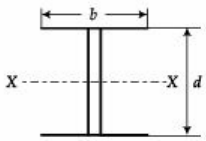
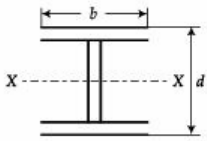
$b = 100$ mm horizontal length

$d = 150$ mm vertical length

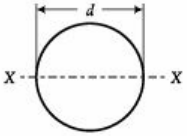
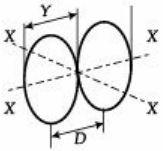
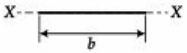
Table 12.3 Properties of Weld-Treating Weld as line

Outline of welded joint *b = width, d = depth*	*Bending* *(about horizontal axis x – x)*	*Twisting*
	$Z_w = \frac{d^2}{6}$	$J_w = \frac{d^3}{12}$
	$Z_w = \frac{d^2}{3}$	$J_w = \frac{d(3b^2 + d^2)}{6}$
	$Z_w = bd$	$J_w = \frac{b^3 + 3bd^2}{6}$
	$Z_w = \frac{4bd + d^2}{6}_{top} = \frac{d^2(4b+d)}{6(2b+d)}_{bottom}$	$J_w = \frac{(b+d)^4 - 6b^2d^2}{12(b+d)}$
	$Z_w = bd + \frac{d^2}{6}$	$J_w = \frac{(2b+d)^3}{12} - \frac{b^2(b+d)^2}{(2b+d)}$

Contd.

	$Z_w = \underset{top}{\frac{2bd+d^2}{3}} = \underset{bottom}{\frac{d^2(2b+d)}{3(b+d)}}$	$J_w = \frac{(b+2d)^3}{12} - \frac{d^2(b+d)^2}{(b+2d)}$
	$Z_w = bd + \frac{d^2}{3}$	$J_w = \frac{(b+d)^3}{6}$
	$Z_w = \underset{top}{\frac{2bd+d^2}{3}} = \underset{bottom}{\frac{d^2(2b+d)}{3(b+d)}}$	$J_w = \frac{(b+2d)^3}{12} - \frac{d^2(b+d)^2}{(b+2d)}$
	$Z_w = \underset{top}{\frac{4bd+d^2}{3}} = \underset{bottom}{\frac{4bd^2+d^3}{6b+3d}}$	$J_w = \frac{d^3(4b+d)}{6(b+d)} + \frac{b^3}{6}$
	$Z_w = bd + \frac{d^2}{3}$	$J_w = \frac{b^3+3bd^2+d^3}{6}$
	$Z_w = 2bd + \frac{d^2}{3}$	$J_w = \frac{2b^3+6bd^2+d^3}{6}$

Contd.

	$Z_w = \frac{\pi d^2}{4}$	$J_w = \frac{\pi d^3}{4}$
	$Z_w = \frac{\pi d^2}{2} + \pi D^2$	
	–	$J_w = \frac{b^3}{12}$

Note: Multiply the values J_w by the size of the weld w to obtain polar moment inertia J of the weld.

Step 1: Ref. Fig. 4, Table 12.3, select equation for c_x and c_y to find c.g.

$$c_x = \frac{d^2}{2(b+d)} = \frac{150^2}{2(100+150)} = 45 \text{ mm}$$

$$c_y = \frac{b^2}{2(b+d)} = \frac{100^2}{2(100+150)} = 20 \text{ mm}$$

Step 2: Diagonal $r = \sqrt{x^2 + y^2}$

where $x = 100 - C_y = 100 - 20 = 80$ mm

$y = C_x = 45$ mm

$\therefore$ $r = \sqrt{80^2 + 45^2} = 91.79$ mm

$$\cos\theta = \frac{x}{r} = \frac{80}{91.79} = 0.872$$

e = distance from cg to load = 250 + x

$e = 250 + 80 = 330$ mm.

Step 3: Direct load/unit length of weld

$$P_d = \frac{P}{l} = \frac{20000}{250} = 80 \text{ N/mm}$$

Step 4: Normal load/unit length of weld

$$P_n = \frac{Per}{J}$$

Again from same Fig. 4, Table 12.3, polar *MI* of weld $J = \dfrac{(b+d)^4 - 6b^2d^2}{12(b+d)}$

$$J = \frac{(100+150)^4 - 6\times 100^2 \times 150^2}{12(100+150)} = 852.08 \times 10^3 \text{ mm}^4/\text{mm}$$

$\therefore$ $$P_n = \frac{20\times 10^3 \times 330 \times 91.79}{852.08 \times 10^3} = 710.98 \text{ N/mm}$$

Step 5: Resultant load/unit length of weld

$$P_e = \sqrt{P_d^2 + P_n^2 + 2P_dP_n\cos\theta}$$

$$= \sqrt{80^2 + 710.98^2 + 2\times 80 \times 710.98 \times 0.872}$$

$$= 781.72 \text{ N/mm}$$

Step 6: $\therefore$ Thickness of weld $t = \dfrac{P_R}{\tau} = \dfrac{781.72}{75} = 10.42$ mm

$\therefore$ Size of weld, $w = \dfrac{t}{0.707} = \dfrac{10.42}{0.707} = 14.74 \simeq 15$ mm.

Problem 15: Determine the size of the weld to be used for a bracket welded as shown in Fig. 12.25. The load is 30 kN.

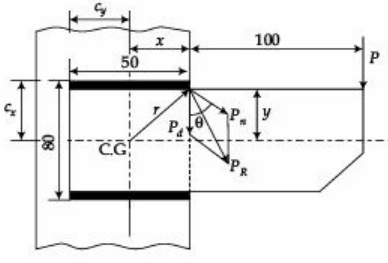


Fig. 12.25

Given data: Load $P = 30 \times 10^3$ N

$$b = 50 \text{ mm} \quad d = 80 \text{ mm}$$

From Table 12.2, select allowable shear stress for weld $\tau = 75.8$ MPa

Step 1: To locate c.g.

Refer Table 12.3, Figure 2, due to symmetry,

$$c_x = \frac{d}{2} = \frac{80}{2} = 40 \text{ mm}$$

$$c_y = \frac{b}{2} = \frac{50}{2} = 25 \text{ mm}$$

Step 2:

$$e = 100 + x$$

$$x = 50 - C_y = 50 - 250 = 25 \text{ mm}$$

$$y = C_x = 40 \text{ mm}$$

$$e = 100 + 25 = 125 \text{ mm}$$

$$r = \sqrt{x^2 + y^2} = \sqrt{25^2 + 40^2} = 47.17 \text{ mm}$$

$$\cos\theta = \frac{x}{r} = \frac{25}{47.17} = 0.53$$

Total length of weld, $l = 2 \times 50 = 100$ mm

Step 3: Direct load/unit length of weld

$$P_d = \frac{P}{l} = \frac{30000}{100} = 300 \text{ N/mm}$$

Step 4: Normal load/unit length of the weld

$$P_n = \frac{Per}{J}$$

From same Fig. 2, Table 12.3, polar MI for weld $J = \dfrac{d(3b^2 + d^2)}{6}$

$$J = \frac{80(3 \times 50^2 + 80^2)}{6} = 185.333 \times 10^3 \text{ mm}^4/\text{mm}$$

$\therefore$

$$P_n = \frac{30000 \times 125 \times 47.17}{185.333 \times 10^3} = 954.43 \text{ N/mm}$$

Step 5: Resultant load/unit length of weld

$$P_R = \sqrt{P_2^d + P_n^2 + 2P_d P_n \cos\theta}$$

$$= \sqrt{300^2 + 954.43^2 + 2 \times 300 \times 954.43 \times 0.53}$$

$$= 1142.12 \text{ N/mm}$$

Step 6: $\therefore$ Thickness of the weld $t = \dfrac{P_R}{\tau}\dfrac{1142.12}{75.8} = 15.07$ mm

$\therefore$ Size of the weld, $w = \dfrac{t}{0.707} = \dfrac{15.07}{0.707} = 21.31 \simeq 22$ mm.

Problem 16: Determine the size of the weld for a bracket welded as shown in Fig. 12.26. Allowable shear stress in weld is 90 MPa.

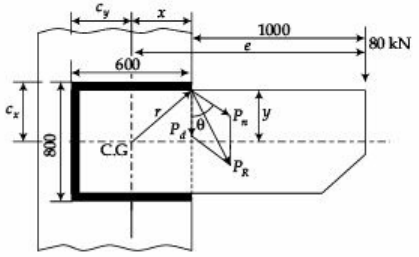


Fig. 12.26

Given data: Load, $F = 80 \times 10^3$ N

Allowable shear stress, $\tau = 90$ MPa

$b = 600$ mm $\quad d = 800$ mm

$\therefore$ length of weld, $l = 2b + d = 2 \times 600 + 800 = 2000$ mm

Step 1: To locate c.g.

$$C_x = \frac{d}{2} = \frac{800}{2} = 400 \text{ mm (symmetric)}$$

$$C_y = \frac{b^2}{2b+d} = \frac{600^2}{2\times 600 + 800} = 180 \text{ mm}$$

(Ref. Fig. 5, Table 12.3 for c.g. equation)

Step 2: $x = 600 - C_y = 600 - 180 = 420$ mm

$$y = C_x = 400 \text{ mm}$$

$$r = \sqrt{x^2 + y^2} = \sqrt{420^2 + 400^2} = 580 \text{ mm}$$

$\therefore$ $e = 1000 + x = 1000 + 420 = 1420$ mm

$$\cos\theta = \frac{x}{r} = \frac{420}{580} = 0.724$$

Step 3: Direct load/unit length of weld

$$P_d = \frac{P}{l} = \frac{80000}{2000} = 40 \text{ N/mm}$$

Step 4: Normal load/unit length of weld

$$P_n = \frac{Per}{J}$$

From Fig. 5, Table 12.3, $$J = \frac{(2b+d)^3}{12} - \frac{b^2(b+d)^2}{(2b+d)}$$

$$J = \frac{(2\times 600+800)^3}{12} - \frac{600^2(600+800)^2}{(2\times 600+800)}$$

$$= 313.87 \times 10^6 \text{ mm}^4/\text{mm}$$

$\therefore$ $$P_n = \frac{80\times 10^3 \times 1420 \times 580}{313.87\times 10^6} = 209.92 \text{ N/mm}$$

Step 5: Resultant load/unit length of weld

$$P_R = \sqrt{P_d^2 + P_n^2 + 2P_dP_n\cos\theta}$$

$$= \sqrt{40^2 + 209.92^2 + 2\times 40\times 209.92\times 0.724}$$

$$= 240.47 \text{ N/mm}$$

Step 6: $\therefore$ Thickness of the weld, $t = \frac{P_R}{\tau} = \frac{240.47}{90} = 2.67$ mm

$\therefore$ Size of the weld, $w = \frac{t}{0.707} = 3.78 \simeq 4$ mm.

Problem 17: Determine the load carrying capacity of a welded joint located as shown in Fig. 12.27. The size of the weld is 12 mm and allowable shear stress in weld is 81 MPa.

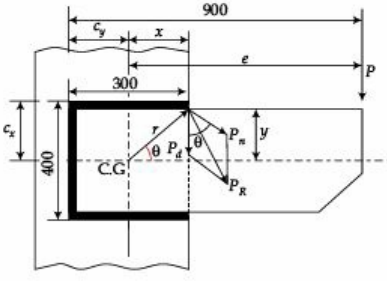


Fig. 12.27

Given data: Size of weld, $w = 12$ mm

Allowable shear stress, $\tau = 81$ MPa

$b = 300$ mm $\quad d = 400$ mm

Step 1: To locate c.g.

Refer Figure 5, Table 12.3,

$$C_y = \frac{b^2}{2b+d} = \frac{300^2}{2\times 300+400} = 90 \text{ mm}$$

$$C_x = \frac{400}{2} = 200 \text{ (symmetric)}$$

$\therefore$ $x = 300 - C_y = 300 - 90 = 210$ mm

$y = C_x = 200$ mm

Step 2: $r = \sqrt{x^2+y^2} = \sqrt{210^2+200^2} = 290$

$$\cos\theta = \frac{x}{r} = \frac{210}{290} = 0.724$$

$e = 900 - C_y = 900 - 210 = 690$ mm

Total length of weld, $l = 300 + 400 + 300 = 1000$ mm

Step 3: Direct load/unit length of weld

$$P_d = \frac{P}{l} = \frac{P}{1000} = 0.001P$$

Step 4: Normal load/unit length of weld

$$P_n = \frac{Per}{J}$$

where J = polar MI of weld

From same Fig. 5, Table 12.3,

$$J = \frac{(2b+d)^3}{12} - \frac{b^2(b+d)^2}{(2b+d)}$$

$$J = \frac{(2\times 300+400)^3}{12} - \frac{300^2(300+400)^2}{2\times 300+400} = 39.233\times 10^6 \text{ mm}^4/\text{mm}$$

$\therefore$ $$P_n = \frac{P\times 690\times 290}{39.233\times 10^6} = 0.0051 \text{ P}$$

Step 5: Resultant load per unit length of the weld

$$P_R = \sqrt{P_d^2 + P_n^2 + 2P_dP_n\cos\theta}$$

$$= \sqrt{(0.001P)^2 + (0.0051P)^2 + 2(0.001P)(0.0051P)(0.724)}$$

$$= 0.005865P$$

Step 6: $\therefore$ Thickness of the weld $t = \dfrac{P_R}{\text{stress}}$

But thickness $t = w\cos 45 = 12 \times 0.707$

$t = 8.484$ mm

$\therefore$ $8.484 = \dfrac{0.005865P}{81}$

$\therefore$ Load capacity $P = \dfrac{8.484 \times 81}{0.005865} = 117.172 \times 10^3$ N.

Problem 18: Determine the load carrying capacity of a welded joint dimensioned in mm as shown in Fig. 12.27A. The size of weld is 10 mm and allowable shear stress in weld is 66 MPa.

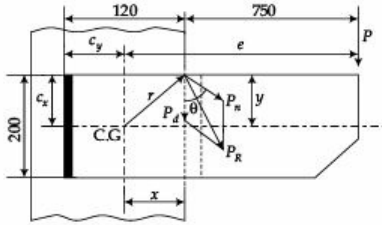


Fig. 12.27A

Given data: Size of weld, $w = 10$ mm

Allowable shear stress, $\tau = 66$ MPa

Length of weld, $l = 2b = 200 + 200$

$l = 400$ mm

Step 1: To locate c.g.

Due to symmetry $C_x = \dfrac{d}{2} = \dfrac{200}{2} = 100$ mm

$C_y = \dfrac{b}{2} = \dfrac{120}{2} = 60$ mm

$\therefore$ $x = 120 - C_y = 120 - 60 = 60$ mm

$y = C_x = 100$ mm

Step 2: $r = \sqrt{x^2 + y^2} = \sqrt{100^2 + 60^2} = 116.62$ mm

$$\cos\theta = \frac{x}{r} = \frac{60}{116.62} = 0.5145$$

$$e = 750 + x = 750 + 60 = 810 \text{ mm}$$

Step 3: Direct load/unit length of weld

$$P_d = \frac{P}{l} = \frac{P}{400} = 2.5 \times 10^{-3} P$$

Step 4: Normal load/unit length of weld

$$P_n = \frac{Per}{J}$$

From Fig. 3, Table 12.1, for J equation

$$J = \frac{b^3 + 3bd^2}{6} = \frac{120^3 + 3 \times 120 \times 200^2}{6}$$

$$J = 2.688 \times 10^6 \text{ mm}^4/\text{mm}$$

$\therefore$ $$P_n = \frac{P \times 810 \times 116.62}{2.688 \times 10^6} = 0.03514P \text{ N/mm}$$

Step 5: Resultant load/unit length of weld

$$P_R = \sqrt{P_d^2 + P_n^2 + 2P_dP_n \cos\theta}$$

$$P_R = \sqrt{(0.0025P)^2 + (0.03514P)^2 + 2(0.0025P)(0.03514P) \times 0.5145}$$

$$= 0.0365P.$$

Step 6: $\therefore$ Thickness of weld, $t = \frac{P_R}{\text{stress}}$

But thickness $t = w \cos 45 = 10 \times 0.707 = 7.07$

$$7.07 = \frac{0.0365P}{66}$$

$\therefore$ Load capacity, $P = 12.784 \times 10^3$ N

12.5 ECCENTRIC LOADING OF WELDED BRACKET TYPE –II

Figure 12.28 shows an eccentrically loaded bracket.

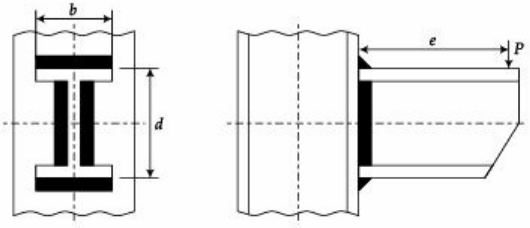


Fig. 12.28

...(Fig 12-6/P 12.6)

Procedure

Step 1: Total length of weld, l = (Add all the lengths given in problem)

Direct load/unit length of weld $P_d = \dfrac{P}{l}$ N/mm ...(12-17/P 12.2)

Step 2: Normal load/unit length of weld, $P_n = \dfrac{Pey}{I}$ ...(12-18)

$$P_n = \frac{Pe}{\left(\frac{I}{y}\right)} = \frac{Pe}{z} \text{ N/mm}$$

where z = section modulus of weld = $\dfrac{I}{y}$

Refer Table 12.3, for equations of z for weld. If two equations are given, then find both $z_{w\,\text{top}}$ and $z_{w\,\text{bottom}}$ and use the smaller value of z_w in the above equation.

Step 3: Resultant load/unit length of weld $P_R = \sqrt{P_d^2 + P_n^2}$ N/mm ...(12-19)

Step 4: Thickness of weld

$$t = \frac{P_R}{\text{stress}}$$

but $t = w \cos 45$ ...(12-1 a/P 12.1)

$\therefore$ Size of weld, $w = \dfrac{t}{0.707}$

Note: If size of the weld is given, then find load capacity P.

Problem 19: Determine the size of weld for a bracket loaded as shown in Fig. 12.29. The allowable shear stress for the weld is 75 MPa.

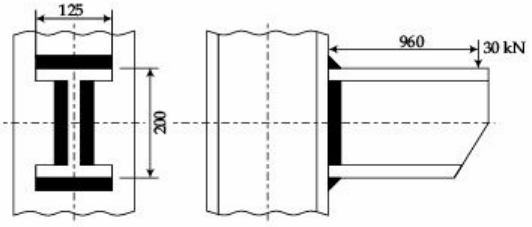


Fig. 12.29

Given data: Load, $P = 30 \text{ kN} = 30 \times 10^3 \text{ N}$

$e = 960 \text{ mm}$

$b = 125 \text{ mm}$

$d = 200 \text{ m}$

Total length of weld, $l = 2(125 + 200)$

$l = 650 \text{ mm}$

Step 1: Direct load/unit length of weld

$$P_d = \frac{P}{l} = \frac{30 \times 10^3}{650} = 46.15 \text{ N/mm}$$

Step 2: Normal load/unit length of weld

$$P_n = \frac{Pey}{I} = \frac{Pe}{\left(\frac{I}{y}\right)} = \frac{Pe}{z}$$

Refer Fig. 10, Table 12.3,

$$z = z_w = bd + \frac{d^2}{3}$$

$$= 125 \times 200 + \frac{200^2}{3} = 38.33 \times 10^3 \text{ mm}^3/\text{mm}$$

$\therefore$ $$P_n = \frac{30 \times 10^3 \times 960}{38.33 \times 10^3} = 751.36 \text{ N/mm}$$

Step 3: Resultant load/unit length of weld

$$P_R = \sqrt{P_d^2 + P_n^2}$$

$$= \sqrt{46.15^2 + 751.36^2}$$

$$= 752.79 \text{ N/mm}$$

Step 4: ∴ Thickness of weld $t = \dfrac{P_R}{\text{stress}}$

$$t = \frac{752.79}{75} = 10.037 \text{ mm}$$

∴ Size of weld $w = \dfrac{t}{0.707} = 14.196 \simeq 15 \text{ mm}$

$$w = 15 \text{ mm}.$$

Problem 20: A bracket is welded to a support as shown in Fig. 12.30. Determine the size of the weld to be used taking allowable stress as 50 MPa.

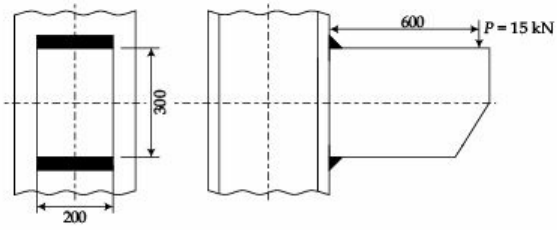


Fig. 12.30

Given data: Load $P = 15 \times 10^3$ N

stress, $\tau = 60$ MPa

$e = 600$ mm

$b = 200$ mm

$d = 300$ mm

weld length, $l = 200 + 200 = 400$ mm

Step 1: Direct load/unit length of weld

$$P_d = \frac{P}{l} = \frac{15000}{400} = 37.5 \text{ N/mm}$$

Step 2: Normal load/unit length of the weld

$$P_n = \frac{Pey}{I} = \frac{Pe}{z}$$

Ref. Fig. 3, Table 12.3, $z = z_w = bd = 200 \times 300$

$$= 60000 \text{ mm}^3/\text{mm}$$

∴ $$P_n = \frac{15000 \times 600}{60000} = 150 \text{ N/mm}$$

Step 3: Resultant load/unit length of the weld

$$P_R = \sqrt{P_d^2 + P_n^2} = \sqrt{37.5^2 + 150^2} = 154.62 \text{ N/mm}$$

Step 4: Thickness of weld $t = \dfrac{P_R}{\tau} = \dfrac{154.62}{60} = 2.577$

$\therefore$ Size of weld, $w = \dfrac{t}{0.707} = 3.65 \simeq 4$ mm.

Problem 21: Determine the size of the weld for a bracket loaded as shown in Fig. 21.31. The allowable stress in the weld is 72 MPa.

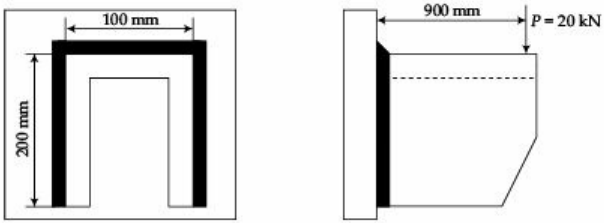


Fig. 12.31

Given data: Load, $P = 20$ kN

$e = 900$ mm

$b = 100$ mm

$d = 200$ mm

$\therefore$ Length of weld, $l = b + 2d = 100 + 2 \times 200 = 500$ mm

$\tau = 72$ MPa

Step 1: Direct load/unit length of weld

$$P_d = \frac{P}{l} = \frac{20000}{500} = 40 \text{ N/mm}$$

Step 2: Normal load/unit length of weld

$$P_n = \frac{Pey}{I} = \frac{P.e}{z}$$

Refer Fig. 6, Table 12.3

$$z_{top} = \frac{2bd + d^2}{3} = \frac{2 \times 100 \times 200 + 200^2}{3} = 26{,}666.67 \text{ mm}^3/\text{m}$$

$$z_{bottom} = \frac{d^2(2b + d)}{3(b + d)} = \frac{200^2(2 \times 100 + 200)}{3(100 + 200)} = 17{,}777.78 \text{ mm}^3/\text{mm}$$

Select smaller value of section modulus,

$$z = 17{,}777.78$$

$$\therefore \quad P_n = \frac{20{,}000 \times 900}{17{,}777.78} = 1012.5 \text{ N/mm}$$

Step 3: Resultant load/unit length of weld

$$P_R = \sqrt{P_d^2 + P_n^2} = \sqrt{40^2 + 1012.5^2} = 1013.29 \text{ N/mm}$$

Step 4: Thickness of weld $\quad t = \frac{P_R}{\tau} = \frac{1013.29}{72} = 14.07$

$$\therefore \quad \text{Size of weld, } w = \frac{t}{0.707} = 19.9 \simeq 20 \text{ mm.}$$

Problem 22: Determine the size of the weld for a bracket welded of shown in Fig. 12.32.

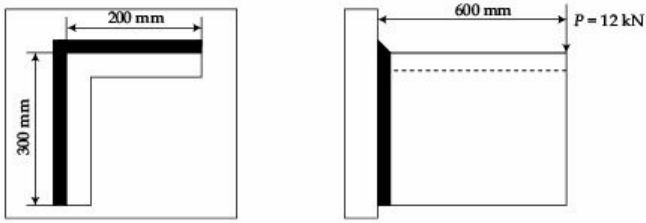


Fig. 12.32

Given data:

$$P = 12000 \text{ N}$$

$$e = 600 \text{ mm}$$

$$b = 200 \text{ mm}$$

$$d = 300 \text{ mm}$$

Length of weld, $l = b + d = 200 + 300 = 500$ mm

From Table 12.2, select allowable shear stress as 75.8 MPa

$$\tau = 75.8 \text{ MPa}$$

Step 1: Direct load/unit length of weld

$$P_d = \frac{P}{l} = \frac{12000}{500} = 24 \text{ N/mm}$$

Step 2: Normal load/unit length of weld

$$P_n = \frac{Pey}{I} = \frac{P.e}{(I/y)} = \frac{P.e}{z}$$

Ref. Fig. 4, Table 12.3,

$$z_{top} = \frac{4bd + d^2}{6} = \frac{4 \times 200 \times 300 + 300^2}{6} = 55000 \text{ mm}^3/\text{mm}$$

$$z_{bottom} = \frac{d^2(4b + d)}{6(2b + d)} = \frac{300^2(4 \times 200 + 300)}{6(2 \times 200 + 300)} = 23.57 \times 10^3 \text{ mm}^3/\text{mm}$$

$\therefore$ $$z = z_{min} = 23.57 \times 10^3$$

$\therefore$ $$P_n = \frac{12000 \times 600}{23.57 \times 10^3} = 305.46 \text{ N/mm}$$

Step 3: Resultant load/unit length of weld

$$P_R = \sqrt{P_d^2 + P_n^2} = \sqrt{24^2 + 305.46^2} = 306.396 \text{ N/mm}$$

Step 4: Thickness of weld $$t = \frac{P_R}{\tau} = \frac{306.396}{75.8}$$

$$t = 4.042 \text{ mm}$$

$\therefore$ Size of weld, $$w = \frac{t}{0.707} = 5.717 \simeq 6 \text{ mm}$$

$$w = 6 \text{ mm}$$

Problem 23: Determine the safe load that can be applied to a bracket welded as shown in Fig. 12.33. The allowable stress in the weld is 60 MPa. Size of weld is 12 mm.

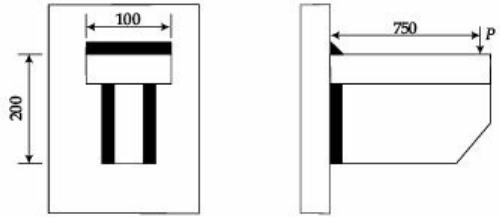


Fig. 12.33

Given data: Size of the weld, $w = 12$ mm

$\therefore$ $$t = w \times 0.707 = 12 \times 0.707 = 8.484$$

Allowable stress $\tau = 60$ MPa

Length of the weld, $l = 200 + 100 + 200 = 500$ mm

$b = 100$ mm $\quad d = 200$ mm $\quad e = 750$ mm

Step 1: Direct load/unit length of weld

$$P_d = \frac{P}{500} = 0.002P$$

Step 2: Normal load/unit length of weld

$$P_n = \frac{Pey}{I} = \frac{P.e}{z}$$

Ref. Fig. 8, Table 12.3,

$$z_{top} = \frac{2bd + d^2}{3} = \frac{2 \times 100 \times 200 + 200^2}{3} = 26{,}666.67 \text{ mm}^3/\text{mm}$$

$$z_{bottom} = \frac{d^2(2b + d)}{3(b + d)} = \frac{200^2(2 \times 100 + 200)}{3(100 + 200)} = 17{,}777.78 \text{ mm}^3/\text{mm}$$

$\therefore$ $$z = z_{min} = 17{,}777.78 \text{ mm}^3/\text{mm}$$

$\therefore$ $$P_n = \frac{P \times 750}{17{,}777.78} = 0.0422P \text{ N/mm}$$

Step 3: Resultant load/unit length of weld

$$P_R = \sqrt{P_d^2 + P_n^2} = \sqrt{(0.002P)^2 + (0.04228)^2} = 0.04223P$$

Step 4: Thickness of the weld $t = \frac{P_R}{\tau}$

$$8.484 = \frac{0.0422P}{60} \Rightarrow \text{Load } P = 12.05 \times 10^3 \text{ N}$$

Problem 24: Determine the load carrying capacity of a welded joint loaded as shown in Fig. 12.34. The allowable shear stress for 10 mm weld used is 50 MPa.

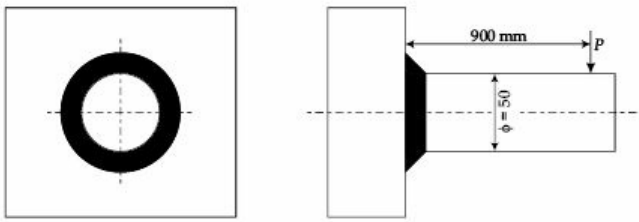


Fig. 12.34

Given data: Size of weld $w = 10$ mm

$\therefore$ $t = w \times 0.707 = 10 \times 0.707$ mm

$e = 900$ mm

$d = 50$ mm

$\tau = 50$ MPa

length of the weld $= \pi d = \pi(50)$

Step 1: Direct load/unit length of weld

$$P_d = \frac{P}{l} = \frac{P}{\pi d}$$

$$P_d = \frac{P}{\pi \times 50} = 6.366 \times 10^{-3}\, P$$

Step 2: Normal load/unit length of the weld

$$P_n = \frac{Pey}{I} = \frac{P.e}{z}$$

Ref. Fig. 12, Table 12.3,

$$z = \frac{\pi d^2}{4} = \frac{\pi \times 50^2}{4} = 1963.5 \text{ mm}^3/\text{mm}$$

$\therefore$ $$P_n = \frac{P \times 900}{1963.5} = 0.4584P$$

Step 3: Resultant load/unit length of weld

$$P_R = \sqrt{P_d^2 + P_n^2} = \sqrt{(6.366 \times 10^{-3} P)^2 + (0.4584P)^2}$$

$$P_R = 0.4584 \text{ P}$$

Step 4: Thickness of the weld $t = \dfrac{P_R}{\text{stress}}$

$$7.07 = \frac{0.4584P}{50}$$

$\therefore$ Load capacity, $P = 771.16$ N.

Problem 25: Determine the safe load that can be applied to a bracket loaded as shown in Fig. 12.35. The size of the weld is 8 mm and allowable shear stress for weld is 70 MPa.

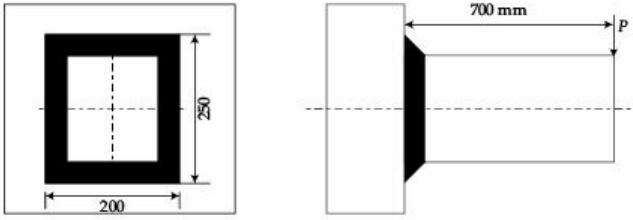


Fig. 12.35

Given data:

$b = 200$ mm

$e = 700$ mm

$d = 250$ mm

$\tau = 70$ MPa

$w = 8$ mm

$t = 0.707w = 0.707 \times 8 = 5.656$ mm

Total length of weld, $l = 200 + 250 + 200 + 250 = 900$ mm

Step 1: Direct load/unit length of weld

$$P_d = \frac{P}{l} = \frac{P}{900} = 1.111 \times 10^{-3} P \quad \text{N/mm}$$

Step 2: Normal load/unit length of weld

$$P_n = \frac{Pey}{I} = \frac{P.e}{z}$$

From Fig. 7, Table 12.3,

$$z = bd + \frac{d^2}{3} = 200 \times 250 + \frac{250^2}{3} = 70{,}833.33 \text{ mm}^3/\text{mm}$$

$\therefore$

$$P_n = \frac{P \times 700}{70{,}833.33} = 9.8823 \times 10^{-3} P \quad \text{N/mm}$$

Step 3: Resultant load/unit length of weld

$$P_R = \sqrt{P_d^2 + P_n^2} = \sqrt{(1.111 \times 10^{-3} P)^2 + (9.8823 \times 10^{-3} P)^2}$$

$$= 9.9446 \times 10^{-3} P \quad \text{N/mm}$$

Step 4: Thickness of the weld $t = \dfrac{P_R}{\text{stress}}$

$$5.656 = \frac{9.9466 \times 10^{-3} P}{70}$$

$\therefore$ Load capacity, $P = 39.812 \times 10^3$ N.

EXERCISE

1. Determine the load carrying capacity of a welded joint welded as shown in Fig. 12.36. The allowable tensile and shear stresses in the weld are 100 MPa and 60 MPa. Size of weld is 10 mm.

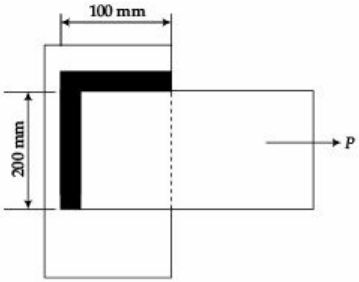


Fig. 12.36

2. Determine the length of welds required for a joint loaded as shown in Fig. 12.37. The allowable tensile and shear stresses for the weld are 90 MPa and 54 MPa. The thickness of plate is 12 mm.

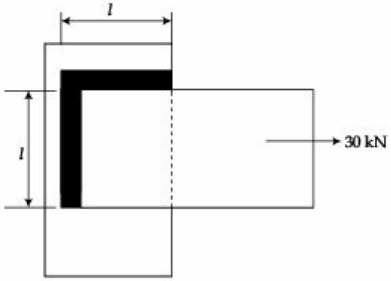


Fig. 12.37

3. An angle 120 mm × 90 mm × 10 mm is welded to a steel plate by means of parallel fillet welds along the braider side of the angle. Find the lengths of welds required taking the allowable shear stress in weld is 75 MPa.
4. Determine the size of weld for a bracket loaded as shown in Fig. 12.38. The allowable stress in the weld is 60 MPa.

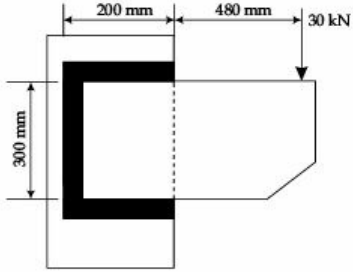


Fig. 12.38

5. Determine the safe load that can be carried by a welded bracket shown in Fig. 12.39. The size of the weld is 6 mm.
6. Determine the load carrying capacity of the welded joint as shown in Fig. 12.40. The size of weld is 9 mm.

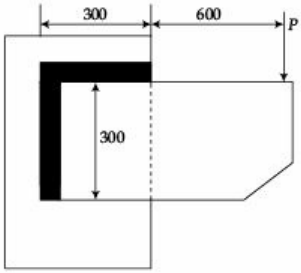


Fig. 12.39

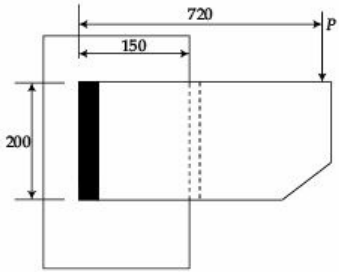


Fig. 12.40

7. Determine the size of the weld for a bracket welded as shown in Fig. 12.41. Take allowable stress for weld as 84 MPa.
8. Determine the size of the weld to be used for a welded joint loaded as shown in Fig. 12.42. The allowable shear stress for the weld is 100 MPa.

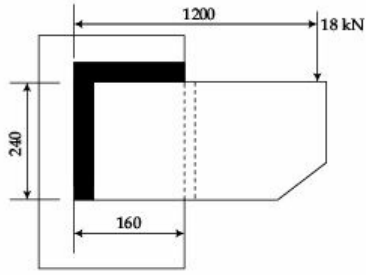


Fig. 12.41

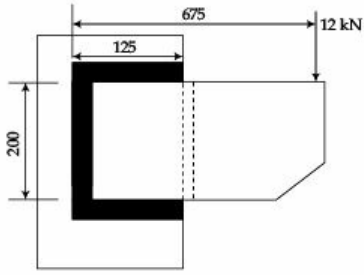


Fig. 12.42

9. Determine the size of the weld for a bracket welded and loaded as shown in Fig. 12.43.

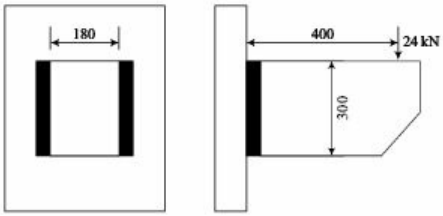


Fig. 12.43

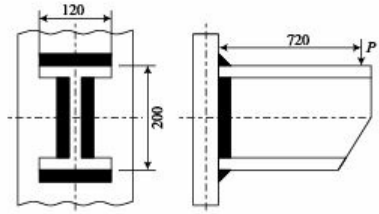


Fig. 12.44

10. Determine the safe load that can be applied at P taking the allowable stress in 10 mm weld used as 60 MPa.
11. Determine the size of weld required for a bracket shown in Fig. 12.45 if load at P is 9 kN.

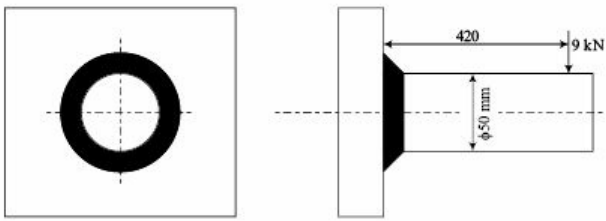


Fig. 12.45

12. Determine the load carrying capacity of a welded bracket shown in Fig. 12.46. Take allowable stress in the weld as 90 MPa.

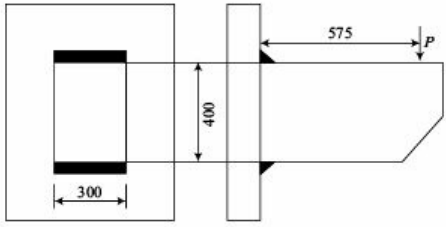


Fig. 12.46

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